Introduction to Statics

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Unit 13

Center of Mass and Center of Pressure

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Unit 13
Center of Mass
and Center of Pressure

Frame 13-1

Introduction

This unit will teach you to locate centers of gravity and centers of pressure. It will also show you how to use them, once you have located them. You will find the work to be very similar to that you did in finding centroids.

In fact you will find you can do a great deal of this unit simply by analogy.

Go to the next frame.

Correct response to preceding frame

No response

Frame 13-2

Center of Gravity

Just as the centroid of an area is a point at which the entire area can be considered to be concentrated, the center of gravity of a body is a point where the body's weight can be considered to be concentrated. Centroids are a two dimensional analogy to the three dimensional center of gravity. To locate a centroid you must find two coordinates; to locate a center of gravity you must find three. The method of operation is the same.

As a review and to see the importance of the center of gravity, read the first section of your notebook.

Correct response to preceding frame

No response

Frame 13-3

Vocabulary

Centers of gravity are often called mass centers or centers of mass. Actually there is a negligible difference in their locations due to the fact that the weight of a particle varies as its distance from the center of the earth.

1. Is the mass center the same as the center of gravity?

☐ Yes ☐ No

2. May the two be used interchangeably for most purposes?

☐ Yes ☐ No

Correct response to preceding frame

1. No
2. Yes

Frame 13-4

Center of Mass

Centroids deal with elements of area. Centers of mass deal with elements of mass.

Since for an area

$$x_G = \frac{\int x \, da}{A}$$

You would expect that, for a mass

$$x_G = \frac{\int \_\_\_\_}{M}$$

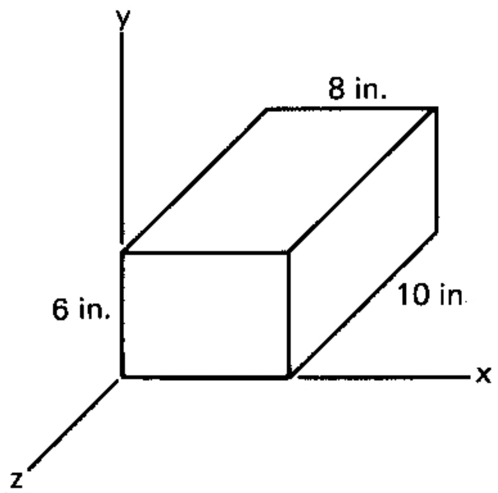
Correct response to preceding frame

$$x_G = \frac{\int x \; dM}{M}$$

(Since, in a constant gravitational field, weight is proportional to mass, this statement reduces to the one you just read in your notebook.)

Frame 13-5

Center of Gravity



For the simplest sorts of shapes, the center of gravity can be located by inspection. Guess the coordinates of the center of gravity of the solid shown. Assume it to be homogeneous.

x_G = ____________________

y_G = ____________________

z_G = ____________________

Correct response to preceding frame

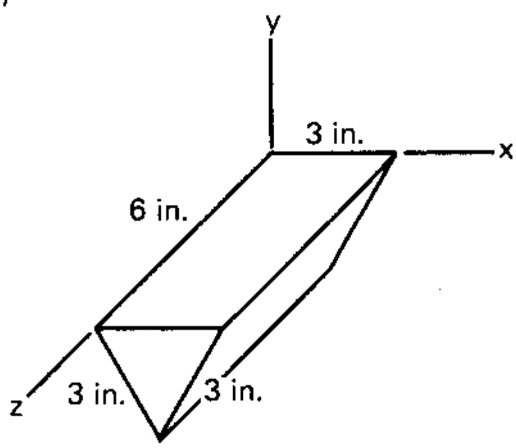
$x_G = +4$ in.
$Y_G = +3$ in.
$Z_G = -5$ in.

Frame 13-6

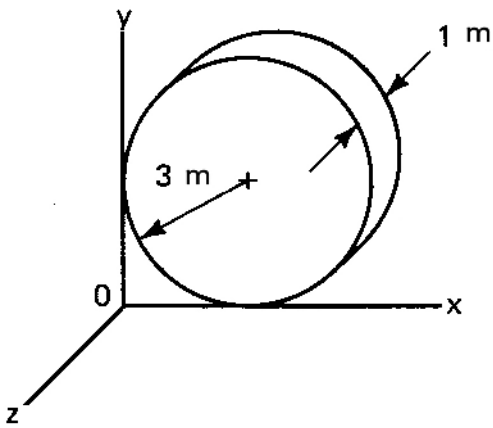
Centers of Gravity

If a body is of uniform thickness, two of the coordinates of the center of gravity are the same as those of the centroid of the end area, the third is equal to one-half the thickness. Locate the centers of gravity of the homogeneous bodies shown.

a)



b)



Correct response to preceding frame

a) $x_G = 1.5$ in.

$y_G = -1/3\ (3\ \sqrt{3}/2) = -.866$ in.

$z_G = 3$ in.

b) $x_G = 3$ m

$y_G = 3$ m

$z_G = -1/2$ m

Frame 13-7

Mass Center

When a body is not of uniform thickness, the x coordinate of its mass center must be found by using

$$x_G = \frac{\int x\ dM}{M}$$

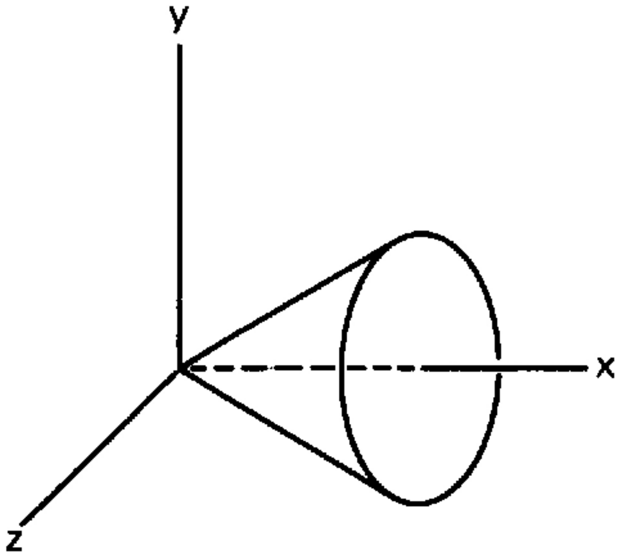
Remembering similar problems in centroids we would expect to choose dM so that it has the same __________ coordinate throughout.

Correct response to preceding frame

x

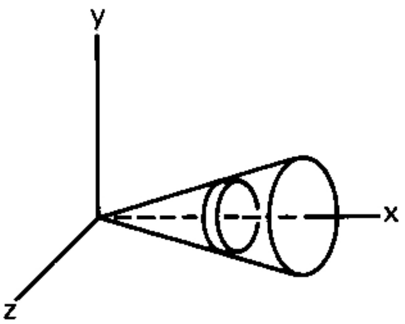
Frame 13-8

Mass Centers



The cone shown is homogeneous. Two of the coordinates of its mass center may be found by inspection. Draw a proper element to use to find the third coordinate.

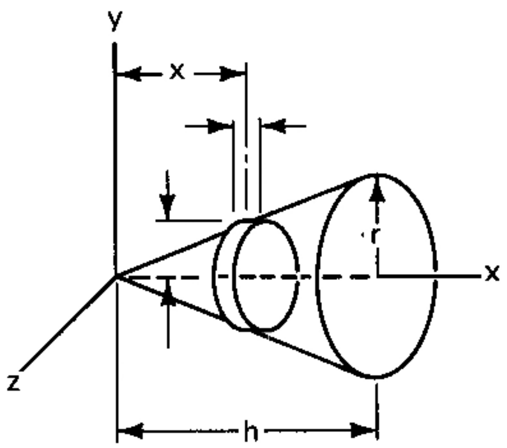
Correct response to preceding frame



($z_G = y_G = 0$ from symmetry)

Frame 13-9

Mass Center

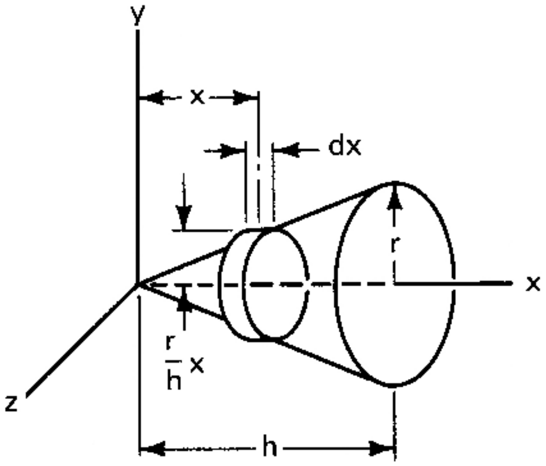


Give each of the two missing dimensions in terms of x.

Using your dimensional element write an expression for dM. (Assume a density of ρ.)

dM = ________________________________

Correct response to preceding frame



$$dM = \rho \pi \frac{r^2 x^2}{h^2} dx$$

Frame 13-10

Mass Center

Using your dM from the preceding frame and the expression

$$x_G = \frac{\int x \, dM}{M}$$

find x_G

x_G = ______________________

Correct response to preceding frame

$x_G = \frac{3}{4} h$

Solution:

$$x_G = \frac{\int x dM}{\int dM}$$

$$\int x dM = \int_0^h \frac{\rho \pi r^2 x^3 dx}{h^2} = \frac{\rho \pi r^2}{h^2} \frac{h^4}{4}$$

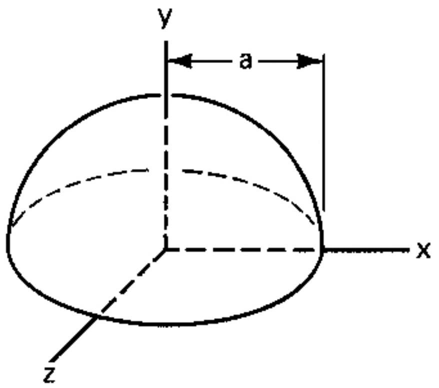
$$\int dM = \int_0^h \frac{\rho \pi^2 r^2 x^2 dx}{h^2} = \frac{\rho \pi r^2}{h^2} \frac{h^3}{3}$$

$$x_G = \frac{\int x dM}{\int dM} = \frac{\frac{h^4}{4}}{\frac{h^3}{3}}$$

Frame 13-11

Mass Center

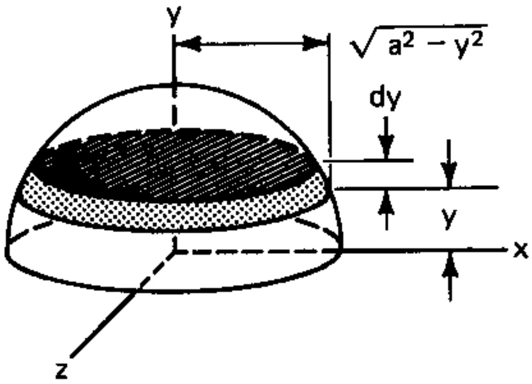
Get ready to locate the mass center of a homogeneous hemisphere of density ρ by drawing the appropriate dM and dimensioning it.



Write your expression for dM.

DM = ________________________________

Correct response to preceding frame



$$dM = \rho \pi (a^2 - y^2)dy$$

Frame 13-12

Mass Center

Turn to Page 13-2 in your notebook and complete the problem.

Correct response to preceding frame

$x_G = 0$

$y_G = \frac{3a}{8}$

$z_G = 0$

<u>Solution</u>:

$$dM = \rho\ \pi(a^2 - y^2)\ dy$$

$$\int dM = \int_0^a \rho\ \pi\ (a^2 - y^2)dy = \frac{2}{3}\pi\, a^3 \rho$$

$$\int ydM = \int_0^a \rho\ \pi\ (a^2 - y^2)ydy = \frac{1}{4}\ \pi\ a^4 \rho$$

$$y_G = \frac{\int ydM}{\int dM} = \frac{3}{8}\ a$$

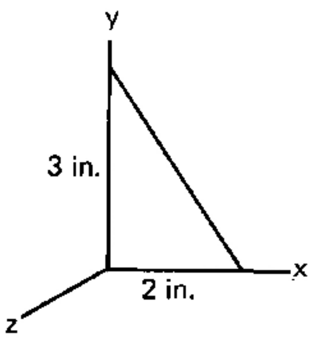
$x_G = 0$ and $z_G = 0$ by symmetry

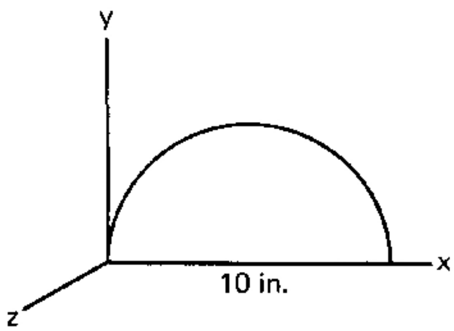
Frame 13-13

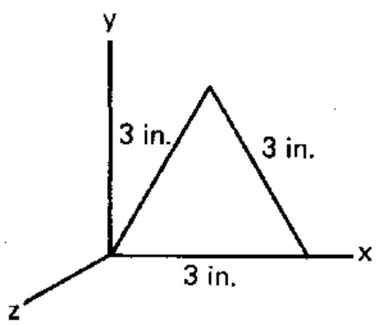
Mass Center

The weights of all the particles of a body may be replaced by a single resultant weight through the center of gravity of the body. For engineering purposes the center of gravity coincides with the mass center.

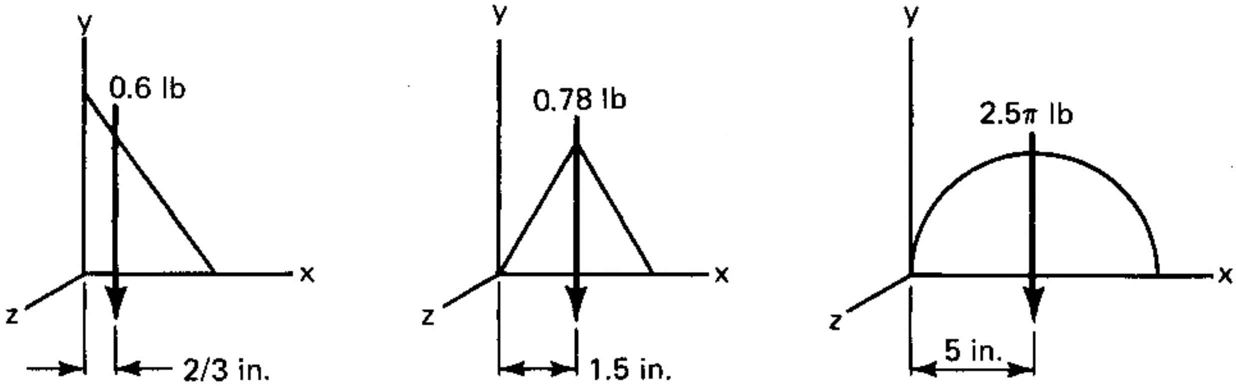
The bodies shown are composed of a plate weighing .2 lb/in^2. and 1/4 in. thick. Find the weight of each and locate the point through which it acts. Show the line of action of the weight on each.







Correct response to preceding frame

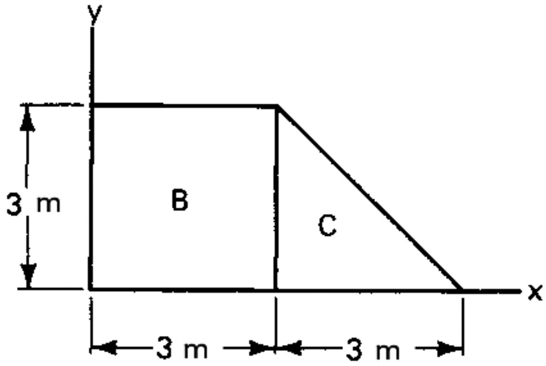


Since W acts in the y direction, the y coordinate of the mass center is not needed.

Frame 13-14

Mass Center

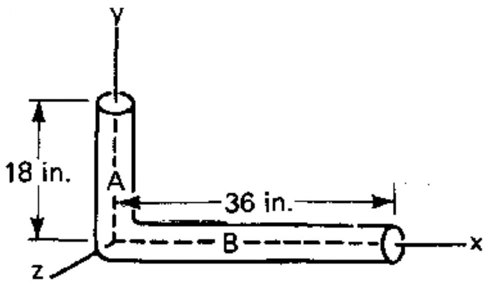
In your study of centroids you learned to work out a tabular solution for the coordinates of the centroid.



Part	A_p	x_{GP}	$A_p x_{GP}$
B	9	3/2	13.5
C	9/2	4	18
Total	13.5		31.5

$$x_G = \frac{\sum A_p x_{GP}}{\sum A_p} = \frac{31.5}{13.5} = 2.33 \text{ m}$$

A similar tabular method may be used to locate the center of mass of an object. Complete the table below and find the center of mass of the angle. The bar weighs 5 lb/ft.



Part	W	x_{GP}	Wx_{GP}	y_{GP}	Wy_{GP}	z_{GP}	Wz_{GP}
A	7.5	0	0	9	67.5		
B						0	0
Total		X		X		X	

$x_G = \dfrac{\sum Wx_{Gp}}{\sum W}$ = ____________ $\quad y_G$ = ____________ $\quad z_G$ = ____________

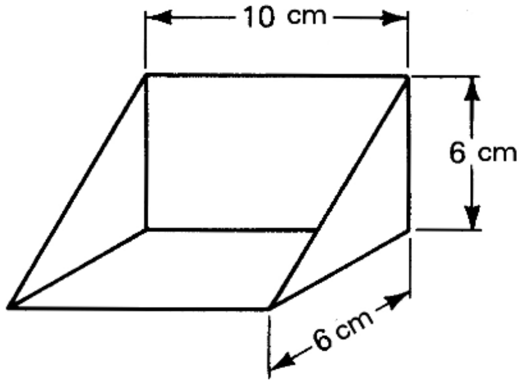
Correct response to preceding frame

x_G = 12.0 in. y_G = 3.0 in. z_G = 0

Part	W	x_{GP}	Wx_{GP}	y_{GP}	Wy_{GP}	z_{GP}	Wz_{GP}
A	7.5	0	0	9	67.5	0	0
B	15	18	270	0	0	0	0
Total	22.5	X	270	X	67.5	X	0

Frame 13-15

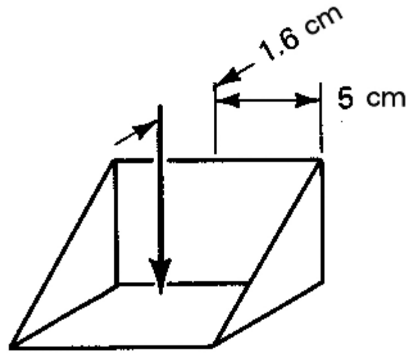
Mass Center



The figure shows a simple scoop made of plates weighing .002 kg/cm^2 . The plate is very thin. Find its total weight and locate its mass center. Show the weight acting on the scoop in its proper position.

Part	W	x_{GP}	Wx_{GP}	y_{GP}	Wy_{GP}	z_{GP}	Wz_{GP}
Total		X		X		X	

Correct response to preceding frame



W = 0.312 kg
Using left rear corner as origin, mass center is located at (5, 1.61, 1.61)

Frame 13-16

Center of Mass

Complete Problem 13-2 in your notebook. The results of your calculations for the mass centers of the individual parts are found in the responses to Frames 13-10 and 13-12.

Correct response to preceding frame

Mass center is on the geometric axis (x as drawn in your notebook) 3.83 ft from the tip of the cone.

Frame 13-17

Transition

You should now have a good working knowledge of what a mass center is and how to go about finding it. A particular body may present problems of more or less complexity but the system of solution will always be the same.

The remainder of this unit will be devoted to a closely-related topic--centers of pressure.

This is a good place to take a break. When you are ready to settle down once more, go to the next frame.

Correct response to preceding frame

No response

Frame 13-18

Pressure

When a force is exerted on a small area it is called a concentrated force. When it is distributed over a large area, it is called a distributed force or a pressure. Which of the following would constitute a distributed force system.

- ☐ (1.) water against a dam
- ☐ (2.) a pile of coal on a floor
- ☐ (3.) a truck tire on a bridge floor
- ☐ (4.) wind against a building
- ☐ (5.) a post standing on top of a concrete floor

Correct response to preceding frame

(1.) water against a dam
(2.) a pile of coal on a floor
(4.) wind against a building

Frame 13-19

Pressure

Pressure can be considered to be a number of concentrated forces, each acting on a very small area of the surface in a direction normal to it.

The pressure on a surface results in a system of parallel forces when the surface is

____________________ .

Correct response to preceding frame

when the surface is ***a plane***

In these units we will only treat pressures on plane areas. If you take fluid mechanics courses you will learn to handle more complicated cases.

Frame 13-20

Pressure

Which units seem appropriate for describing pressure?

American Customary Units (ACU)

- ☐ pounds
- ☐ pounds per foot
- ☐ pounds per square foot (psf)
- ☐ pounds per cubic foot
- ☐ pounds per inch
- ☐ pounds per square inch (psi)
- ☐ pounds per cubic inch

System Internationale (SI)

- ☐ Newtons
- ☐ Newtonsper meter
- ☐ Pascals (1 Pascal is defined as 1 Newton per square meter)
- ☐ Newtonsper cubic meter

Correct response to preceding frame

ACU - Pounds per square foot (psf) and pounds per square inch (psi) are appropriate, since they are a force distributed over an area. While gas and fluids problems in machines and pipes are usually expressed in psi, structural loads are commonly expressed in psf.

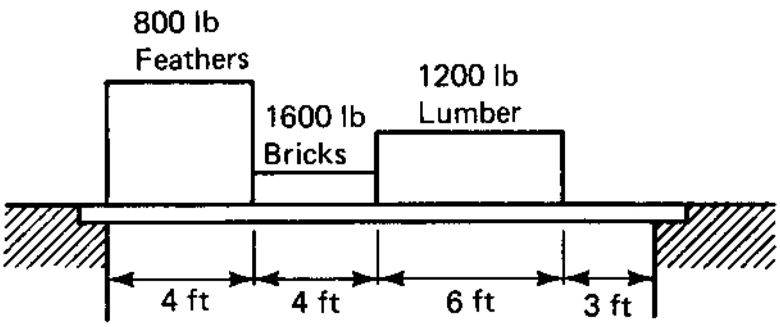
SI - Pascals (Pa) are appropriate for describing pressure. The Pascal is the approved unit for pressure in SI. You may encounter other units are in use in old "metric system" texts, references, ordesign books.
A Pascal is a very small quantity. 1 Pa = 0.0001450 psi
In SI pressures are commonly expressed in kPa and MPa
1 kPa = 1000 Pa and 1 MPa = 10^6 Pa

Frame 13-21

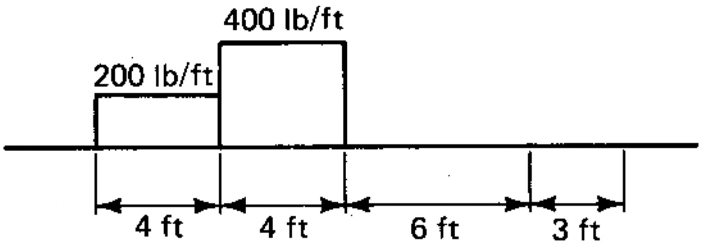
Load Diagrams

A concept called the load diagram, or pressure diagram, is often used to indicate loads on structures.

An engineer doing calculations for the beam loaded like this

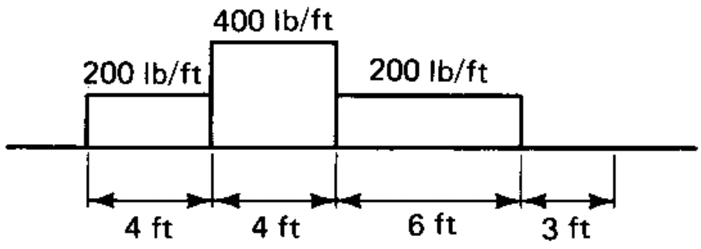


might represent the first two loads as shown.



Show how the stack of lumber might be shown with this system.

Correct response to preceding frame

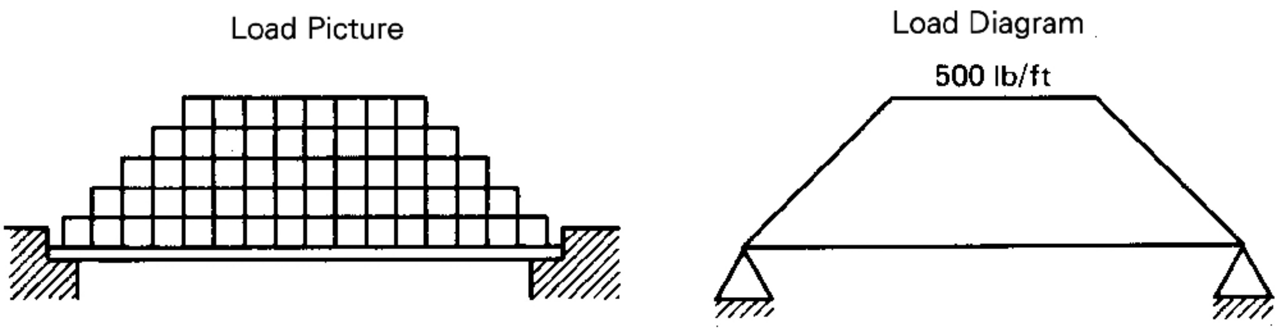


Frame 13-22

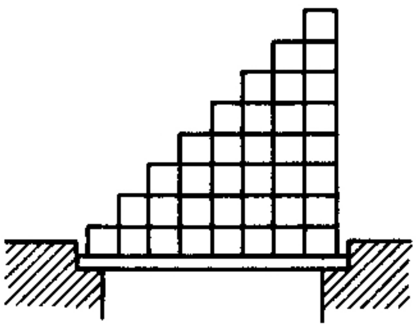
Load Diagrams

When a structure is to be loaded in a non-uniform manner it is customary to show the expected load by means of a "load diagram."

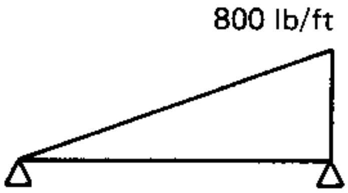
For example a load of 100 lb blocks each one foot wide might be like this.



Draw a load diagram for this load.



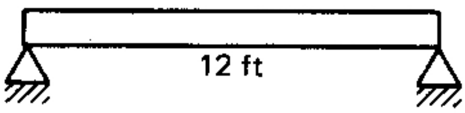
Correct response to preceding frame



Frame 13-23

Load Diagrams

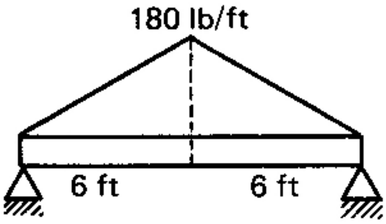
The beam shown supports a pile of grain. The load intensity varies uniformly from zero at each end to 180 lb/ft at the midpoint. Draw the load diagram.



Would the shape of the grain pile be the same as that of the load diagram?

☐ Yes ☐ No

Correct response to preceding frame



Yes. However, in order to get this shape the grain would have to be dumped in a long pile of constant maximum height rather than the usual conical

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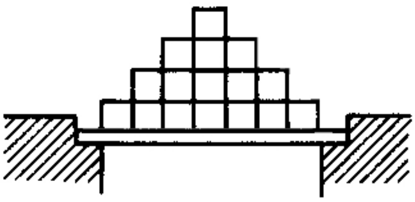
Frame 13-24

Load Diagrams

In American units we give both weight and force in the same units – pounds.

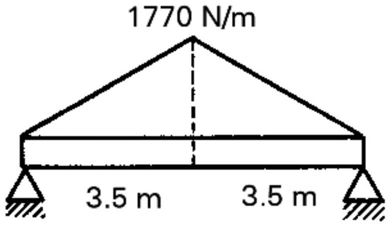
In SI, or other metric systems, you might encounter loads expressed in either terms of mass (kg/m) or force (N/m).



Suppose that the picture shows a beam carrying a stack of similar crates. Each crate ia 1 meter wide, and weighs 45 kilograms.

Draw a load diagram for the beam using Newtons per meter for the magnitude of the load.

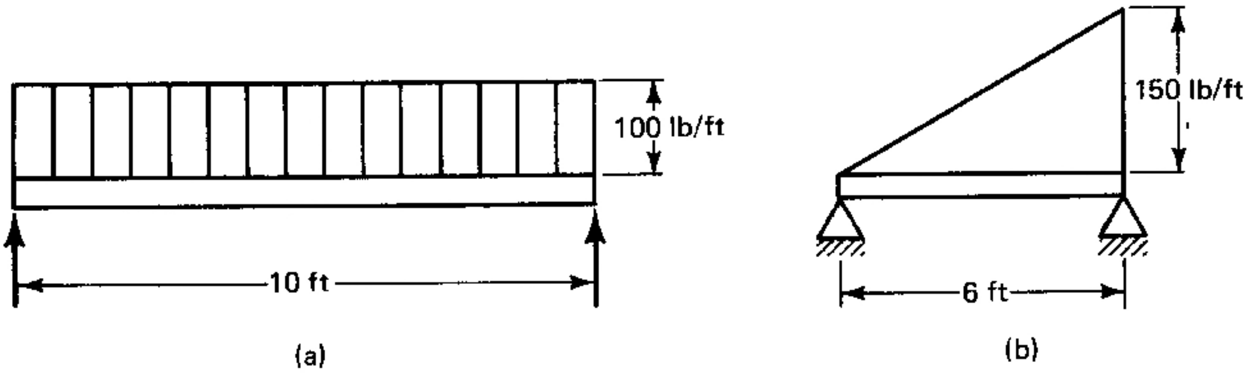
Correct response to preceding frame



Frame 13-25

Load Diagrams

In a load diagram the intensity of the load is shown as a dimension perpendicular to the plane on which it acts. If the pressure varies in only one direction, the load intensity is given in units of force per unit of length.



1. What is the intensity of the load at the midpoint of each beam shown above?

2. Assuming each beam to have a uniform breadth of 6 inches, find the pressure at each midpoint.

Correct response to preceding frame

1. (a) 100 lb/ft (b) 75 lb/ft
2. (a) 200 lb/ft^2 (b) 150 lb/ft^2

Frame 13-26

Center of Pressure

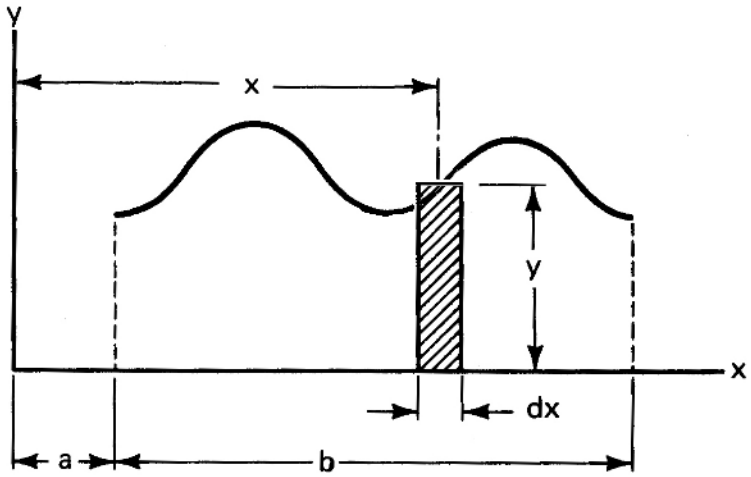
Read the first section of Page 13-3 in your notebook.

Correct response to preceding frame

No response

Frame 13-27

Review of Calculus



Forgetting about forces for a minute, and thinking about any mathematical curve, $y\,dx$ is an expression for the area of the shaded element.

What does

$$\int_a^{a+b} y\,dx$$

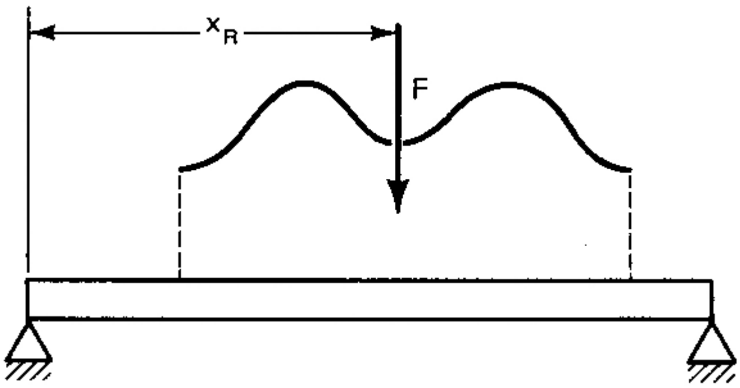
represent geometrically? ______________________________

Correct response to preceding frame

the area under the curve (Or equivalent response)

Frame 13-28

Total Force



Both w and y stand for the variable height of the curve, plotted to different scales.

$$F = \int_{a}^{a+b} w\,dx \qquad A = \int_{a}^{a+b} y\,dx$$

Examination of the above relationships show us that the area under the load diagram

is proportional to ______________________________________ .

Correct response to preceding frame

area under the load diagram is proportional to force (Or equivalent response)

Frame 13-29

Total Force

To find the total force resulting from a distributed load the steps are:

1. draw the load diagram

2. ____

Correct response to preceding frame

find the area under the curve (Or equivalent response)

Frame 13-30

Total Load

This is probably the first time you have had to be concerned with areas whose units are not all related to length.

On the load diagrams you have seen the "height" of the diagram is measured in

1. ______________________________

the "width" of the area is measured in

2. ______________________________

the area is the product of "height" and "width" and is measured in

3. ______________________________

Correct response to preceding frame

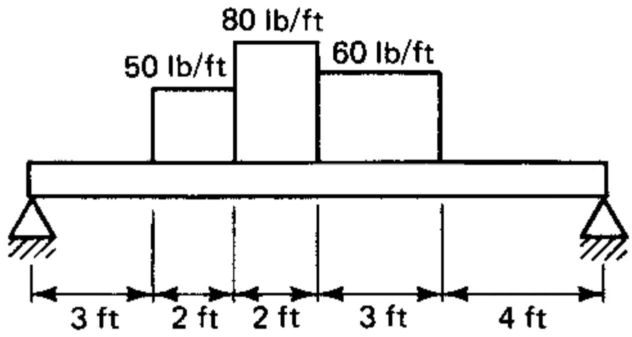
1. force per unit length
2. length
3. force

(Or equivalent response)

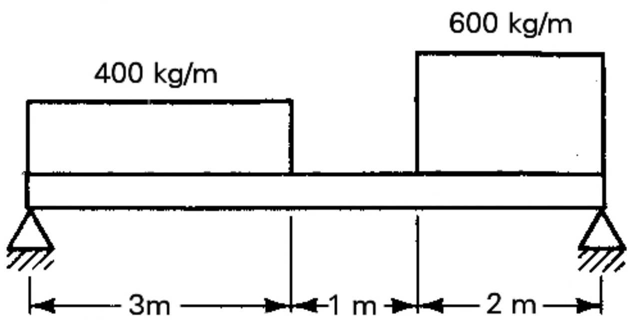
Frame 13-31

Total Load

Determine the total force due to the load of blocks on the beams below.



F = ________________



F = ________________

Correct response to preceding frame

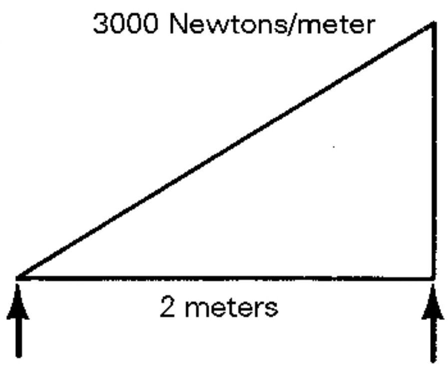
440 pounds
23 500 Newtons

Frame 13-32

Total Load

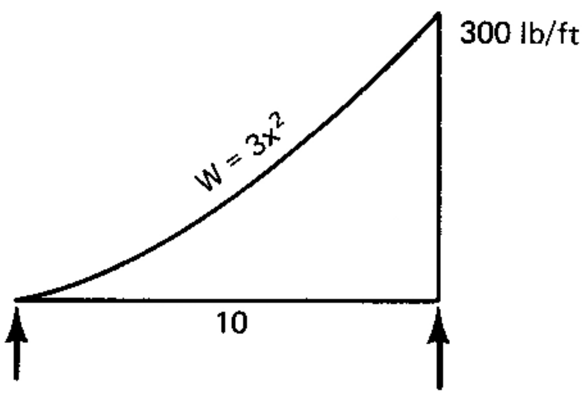
Determine the total force represented by each of the load diagrams below.

1.



F = ______________________

2.



F = ______________________

Correct response to preceding frame

1. F = 3000 N
2. F = 1000 lb

Frame 13-33

Total Load

Fill in the next section on page 13-3 of your notebook.

Correct response to preceding frame

No response

Frame 13-34

Transition

The last few frames have been meant to help you find the magnitude of a load from a load or pressure diagram. The next group of frames will teach you how to locate the line of action of such a load.

Take a short break if you need one but don't make a major interruption here.

Go to the next frame.

Correct response to preceding frame

No response

Frame 13-35

Location of Center of Pressure

The location of the center of pressure from a given origin is given by

$$x_R = \frac{\int_a^{b+a} x \, dF}{F}$$

Since F is proportional to the area under a load diagram,

$$\int_a^{b+a} x \, dF$$

is proportional to the ______________________ of the area under the load diagram.

x_R is equal to the coordinate of the _______________ of the area under the load diagram.

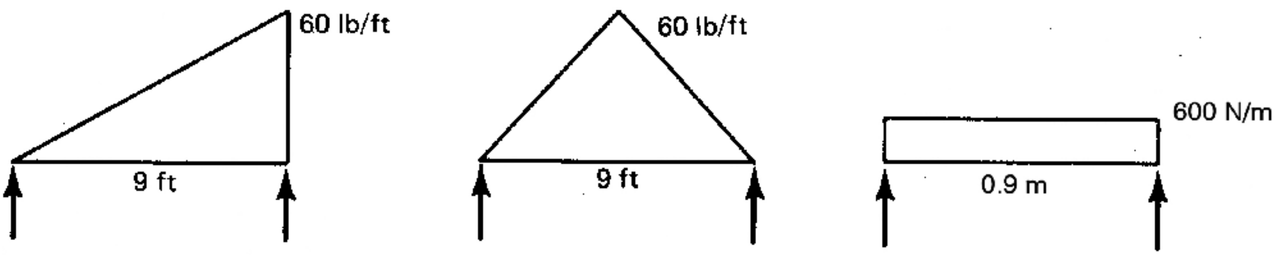
Correct response to preceding frame

first moment
centroid

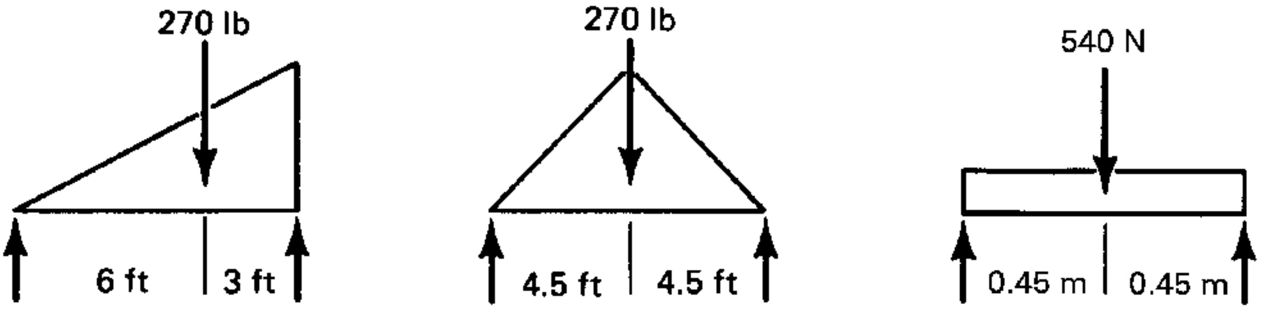
Frame 13-36

Distributed Loads

The line of action of the total force passes through the centroid of the area under the load diagram. Find the resultant forces that correspond to the distributed loads shown and draw them on the diagrams with correctly located lines of action.

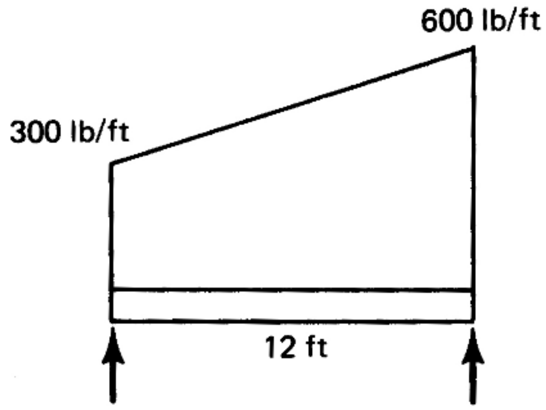


Correct response to preceding frame



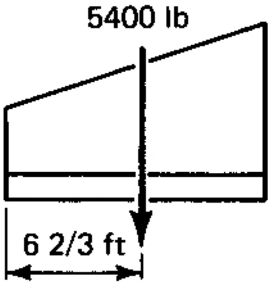
Frame 13-37

Distributed Loads



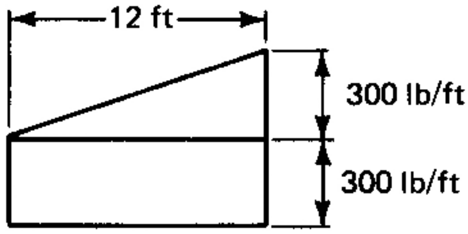
Find and locate the resultant of the load shown by dividing the load diagram into a triangle and rectangle and finding the centroid of the composite area.

Correct response to preceding frame



Solution:

Taking origin
at lower left
corner



Shape	Force (Area)	x_{GP}	Fx_{GP} (Ax_{GP})
◺ (triangle)	1800	8	14400
▭ (rectangle)	3600	6	21600
Total	5400	X	36000

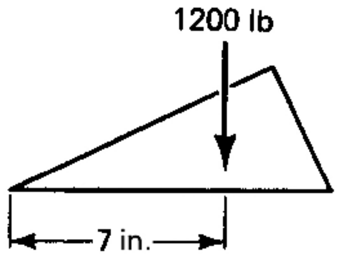
$$x_G = \frac{36000}{5400} = 6.67$$

Frame 13-38

Center of Pressure

Complete the top section of page 13-4 in your notebook and do problem 13-3.

Correct response to preceding frame

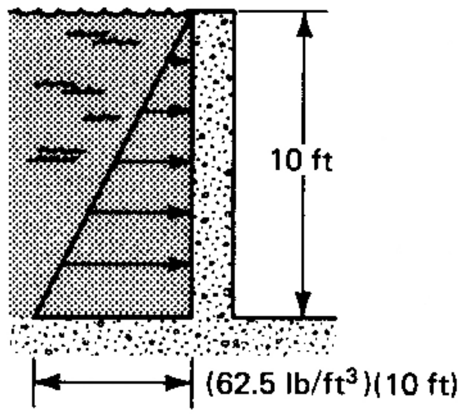


Frame 13-39

Hydrostatic Pressure

As you may remember from physics a fluid exerts a pressure proportional to its density and its depth. Water weighs about 62.5 pounds per cubic foot, therefor it exerts a pressure of 62.5 pounds per square foot for each foot of depth.

The pressure exerted against a dam is shown below. Notice that it bears great resemblance to a load diagram turned sideways.



Find the total force exerted on a section 1 foot in width and locate its line of action.

F = ______________________

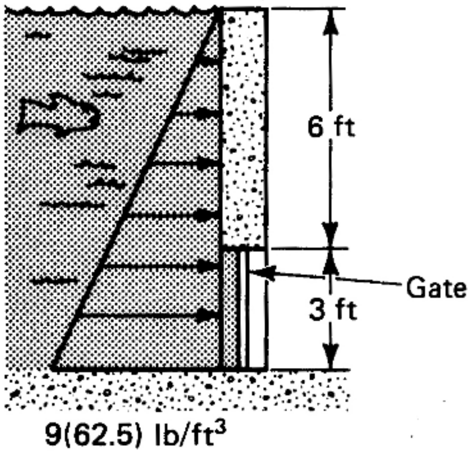
y_F = ______________________

Correct response to preceding frame

F = 3125 pounds acting 3.33 feet above bottom.

Frame 13-40

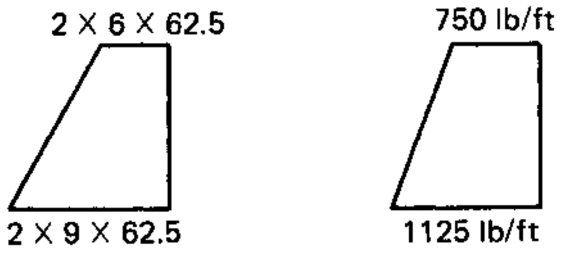
Hydrostatic Pressure



The figure shows a pressure distribution curve for a dam with a 2 foot by 3 foot gate.

Draw a load diagram for just the gate.

Correct response to preceding frame



Frame 13-41

Note

This unit has concerned itself with the rather simple case of pressure which varies in only one direction and acts on a plane of uniform breadth. The situation can become much more complex. (Consider, for example, fluid pressure on a circular gate.) Such problems can be solved by extension of the principles of this unit into three dimensional integrals. Fortunately the need to do so seldom arises.

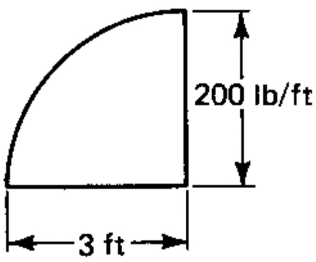
Go to the next frame.

Correct response to preceding frame

2810 pounds acting 1.40 feet above bottom.

Frame 13-42

A Sort of Special Case



A problem seen occasionally is that of a load diagram shaped like a quarter circle. (These don't occur often in the world of nature, but some exam writers dote on them.)

Everyone knows the area of a quarter circle is $\frac{\pi r^2}{4}$.

In order to find the force represented in the figure what would you square?

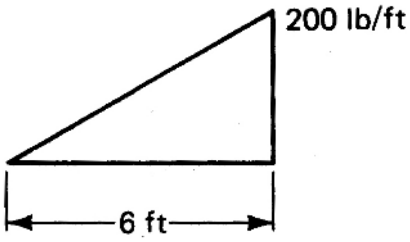
- [ ] 3 ft
- [ ] 200 lb-ft
- [ ] neither

Correct response to preceding frame

Neither. You must take

$$F = \frac{\pi (3) \ (200)}{4}$$

since the two radii are to different scales. This is similar to taking F = 1/2 (6) (200) for the loading below.



Think about it.

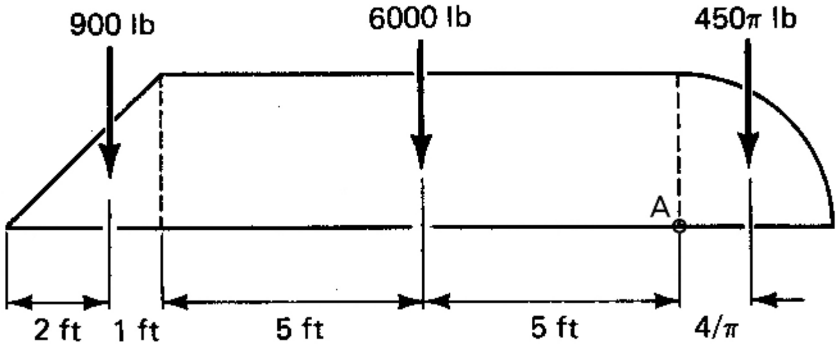
Frame 13-43

Center of Pressure

Do problem 13-4 in your notebook.

Correct response to preceding frame

x = -4.58 feet from A



$$\sum \bar{F} = -8313.7\bar{j} \text{ lb}$$

$$\sum \bar{M}_A = (-11\bar{i}) \times (-900\bar{j}) + (-5\bar{i}) \times (-6000\bar{j}) + (4/\pi\bar{i}) \times (-450\,\pi\,\bar{j})$$

$$= (9900 + 30000 - 1800)\bar{k} = 38100\bar{k}$$

$$(x\,\bar{i}) \times (-8313.7\bar{j}) = 38100\bar{k}$$

$$x = -\frac{38100}{8313.7}$$

Frame 13-44

Summary

In this unit you have used your knowledge of centroids to locate and use centers of pressure. As you begin to study real bodies and real loads you will find that you use this information very frequently, so frequently that you may even be convinced that the location of the centroids of areas and volumes is a useful skill after all.