Introduction to Statics

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Unit 15
Equilibrium of Bodies

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Unit 15
Equilibrium of Bodies

Frame 15-1

Introduction

In an earlier unit you were introduced to the concept of equilibrium of a particle. Since then you have learned to take the moment about a point so that now you will be able to work problems dealing with the equilibrium of rigid bodies. There is actually very little new material in this unit but you will be required to make use of most of what you have learned up to this point.

Begin by going to the next frame.

Correct response to preceding frame

No response

Frame 15-2

Review

1. What is the necessary and sufficient condition for the equilibrium of a particle?

_____ = _____

2. Since all forces acting on a particle must have lines of action which intersect at a point, what is the total moment about that point? _____

About any other point? _____

3. What is the name of the sort of force system described in this frame?

A _____ force system.

Correct response to preceding frame

1. $\overline{R} = 0$ or $\Sigma\overline{F} = 0$
2. The moments of all the forces acting on a particle taken about any point must be zero.
3. concurrent

Frame 15-3

Equilibrium of Rigid Bodies

Read the first portion of page 15-1 of your notebook before answering the following questions.

Do the necessary and sufficient conditions for the equilibrium of a rigid body apply to a particle?

☐ Yes ☐ No

Why or why not? _____

Is the equation $\overline{R} = 0$, which is sufficient for equilibrium of a particle, sufficient for equilibrium of a rigid body?

☐ Yes ☐ No ☐ Sometimes

Correct response to preceding frame

Yes, if a particle is in equilibrium $\Sigma \overline{F} = 0$ and $\Sigma \overline{M}_0 = 0$ ($\Sigma \overline{M}_0 = 0$ is unnecessary but still true.)

Sometimes, if a concurrent force system acts on a rigid body, $\overline{R} = 0$ is sufficient to solve. (Of course, you could treat such a rigid body as a particle if you choose.)

Frame 15-4

Equilibrium of a Rigid Body

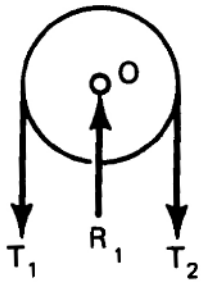
In Problem 15-1 in your notebook write $\Sigma \overline{M}_0 = 0$, taking O at the center of the pulley to prove the first part. Then take $\Sigma \overline{F} = 0$ to solve the second part. (Don't forget your free body.)

Correct response to preceding frame

$T_1 = T_2$

$R_1 = 2T_1$

Solution:



$$\Sigma \bar{M}_o = 0$$

$$[(-r\bar{i}) \times (-T_1\bar{j})] + [(r\bar{i}) \times (-T_2\bar{j})] = 0$$

$$T_1 r\bar{k} - T_2 r\bar{k} = 0$$

$$T_1 r - T_2 r = 0$$

$$T_1 = T_2$$

$$\Sigma \bar{F} = 0$$

$$R_1\bar{j} - T_1\bar{j} - T_2\bar{j}$$

$$R_1\bar{j} - T_1\bar{j} - T_1\bar{j}$$

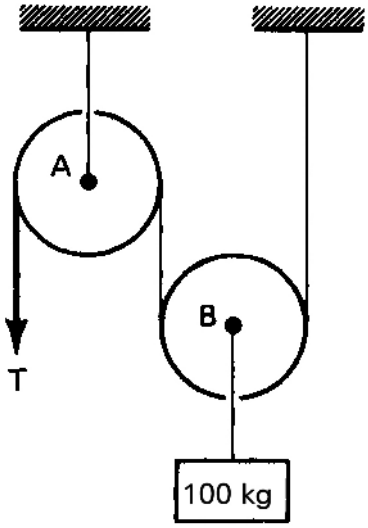
$$R_1 - 2T_1 = 0$$

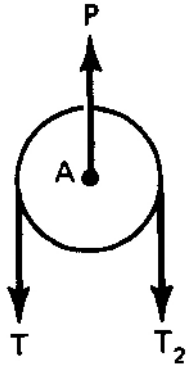
$$R_1 = 2T_1$$

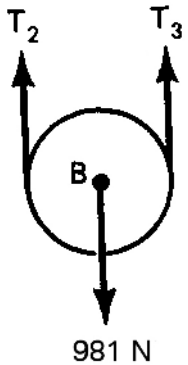
Frame 15-5

Equilibrium of Rigid Bodies

The weight is 100 kilograms. Find T using the free bodies shown. Consider pulleys to be weightless and frictionless unless given other information.



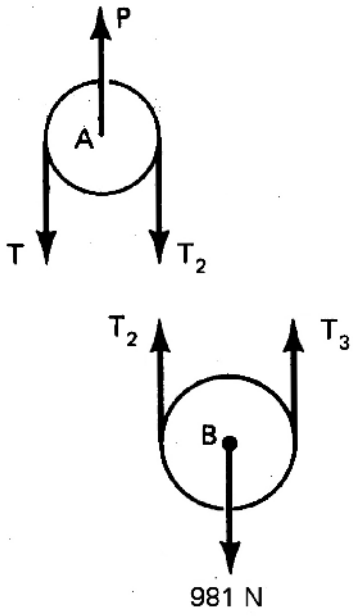




Correct response to preceding frame

T = 491 N

Solution:



For free body diagram of pulley A

$\Sigma \bar{M}_o = 0 \qquad T_2 = T$

For free body diagram of pulley B

$\Sigma \bar{M}_o = 0 \qquad T_3 = T_2$

$\Sigma \bar{F} = 0$

$T_2\bar{j} + T_3\bar{j} - 981\,\bar{j} = 0$

$2T\bar{j} - 981\,\bar{j} = 0$

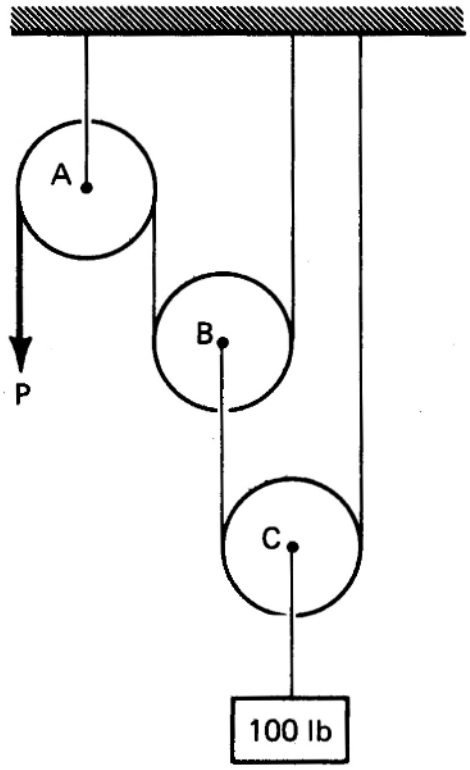
$2T = 981$

$T = 490.5$ N

Frame 15-6

Equilibrium of a Rigid Body

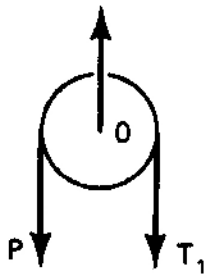
Draw a free body of each pulley and find P.



Correct response to preceding frame

P = 25 lb Solution:

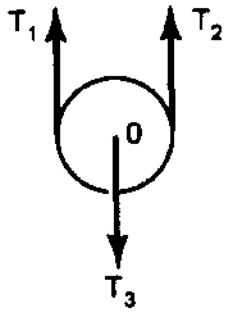
Pulley A



$\Sigma \bar{M}_o = 0$

$P = T_1$

Pulley B



$\Sigma \bar{M}_o = 0$

$T_1 = T_2 = P$

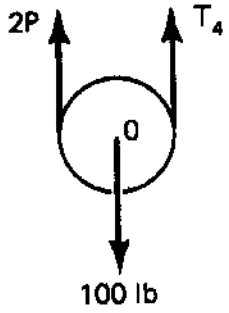
$\Sigma \bar{F} = 0$

$T_1\bar{j} + T_2\bar{j} - T_3\bar{j} = 0$

$P + P - T_3 = 0$

$T_3 = 2P$

Pulley C



$\Sigma \bar{M}_o = 0$

$T_4 = 2P$

$\Sigma \bar{F} = 0$

$2P + T_4 = 100 = 0$

$2P + 2P - 100 = 0$

$P = 25$ lb

Frame 15-7

Equilibrium of Rigid Body

Is the following statement true or false? If it is false, correct it.

When a cord passes around a frictionless pulley, or pulleys, the force in the cord is the same on both sides of the pulley.

☐ True

☐ False _____

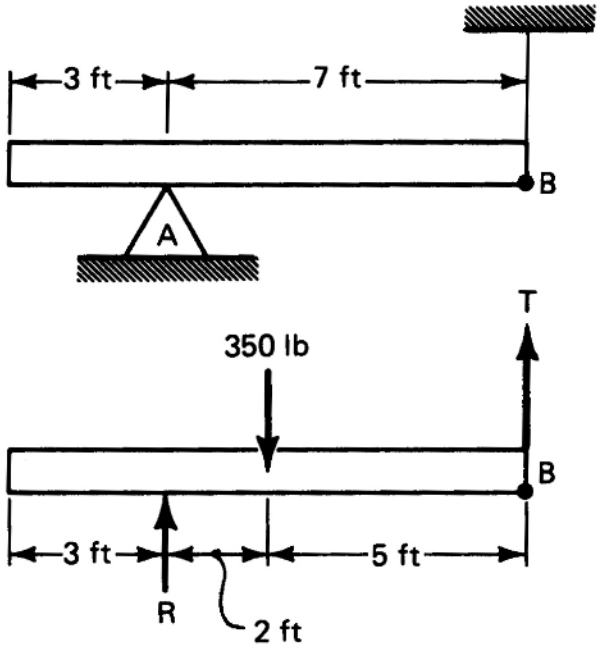
Correct response to preceding frame

True

Frame 15-8

Equilibrium of Rigid Bodies

The uniform beam shown weighs 350 lb. Find the tension in the string and the reaction at the knife edge using the free body shown. Take moments about B for ease in checking.

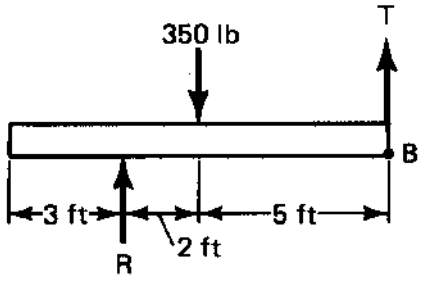


Correct response to preceding frame

R = 250 lb

T = 100 lb

Solution:



$\Sigma M_B = 0$

$[(-7\bar{i}) \times (R\bar{j})] + [(-5\bar{i}) \times (-350\bar{j})] = 0$

$-7R\bar{k} + 1750\bar{k} = 0$

$R = \frac{1750}{7} = 250$ lb

$\Sigma\bar{F} = 0$

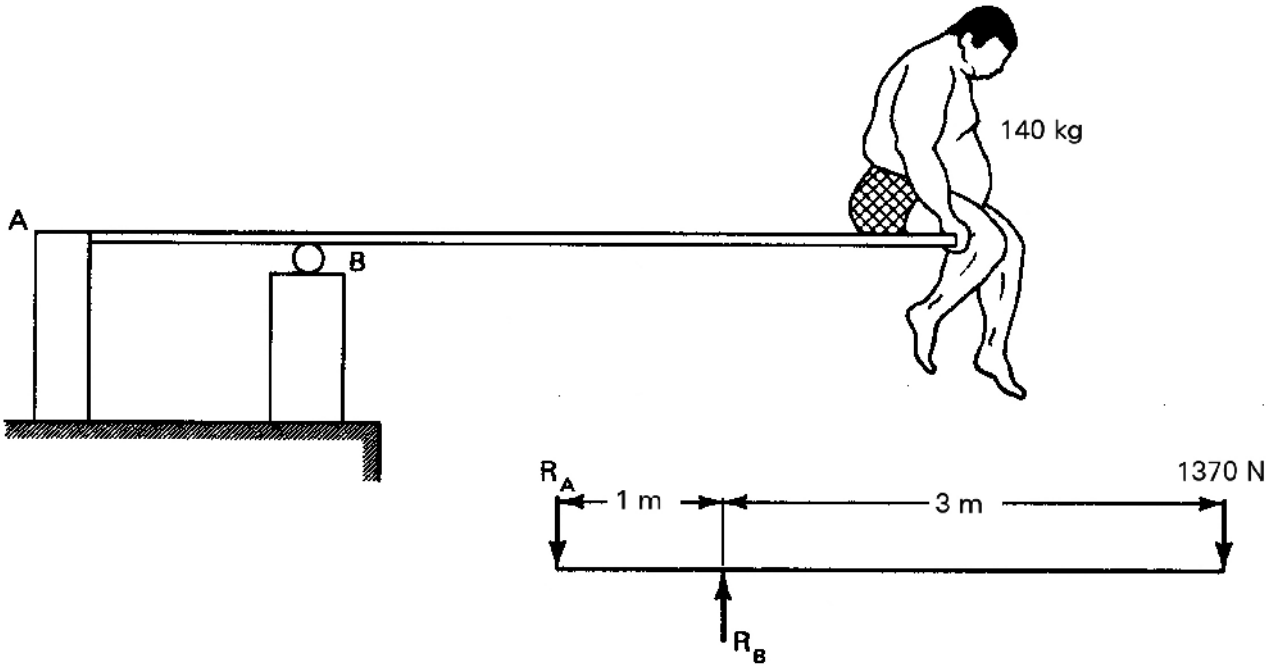
$T\bar{j} + 250\bar{j} - 350\bar{j} = 0$

$T + 250 - 350 = 0$

Frame 15-9

Equilibrium of Rigid Bodies

A diving board is supported by a strap at A and a pipe at B. A 140 kilogram man is sitting on the end trying to make up his mind to slide off. Using the free body given, find $\mathbf{R}_A$ and R_B.

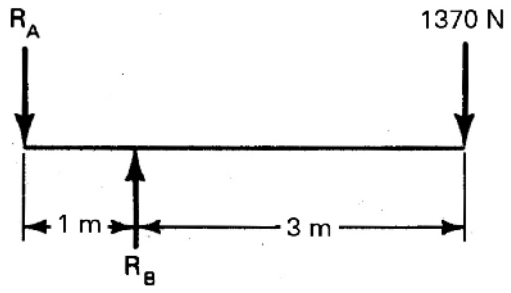


Correct response to preceding frame

$R_A = 4110$ N

$R_B = 5480$ N

Solution:



$\Sigma \bar{M}_B = 0$

$[(-\bar{i}) \times (\bar{R}_A)] + [(3\bar{i}) \times (-1370\bar{j})] = 0$

$R_A\bar{k} - 4110\bar{k} = 0$

$R_A = 4110$ N

$\Sigma \bar{F} = 0$

$-4110\bar{j} + R_B\bar{j} - 1370\bar{j} = 0$

$R_B = 5480$ N

We can check by writing a second moment equation.

Check:

$\Sigma \bar{M}_A = 0$

$[(\bar{i}) \times (R_B\bar{j})] + [(4\bar{i}) \times (-1370\bar{j})] = 0$

$R_B\bar{k} - 5480\bar{k} = 0$

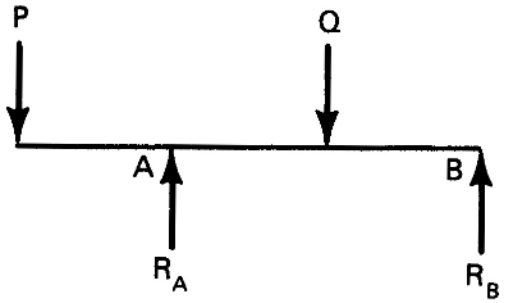
$R_B = 5480$ N

Frame 15-10

Equilibrium of Rigid Bodies

If you solve an equilibrium problem by writing $\Sigma \bar{F} = 0$ and $\Sigma \bar{M}_0 = 0$, you can check your work by writing the sum of the moments about any other point. On the other hand if you solve (and you can if you wish) by taking moments about two points, you can check by taking the sum of the forces.

In the problem shown indicate how you could check most easily.



Solve by	Check by
$\Sigma \bar{M}_A = 0$ $\Sigma \bar{F} = 0$	
$\Sigma \bar{M}_B = 0$ $\Sigma \bar{F} = 0$	
$\Sigma \bar{M}_A = 0$ $\Sigma \bar{M}_B = 0$	

Correct response to preceding frame

Solve by	Check by
$\Sigma\overline{M}_A = 0$ $\Sigma\overline{F} = 0$	$\Sigma\overline{M}_B = 0$
$\Sigma\overline{M}_B = 0$ $\Sigma\overline{F} = 0$	$\Sigma\overline{M}_A = 0$
$\Sigma\overline{M}_A = 0$ $\Sigma\overline{M}_B = 0$	$\Sigma\overline{F} = 0$

Frame 15-11

Equilibrium of Rigid Bodies

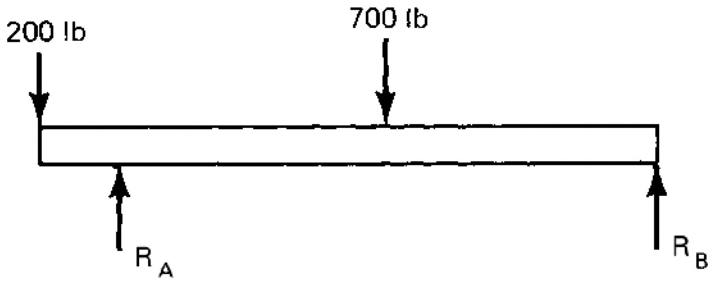
Do Problem 15-2 in your notebook.

Correct response to preceding frame

$R_A = 579$ pounds

$R_B = 321$ pounds

Solution:



$$\Sigma\bar{F} = 0 = -200\bar{j} + R_A\bar{j} - 700\bar{j} + R_B\bar{j}$$

$$R_A + R_B = 900$$

$$\Sigma\bar{M}_A = 0 = (-2\bar{i} \times -200\bar{j}) + (7\bar{i} \times -700\bar{j}) + (14\bar{i} \times R_B\bar{j})$$

$$400\bar{k} - 4900\bar{k} + 14R_B\bar{k} = 0$$

$$R_B = 321$$

$$R_A = 900 - 321 = 579$$

Check:

$$\Sigma\bar{M}_B = 0 = (-16\bar{i} \times -200\bar{j}) + (-14\bar{i} \times R_A\bar{j}) + (-7\bar{i} \times -700\bar{j})$$

$$3200\bar{k} - 14R_A\bar{k} + 4900\bar{k} = 0$$

$$R_A = 579$$

Frame 15-12

Transition

So far in this unit you have worked problems involving parallel, coplanar force systems acting on rigid bodies.

The next group of frames will deal with the equilibrium of rigid bodies acted upon by coplanar force systems which are neither parallel nor concurrent. This is the most general case of two dimensional equilibrium.

The next set of problems will take at least 20 minutes. When you are ready to invest that much time, go to the next frame.

Correct response to preceding frame

No response

Frame 15-13

Equilibrium of Rigid Bodies

In the unit on equilibrium of particles the following procedure was used:

1. Draw the free body diagram.
2. Write all forces as vectors.
3. Write $\sum \overline{F} = 0$.
4. Break the equation from step 3 into coefficient equations and solve.

For rigid body equilibrium we must do one more thing. What is it?

_____

Correct response to preceding frame

Write $\Sigma \overline{M}_0 = 0$

Frame 15-14

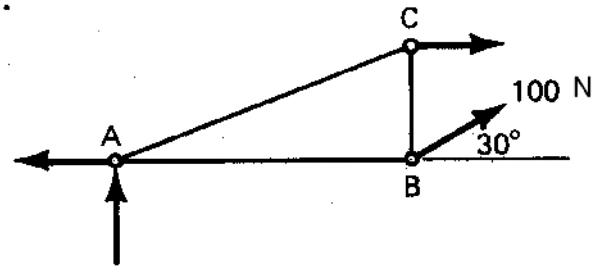
Moments about a Point

When a body is in static equilibrium the sum of the moments of the force system is zero about any point whatsoever. Consequently you may choose any point in the universe and get a correct solution.

However, certain points do have the advantage of giving you an easier solution. The easiest solution will usually result from writing $\Sigma \overline{M}$ about the point through which the largest number of unknown forces passes.

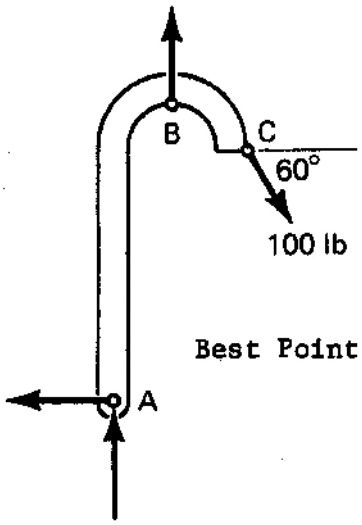
Choose the best point for taking moments on the following free bodies.

1.



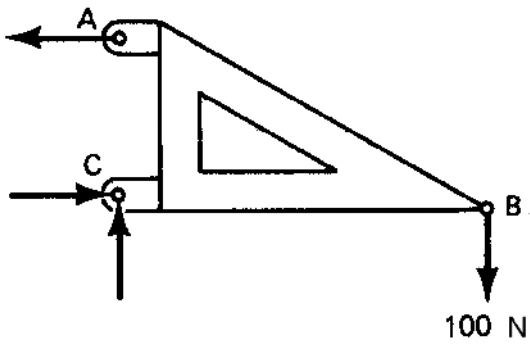
Best Point _____

2.



Best Point _____

3.



Best Point _____

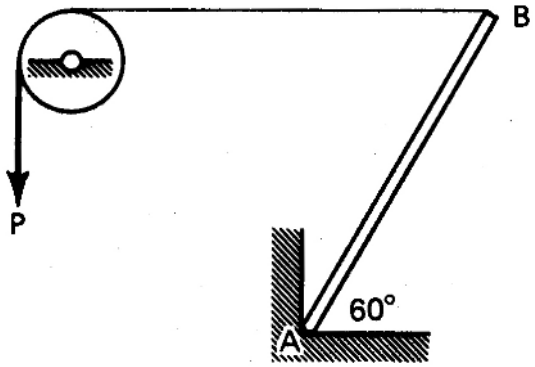
Correct response to preceding frame

1. B or A
2. A
3. C

Frame 15-15

Rigid Body Equilibrium--Coplanar

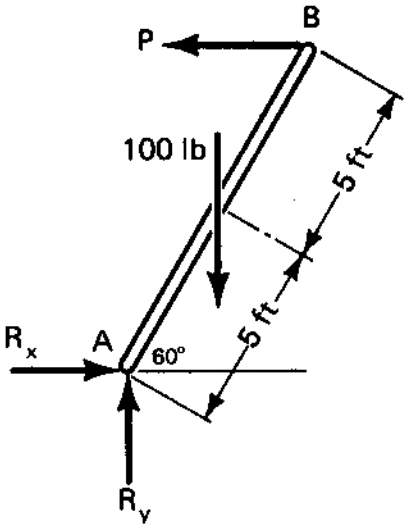
In the figure shown the rod AB is 10 ft long and weighs 100 pounds.



Draw a free body diagram of the rod.

Write $\Sigma \overline{F} = 0$ and $\Sigma \overline{M}_A = 0$.

Correct response to preceding frame



$$\Sigma\bar{F} = 0$$

$$-P\bar{i} - 100\bar{j} + R_x\bar{i} + R_y\bar{j} = 0$$

$$\Sigma\bar{M}_A = 0$$

$$[(8.66\bar{j}) \times (-P\bar{i})] + [(2.5\bar{i}) \times (-100\bar{j})] = 0$$

$$8.66P\bar{k} - 250\bar{k} = 0$$

Frame 15-16

Rigid Bodies Equilibrium--Coplanar

Complete the problem you began in the preceding frame and check your answers by writing $\Sigma\bar{M}_B = 0$.

Correct response to preceding frame

$P = 28.9$ lb

$R_x = 28.9$ lb

$R_y = 100$ lb

Solution:

$8.66P - 250 = 0$

$P = 28.9$ lb

$-P + R_x = 0$

$R_x = P = 28.9$ lb

$-100 + R_y = 0$

$R_y = 100$ lb

Check:

$\Sigma \bar{M}_B = 0$

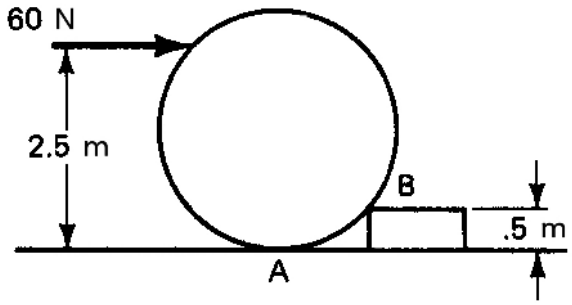
$(-5\bar{i} \times 100\bar{j}) + (-2.5\bar{i} \times -100\bar{j})$

$+ (-8.66\bar{j} \times 28.9\bar{i}) = 0$

$-500\bar{k} + 250\bar{k} + 250\bar{k} = 0$

Frame 15-17

Equilibrium of Rigid Bodies--Coplanar



The cylinder shown weighs 50 kilograms. The diameter is 3 meters. Find all unknown forces acting on it.

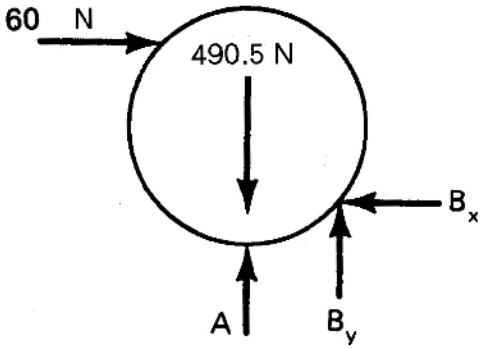
Correct response to preceding frame

$B_x = 60$ N

$B_y = 190$ N

$A = 381.5$ N

Solution:



$\Sigma \bar{M}_B = 0$

$[(-1.11\bar{i} - 0.5\bar{j}) \times (A\bar{j})] + [(-1.11\bar{i} + 1\bar{j}) \times (-490.5\bar{j})]$

$+ [(2\bar{j}) \times (60\ \bar{i})] = 0$

$-1.11A\bar{k} + 539.6\ \bar{k} - 120\ \bar{k} = 0$

$A = 381.5$ N

$\Sigma \bar{F} = 0$

$600\bar{i} - 490.5\bar{j} - B_x\bar{i} + B_y\bar{j} + 381.5\bar{j} = 0$

$-490.5 + B_y + 600.4 = 0$

$B_y = 190$ N

$60 - B_x = 0$

$B_x = 60$ N

Frame 15-18

Equilibrium of Rigid Bodies

The steps in solving equilibrium of rigid bodies are given below. Fill in the missing steps.

1. _____

2. Write all forces as vectors.

3. Write $\Sigma \bar{F} = 0$

4. _____

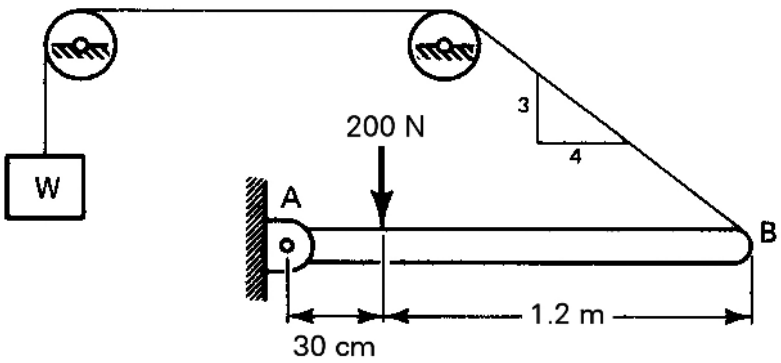
5. Break into coefficient equations and solve.

Correct response to preceding frame

1. Draw the free body diagram.
4. Write $\Sigma \overline{M} = 0$.

Frame 15-19

Equilibrium of Rigid Bodies--Coplanar

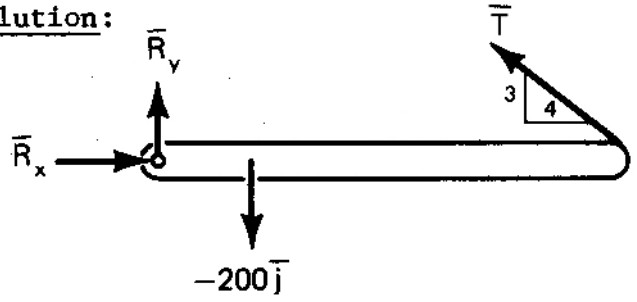


Rod AB is of negligible weight and the pulleys are smooth. Find the weight necessary to hold the rod in equilibrium.

Correct response to preceding frame

W = 6.83 kg

Solution:



$$\bar{T} = T(-.8\bar{i} + .6\bar{j})$$

$$\Sigma \bar{F} = 0 = R_x\bar{i} + R_y\bar{j} - 200\bar{j} - .8T\bar{i} + .6T\bar{j}$$

$$R_x - .8T = 0$$

$$R_y - 200 + .6T = 0$$

$$\Sigma \bar{M}_B = 0 = (-1.5\bar{i} \times R_y\bar{j}) + (-1.2\bar{j} \times -200\bar{j})$$

$$+1.5R_y\bar{k} - 240\bar{k} = 0$$

$$R_y = 160 \text{ N}$$

$$160 - 200 + .6T = 0$$

$$T = \frac{40}{.6} = 67 \text{ N}$$

$$T = 9.81 \text{ W}$$

Frame 15-20

Equilibrium of Rigid Bodies

Do problem 15-3 in your notebook and check your work by writing a second moment equation.

Summarize (from memory if possible) the steps for solving problems dealing with equilibrium of rigid bodies in the space provided in your notebook on page 15-3.

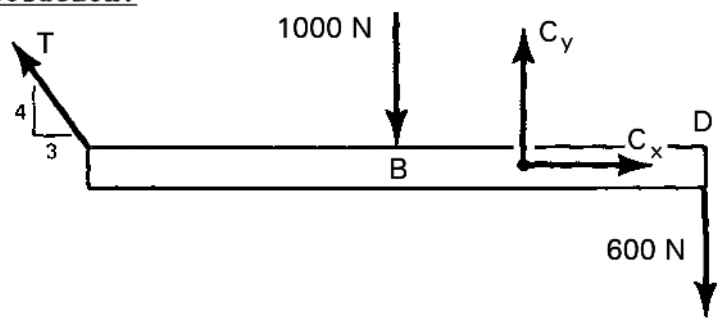
Correct response to preceding frame

$T = 35.7$ Newtons

$C_x = 21.4$ Newtons

$C_y = 1570$ Newtons

Solution:



$$\Sigma\bar{F} = 0 = T(-0.6\bar{i} + 0.8\bar{j}) - 1000\bar{j} + (C_x\bar{i} + C_y\bar{j}) - 600\bar{j}$$

$$-0.6T + C_x = 0$$

$$0.8T - 1000 + C_y - 600 = 0$$

$$\Sigma\bar{M}_C = 0 = (-7\bar{i} \times T[-0.6\bar{i} + 0.8\bar{j}]) + (-2\bar{i} \times -1000\bar{j}) + (3\bar{i} \times -600\bar{j})$$

$$-5.6T + 2000 - 1800 = 0$$

$$T = 35.7$$

$$C_x = 0.6T = 21.4$$

$$C_y = -0.8T + 1000 + 600 = 1570$$

Frame 15-21

Transition

Until now you have been instructed thoroughly in problems you can solve by writing $\Sigma\bar{F} = 0$ and/or $\Sigma\bar{M} = 0$. Unfortunately there are problems which cannot be solved by statics alone. It will save you time if you learn to identify them. Consequently the next (and last) section of this unit will be devoted to the identification of statically indeterminate bodies.

Fifteen minutes more should finish you off--one way or another.

Go to the next frame.

Correct response to preceding frame

No response

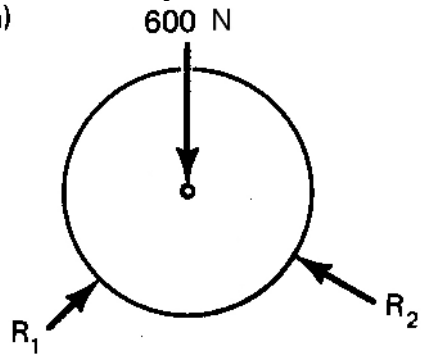
Frame 15-22

Statically Indeterminate Systems

A body is statically indeterminate when the forces acting upon it cannot be found by statics alone. This occurs when the number of unknown forces exceeds the number of independent equations that can be obtained by writing $\Sigma \overline{F} = 0$ and $\Sigma \overline{M}_0 = 0$.

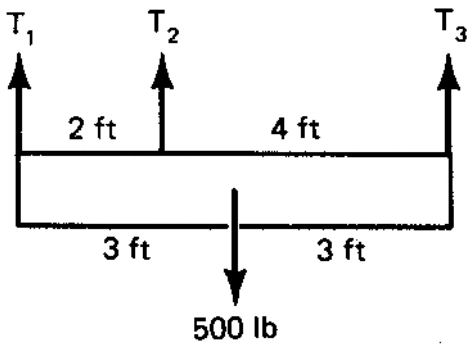
Which of the following is statically indeterminate?

a)



b)

c)



Correct response to preceding frame

Body (b) is statically indeterminate. Both $\Sigma \overline{\mathbf{F}} = 0$ and $\Sigma \overline{\mathbf{M}} = 0$ reduce to $R_1 + R_2 = 300$.

Body (c) is indeterminate. Taking $\Sigma \overline{\mathbf{F}} = 0$ gives $T_1 + T_2 + T_3 = 500$. Taking moments will give only one more independent equation.

Frame 15-23

Statically Indeterminate Systems

1. Draw a coplanar concurrent force system.

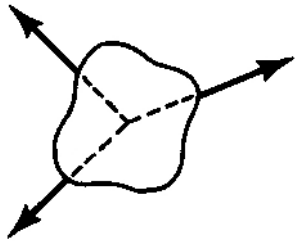
How many unknown forces can be found in a concurrent coplanar force system? ____

2. How many unknown forces can be found in a coplanar system consisting of parallel non-concurrent forces? _____

Sketch such a system.

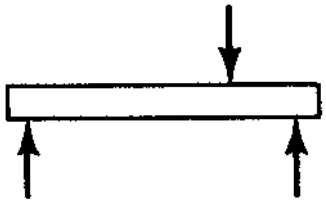
Correct response to preceding frame

a)



Two unknown forces can be found if their directions are known. Only one can be found if it is unknown in both magnitude and direction.

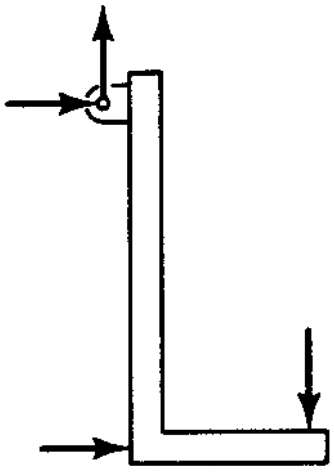
b)



Two unknown forces can be found.

Frame 15-24

Statically Indeterminate Systems



How many unknowns can be found for a coplanar force system such as is shown? _____

Correct response to preceding frame

Three unknowns may be found for a non-concurrent non-parallel coplanar force system. They may all be magnitudes or all directions or some of each, but are most commonly all magnitudes.

Frame 15-25

Statically Indeterminate Systems

What is the largest number of reactions that can be found for a rigid body acted upon by the following force systems?

1. concurrent coplanar _____
2. parallel coplanar _____
3. general coplanar _____
4. parallel concurrent _____

Correct response to preceding frame

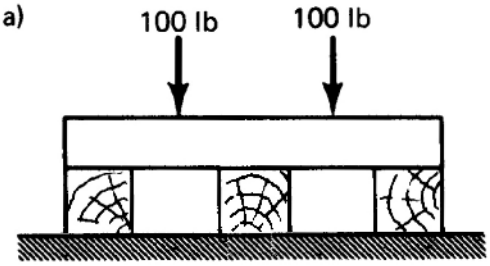
1. concurrent coplanar 2
2. parallel coplanar 2
3. general coplanar 3
4. parallel concurrent 1

Frame 15-26

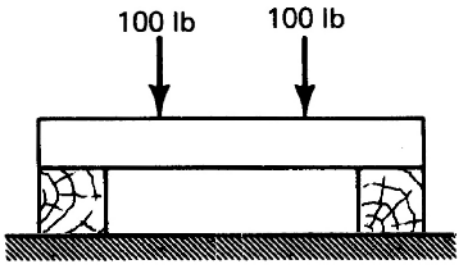
Statically Indeterminate Bodies

Draw a free body for each of the following.

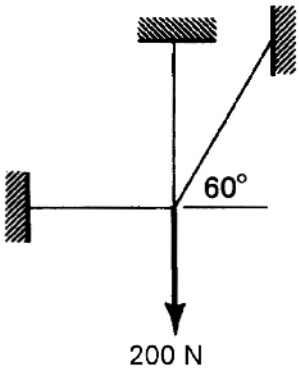
a)



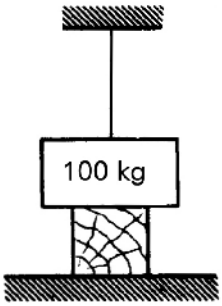
b)



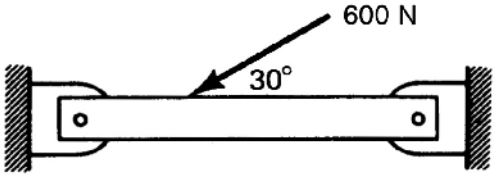
c)



d)

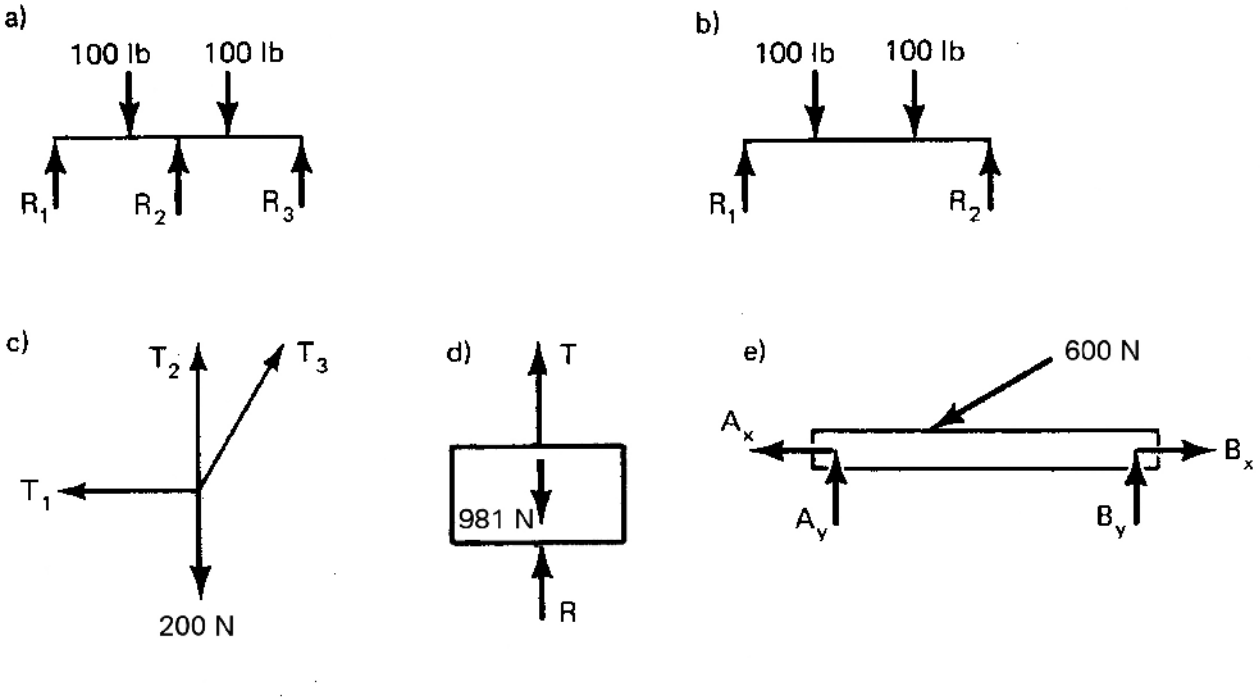


e)



Which are statically indeterminate? _____

Correct response to preceding frame



All except (b) are statically indeterminate.

Frame 15-27

Statically Indeterminate Systems

Summarize what you have learned about statically indeterminate systems in the space provided on page 15-3 in your notebook.

Correct response to preceding frame

No response

Frame 15-28

Closure

In this unit you learned to solve problems involving coplanar force systems acting on rigid bodies, looking first at the case of parallel forces, then at the general case.

Taking this unit with the unit on particle equilibrium, you can now solve any two dimensional problem in static equilibrium--so long as it is statically determinate. (If you can't do it, no one can!) Furthermore, you can identify the ones that are statically indeterminate, so you know your limitations. Further units will deal with more complex problems but no new methods will emerge. When it comes to coplanar systems, in a very real sense you know it all.