Introduction to Statics

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Unit 12
Centroids

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Unit 12
Centroids

Frame 12-1

Introduction

This unit will help you build on what you have just learned about first moments to learn the very important skill of locating centroids.*

First it will deal with the centroids of simple geometric shapes. Then it will consider composite areas made up of such shapes.

As you progress in the study of mechanics you will find that you must locate many centroids quickly and accurately. In learning to do so you need little theory, but a great deal of practice is required.

Go to the next frame.

*If you have skipped Unit 11 do not be alarmed by the occasional calculus frame in this unit. Simply note the answer to such a frame, learn it as a given fact, and go on. It won't happen often.

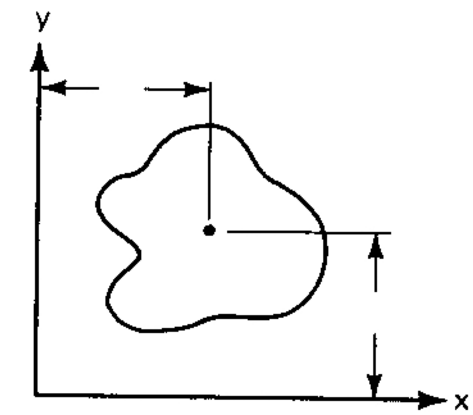
Correct response to preceding frame

No Response

Frame 12-2

Definition

The centroid of an area is the point at which all the area could be concentrated without changing its first moment about any axis.



The "amoeba" shown at the left has an area of 3 cm^2.

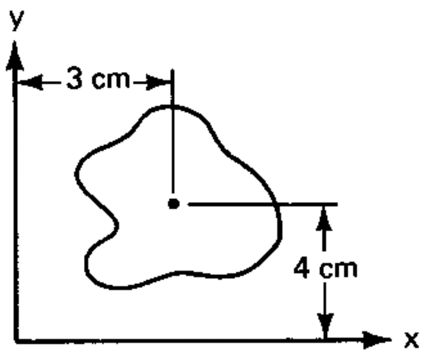
If Q_x = 12 cm^3 and Q_y = 9 cm^3, we can use the definition above to locate the centroid. To find the vertical coordinate

$Q_x = A\,y$
$12 = 3\,y$
so $y = 4$

Determine the horizontal coordinate of the centroid and supply the missing dimensions on the sketch.

x = ____________________________

Correct response to preceding frame

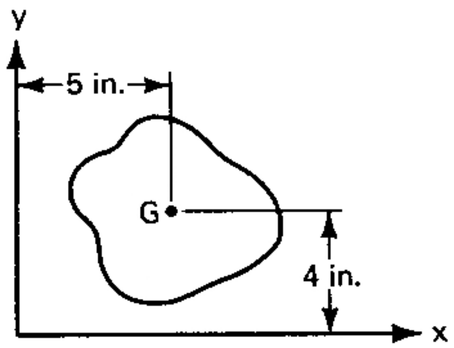


An area of 3 cm^2 "concentrated" at the dot would have a $Q_x = 3\ (4)$ and a $Q_y = 3\ (3)$.

Frame 12-3

Definition

The distance from an axis to the centroid is called "the centroidal distance."



In the figure point G represents the centroid.

The centroidal distance from the y -axis is

________________ in.

It represents the ____ coordinate of the centroid in the coordinate system shown.

Correct response to preceding frame

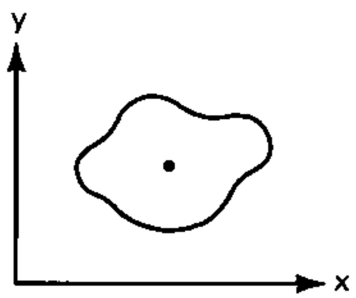
5 in. is the ***x*** -coordinate of the centroid.

Frame 12-4

Nomenclature

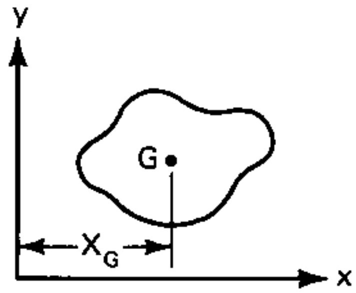
The centroidal distances found in this unit will be designated x_G and y_G. The point representing the centroid will be labelled G.

x_G is the _____ coordinate of the centroid and is the distance from the ____ axis.



Show G and x_G on the sketch.

Correct response to preceding frame

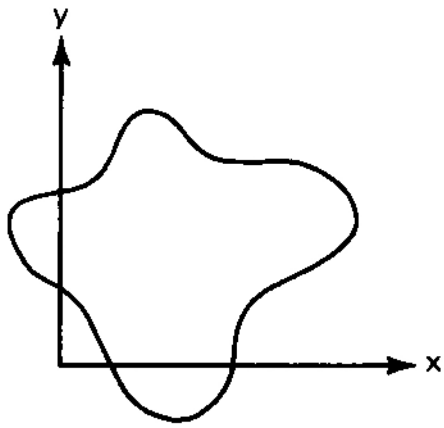


x_G is the x-coordinate and is the distance from the y-axis.

Frame 12-5

Computing Centroidal Distances

The distance from the centroid of a given area to a specified axis may be found by dividing the first moment of the area about the axis by the area.



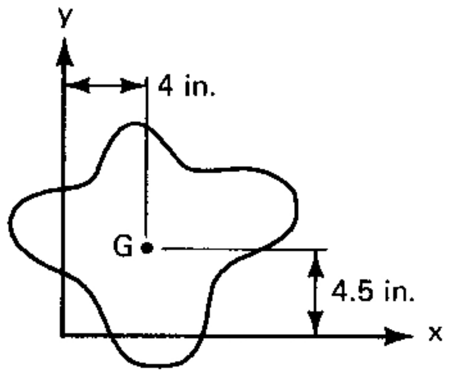
For the area shown $A = 4$ in^2.
$Q_y = 16$ in^3 and $Q_x = 18$ in^3.

Find x_G and y_G and show G by dimensioning the drawing.

$x_G =$ ______________________

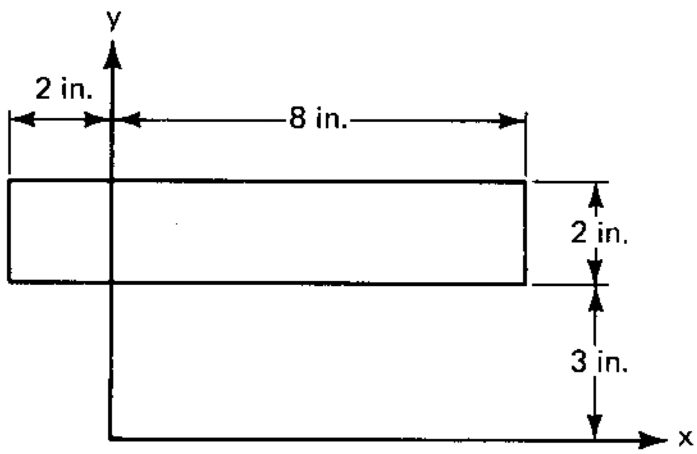
$y_G =$ ______________________

Correct response to preceding frame



Frame 12-6

Computing Centroidal Distances



The first moments of the rectangle are $Q_x = 80$ in^3 and $Q_y = 60$ in^3.

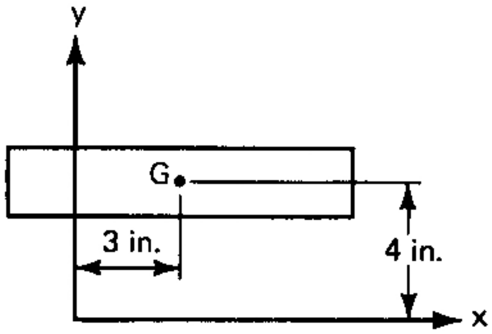
x_G = ________________

y_G = ________________

Locate G on the figure.

Correct response to preceding frame

x_G = 3 in., y_G = 4 in.



Frame 12-7

Transition

By now you should have the general idea that the centroid is a point in the middle of the area and that you have to be able to find its co-ordinates.

In the next few frames we will briefly derive formulas for the centroidal coordinates for three simple shapes.

Rotate this paper rectangle about the axis along its left edge and read the next frame. (i.e. Turn the page!)

Correct response to preceding frame

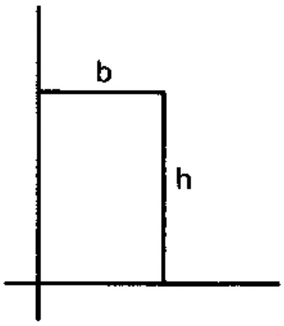
No Response

Frame 12-8

Centroid of a rectangle

The coordinate of the centroid of an area may be found by dividing the first moment of the area by the area thus

$$y_G = \frac{Qx}{A}$$



Set up an integral and find Q_x.

Use it to find y_G.

y_G = ____________________

What would you expect for xG?

x_G = ____________________

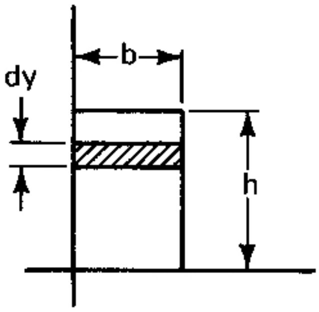
Correct response to preceding frame

$$Q_x = \int_0^h bydy = \frac{bh^2}{2}$$

$$y_G = \frac{h}{2}$$

$$x_G = \frac{b}{2}$$

Solution:

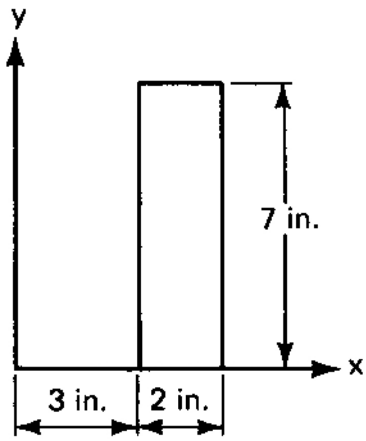


$$Q_x = \int_0^h ydA = \int_0^h ybdy$$

$$y_G = \frac{Q_x}{A} = \frac{\frac{bh^2}{2}}{bh} = \frac{h}{2}$$

Frame 12-9

Centroid of a Rectangle

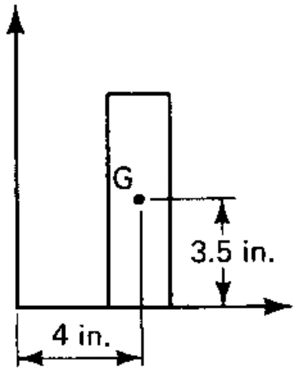


Locate the centroid of the rectangle shown without integrating.

Correct response to preceding frame

$$y_G = \frac{h}{2} = \frac{7}{2}$$

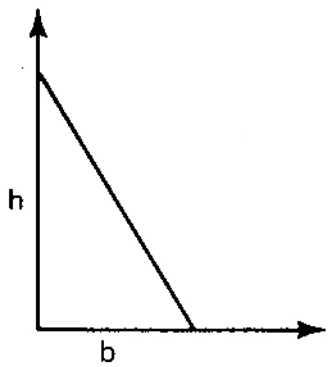
$$x_G = \frac{b}{2} + 3'' = \frac{2}{2} + 3''$$



Frame 12-10

Centroid of a right triangle

In the preceding unit you found the following first moments by integration. Use them to locate the centroid of the triangle.



$$Q_x = \frac{bh^2}{6}$$

$$Q_y = \frac{b^2h}{6}$$

x_G = __________

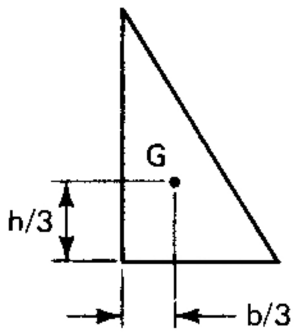
y_G = __________

Show your results on the figure.

Correct response to preceding frame

$x_G = \frac{b}{3}$

$y_G = \frac{h}{3}$



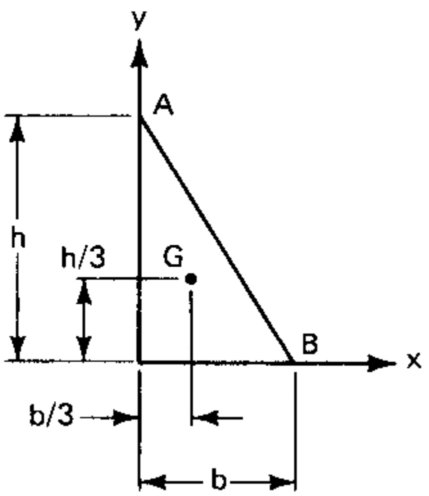
Solution:

$$x_G = \frac{Q_y}{A} = \frac{\frac{b^2h}{6}}{\frac{bh}{2}}$$

$$y_G = \frac{Q_x}{A} = \frac{\frac{bh^2}{6}}{\frac{bh}{2}}$$

Frame 12-11

Centroid of a right triangle



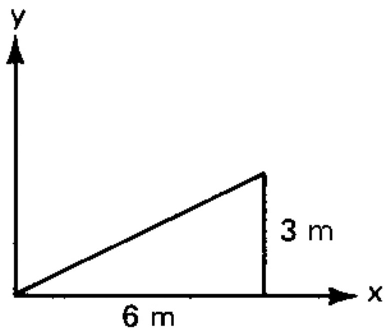
The centroid of a right triangle is shown as measured from the bases. What is its vertical distance from apex A?

Correct response to preceding frame

$\frac{2}{3}$ h

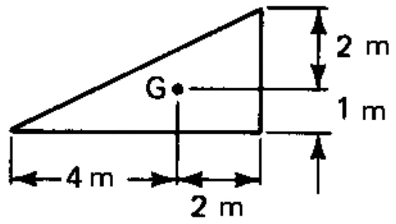
Frame 12-12

Centroid of a right triangle



Locate the centroid of the right triangle shown.

Correct response to preceding frame

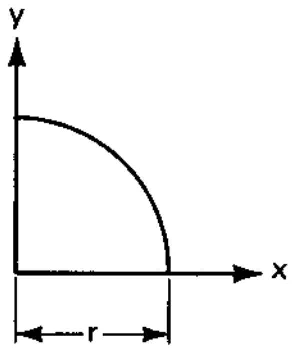


$$x_G = \frac{2}{3}\,b = 4$$

$$y_G = \frac{1}{3}\,h = 1$$

Frame 12-13

Centroid of a Quarter Circle

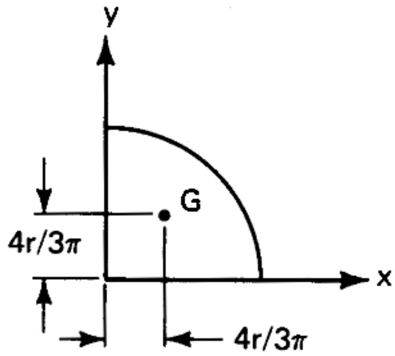


You earlier found that for a quarter circle

$$Q_x = \frac{r^3}{3}$$

Locate the centroid on the drawing giving both coordinates.

Correct response to preceding frame

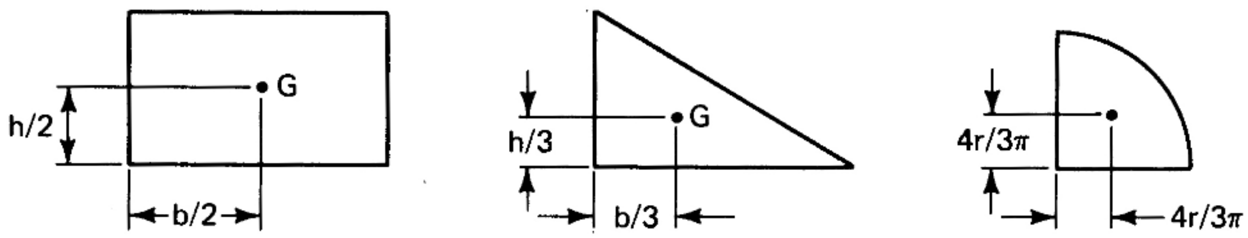


Frame 12-14

Recapitulation

You now know the locations of the centroids of three important shapes. Summarize the preceding frames in your notebook on Page 12-1.

Correct response to preceding frame



Frame 12-15

Transition

You have learned the vocabulary and notation for centroids and have learned to locate the centroids of three basic areas.

Using the areas you have learned plus a couple of simple rules it is often possible to locate the centroid of a figure simply by "eyeballing" its geometry.

The next several frames will be devoted to showing you these "tricks of the trade."

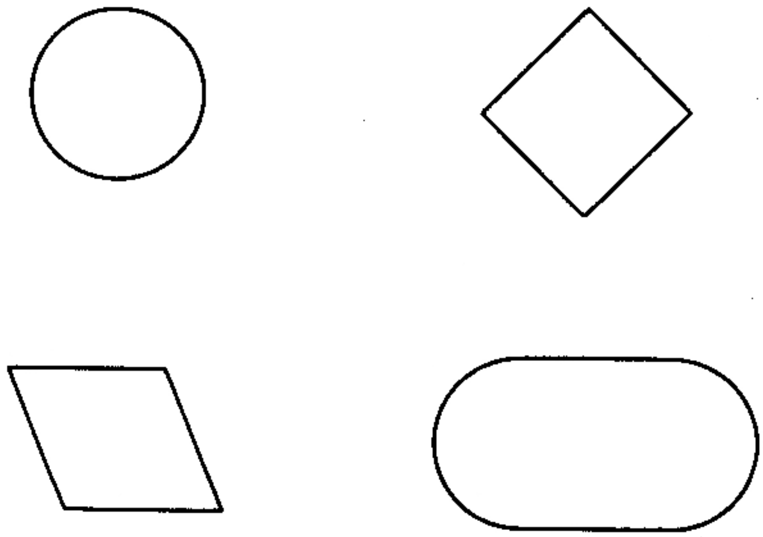
Go to the next frame.

Correct response to preceding frame No Response

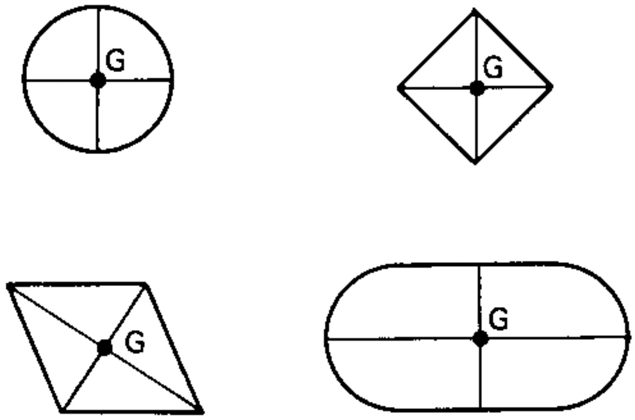
Frame 12-16

Centroids by Symmetry

The centroid of any symmetrical area will fall on every axis of symmetry. Draw two axes of symmetry for each of the areas shown below to locate their centroids.



Correct response to preceding frame



Frame 12-17

Centroids by Symmetry

When an area has two or more axes of symmetry the centroid of the area will lie

____ .

Correct response to preceding frame

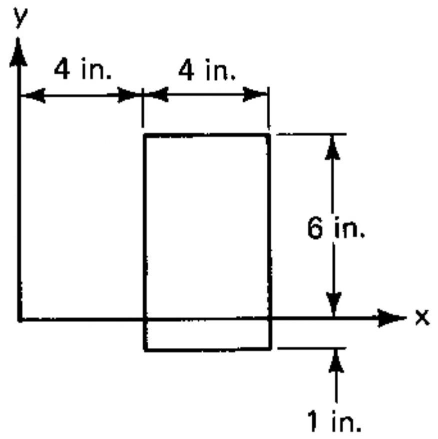
at the intersection of the axes of symmetry. (or equivalent response)

Frame 12-18

Centroids by Symmetry

Use symmetry to find the centroids of the areas shown below.

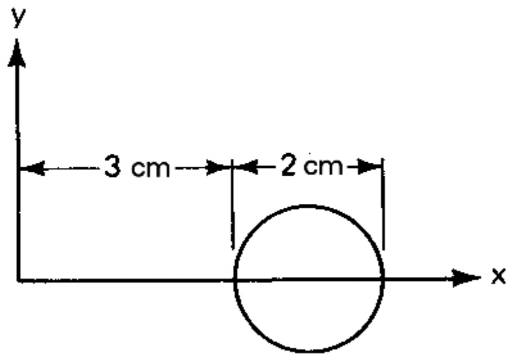
1)



x_G = ____

y_G = ____

2)



x_G = ____

y_G = ____

Correct response to preceding frame

(1.) x_G = 6.00 in. y_G = 2.50 in.
(2.) x_G = 4.00 cm y_G = 0.00 cm

Frame 12-19

Centroids by Symmetry

When an area has only one axis of symmetry only one coordinate of the centroid can be found by inspection.

Determine which of the centroidal coordinates can be found for the figures below and give their values.

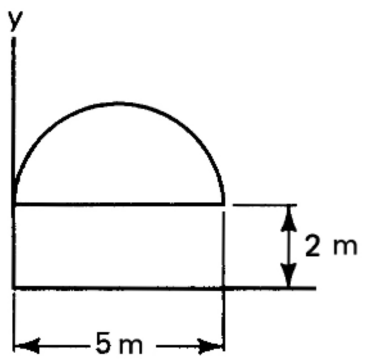


Figure (1)

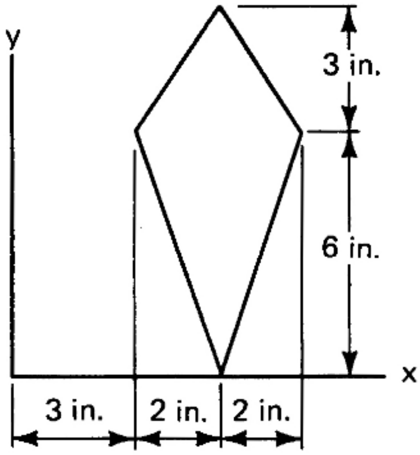


Figure (2)

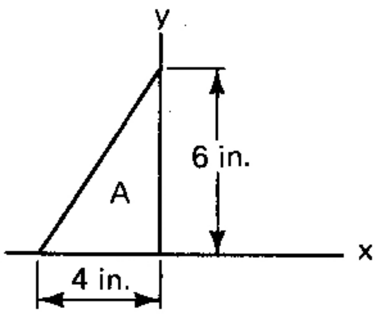
Correct response to preceding frame

fig. (1) x_G = 2.50 cm   fig. (2) x_G = 5.00 in.

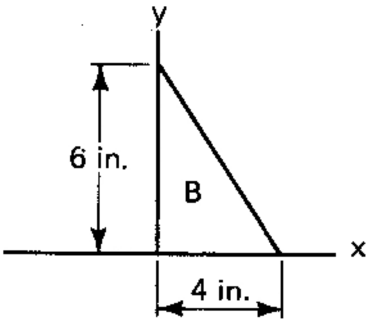
Frame 12-20

Centroids of Parts

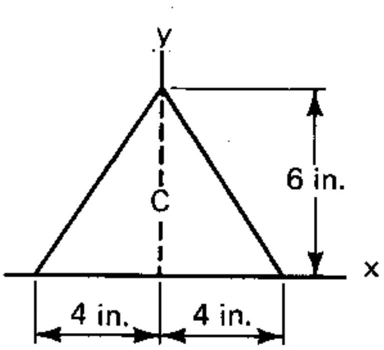
When the centroids of all parts of an area share a common centroidal coordinate, the centroid of the entire area will have that centroidal coordinate.



(1.) What is y_G for triangle A? (Refer to notebook if you need a hint.)



(2.) What is y_G for triangle B?



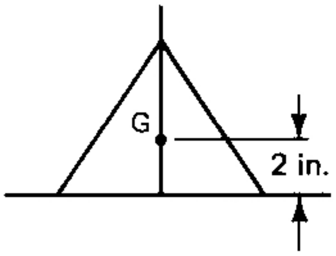
(3.) Locate the centroid of triangle C.

(4.) Give the coordinates of the centroid of triangle C.

X_G = ______________ y_G = ________________

Correct response to preceding frame

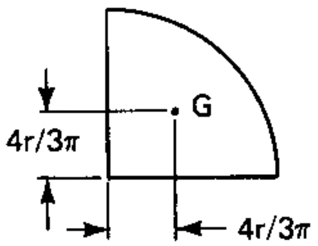
(1.) y_G = 2 in.
(2.) y_G = 2 in.
(3.)



(4.) x_G = 0 in. By symetry y_G = 2 in.

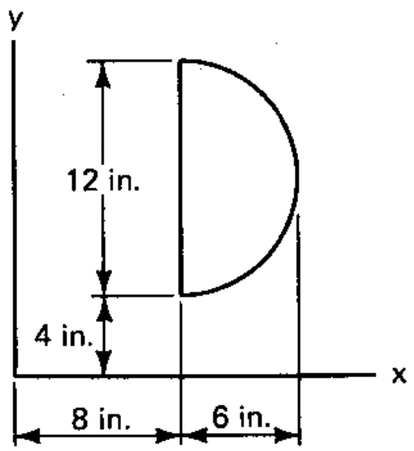
Frame 12-21

Centroids from Parts



Earlier you learned to locate the centroid of a quarter circle as shown.

Now locate the centroid of the semicircle shown below.



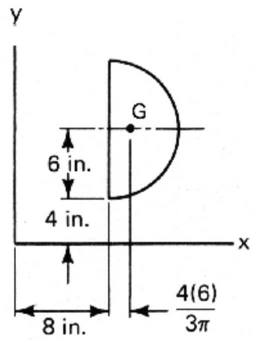
X_G = ______________

y_G = ______________

Correct response to preceding frame

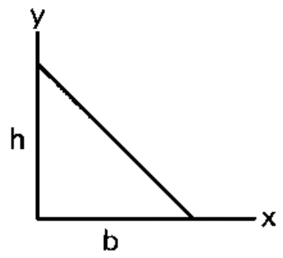
$$x_G = 8 + \frac{8}{\pi} = 10.6''$$

$$y_G = 10''$$



Frame 12-22

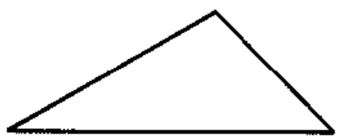
Centroids from Parts



You know that for the right triangle shown

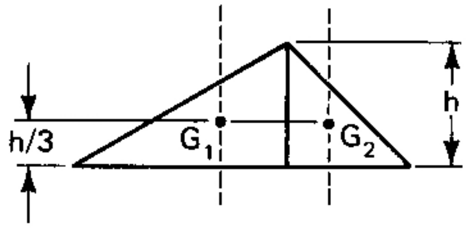
$$x_G = \frac{b}{3} \text{ and } y_G = \frac{h}{3}$$

Use this information to find one coordinate of the centroid of the scalene triangle below. (You cannot find the other as yet.) Name the dimensions as you need to.



Correct response to preceding frame

$y_G = \frac{h}{3}$



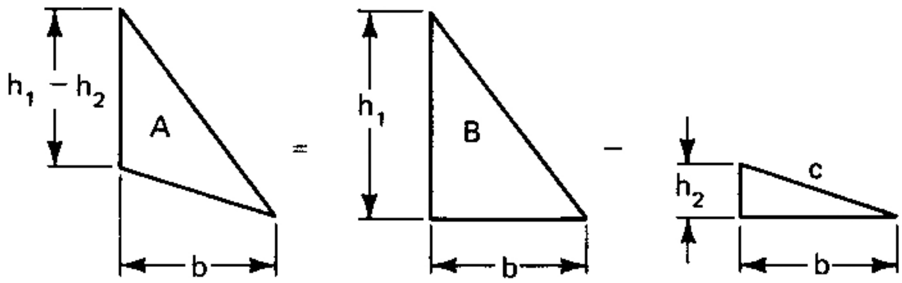
Solution:
Divide the triangle into two right triangles. Locate their centroids, both at one-third the altitude and reason that the centroid of the entire triangle lies one-third the altitude above the base.

Frame 12-23

Centroids from Parts

Consider the scalene triangle below as being the difference of two right triangles.

Use what you know about right triangles to find one coordinate of the centroid of triangle A.



(Triangle A is formed by cutting triangle C away from triangle B.)

Correct response to preceding frame

For triangle A

$$x_G = \frac{b}{3}$$

Frame 12-24

Centroid of a triangle

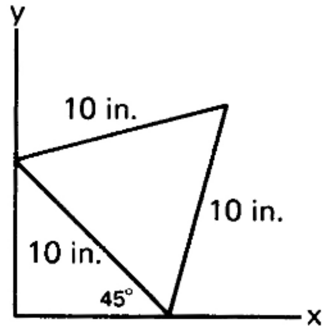
The centroid of any triangle is located ________________ of the __________ distance from any side to the opposite apex.

Correct response to preceding frame

One-third of the ***perpendicular*** distance

Frame 12-25

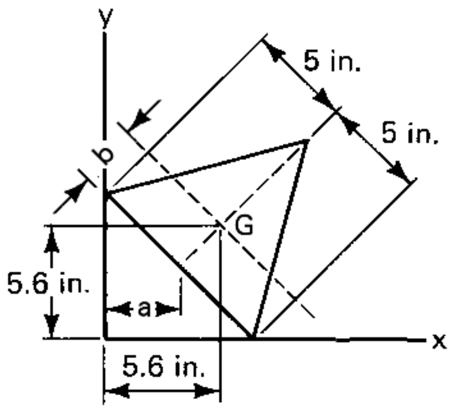
Centroids



Use what you have learned about triangles, symmetry, and plane trig to find the centroid of the equilateral triangle. A recommended first step is to locate the centroid on the drawing.

Correct response to preceding frame

$x_G = y_G = 5.6"$ **Solution:**



$$x_G = a + b \cos 45°$$

$$a = 5 \cos 45° = 5(.707)$$

$$b = \frac{1}{3} 10 \sin 60°$$

$$x_G = 5(.707) + \frac{8.66}{3}(.707)$$

Frame 12-26

Transition

You should now be pretty proficient at locating centroids by inspection. Don't neglect the skill. Properly encouraged, it can save you much time and computation.

Even skillful inspection, however, can take you only so far. The remainder of this unit might be called "What to do after the inspector gives up." and will teach you to solve problems the long, hard, foolproof way--and really it's not so hard.

Go to the next frame.

Correct response to preceding frame

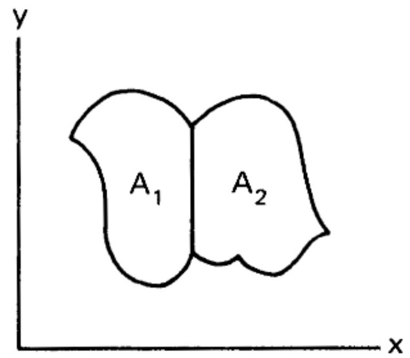
No Response

Frame 12-27

Composite Areas

The basic definition for first moments that we have been using is that the first moment of an area about a given axis is equal to the sum of the first moments of all parts of the area about the axis.

Thus, for the area shown the total first moment is:



$Q_{x\,total} = Q_{x1} + Q_{x2}$

and the total area is

$A_{total} = A_1 + A_{x2}$

The y-coordinate of the centroid is thus:

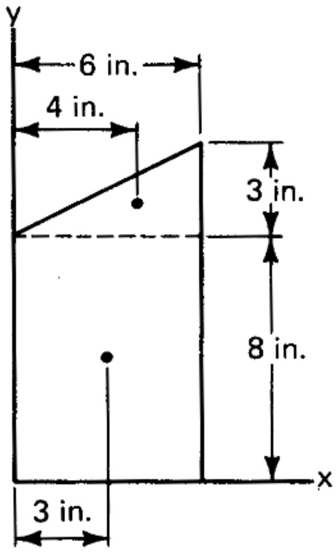
$$y_G = \frac{Q_{x\ total}}{A_{\ total}} = \_\_\_\_\_\_\_\_$$

Correct response to preceding frame

$$\frac{Q_{x1} + Q_{x2}}{A_1 + A_2}$$

Frame 12-28

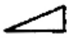
Composite Areas



The figure shows a trapezoid broken into a triangle and a rectangle. The table below shows a systematic way of finding the centroid of such a composite area. It is completed for x_G only.

Finish it to find y_G.

(A_P means "Area of the part" etc.)

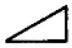
Shape	A_p	x_{Gp}	$A_p x_{Gp}$	y_{Gp}	$A_p y_{Gp}$
	9	4	36		
	48	3	144		
Totals:	57 $= A$		180 $= Q_y$		$= Q_x$

$$x_G = \frac{Q_y}{A} = \frac{180}{57} = 3.16 \text{ in.}$$

$y_G =$ ________

Correct response to preceding frame

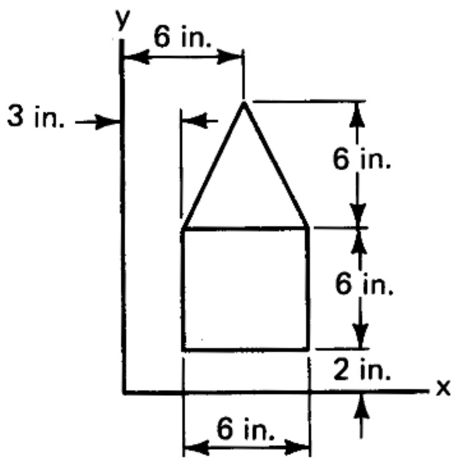
$y_G = 4.8"$

Shape	A_p	y_{Gp}	$A_p y_{Gp}$
	9	9	81
	48	4	192
Total	57 $= A$	X	273 $= Q_x$

$$y_G = \frac{Q_x}{A} = \frac{273}{57}$$

Frame 12-29

Composite Areas



Use the table below to find y_G for the figure shown.

Shape	A_p	y_{Gp}	$A_p y_{Gp}$
Totals:		X	

y_G = ____________

Now find x_G

x_G = ____________

Correct response to preceding frame

y_G = 6.67 in.

x_G = 6.00 in.

Shape	A_p	y_{Gp}	$A_p y_{Gp}$
△	18	10	180
□	36	5	180
Totals	54		360

$A = 54 \qquad Q_x = 360$

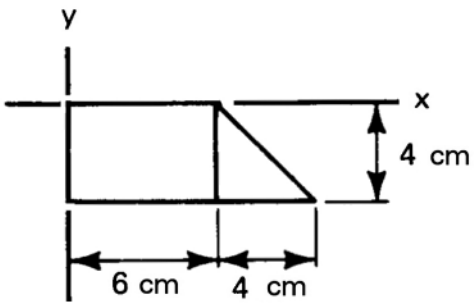
$$y_G = \frac{Q_x}{A} = \frac{360}{54} = 6.67 \text{ in.}$$

x_G = 6 in. by inspection

Frame 12-30

Signs

Sometimes the centroid of part of an area is so located as to have a negative coordinate. In such a case the first moment will be negative and the centroid of the entire area may have a negative coordinate. Complete the following computation and locate the centroid on the sketch.

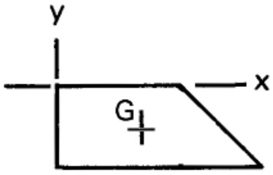


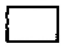
Shape	A_p	x_{Gp}	$A_p x_{Gp}$	y_{Gp}	$A_p y_{Gp}$
▭	24	3	72	−2	−48

Correct response to preceding frame

x_G = 4.08 cm

y_G = −2.16 cm

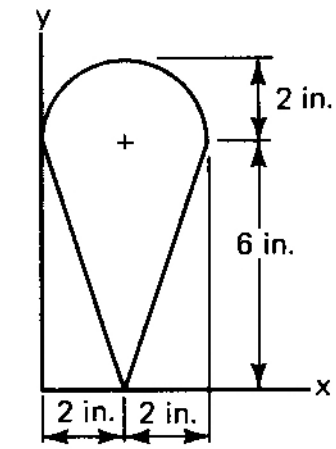


Shape	A_p	x_p	$A_p x_p$	y_p	$A_p y_p$
	24	3	72	−2	−48
	8	7.33	58.67	$-\frac{8}{3}$	−21.3
Totals	32		130.67 = Q_y		−69.3 = Q_x

Frame 12-31

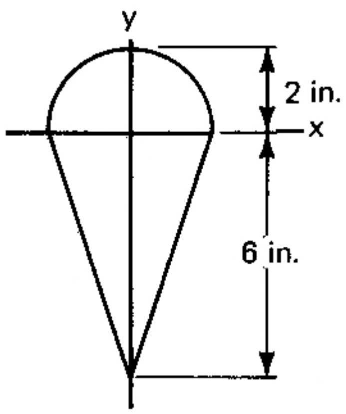
Composite Areas

It is sometimes convenient to work with reference axes which intersect the area, even though this results in some negative coordinates. Compare the two partial solutions below and complete the one that seems easier.



x_G = 2 by inspection

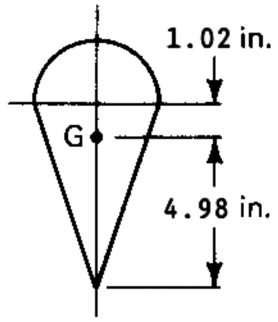
Shape	A_p	y_{Gp}	$A_p y_{Gp}$
	2π	$6 + \frac{8}{3\pi}$	
	12	4	



x_G = 0 by inspection

Shape	A_p	y_{Gp}	$A_p y_{Gp}$
	2π	$\frac{8}{3\pi}$	
	12	−2	

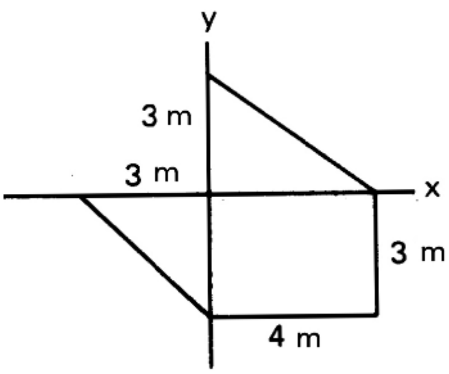
Correct response to preceding frame



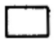
Shape	A_p	y_{Gp}	$A_p y_{Gp}$
semicircle	2π	$\frac{8}{3\pi}$	$\frac{16}{3}$
triangle	12	−2	−24
Totals	18.28		−18.67

Frame 12-32

Composite Areas



Using the indicated axes as references, find the coordinates of the centroid of the area shown.

Part	A_p	x_{Gp}	$A_p x_{Gp}$	y_{Gp}	$A_p y_{Gp}$
	6				
	4.5				
	12				

A_{total} = ____________

Q_x = ____________ y_G = ____________

Q_y = ____________ x_G = ____________

Correct response to preceding frame

$x_G = 1.22$ m

$y_G = -.725$ m

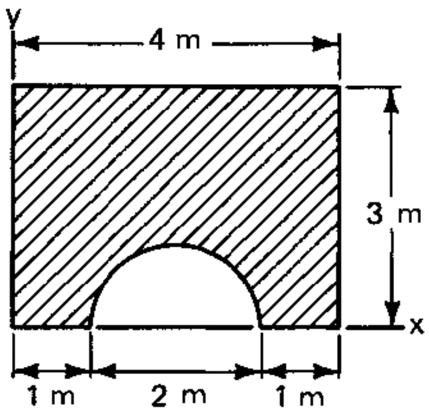
Part	A_p	x_{Gp}	$A_p x_{Gp}$	y_{Gp}	$A_p y_{Gp}$
◺	6	$\frac{4}{3}$	8	1	6
◹	4.5	-1	-4.5	-1	-4.5
□	12	2	24	-1.5	-18

$A_{total} = 22.5$ $Q_y = 27.5$ $Q_x = -16.5$

Frame 12-33

Negative Areas

Some areas have holes in them. Handle a hole as a negative area. Complete the problem below. Find the centroid of the cross-hatched area.



Part	A_p	x_{Gp}	$A_p x_{Gp}$	y_{Gp}	$A_p y_{Gp}$
□	12	2	24		
⌓	$\frac{-\pi}{2}$	2	-3.14		

$A_{total} =$ ________

$Q_y = 20.86$ $Q_x =$ ________

$x_G = 2$ m $y_G =$ ________

Correct response to preceding frame

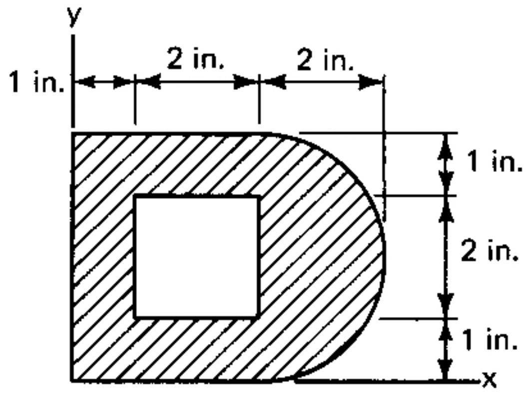
$Q_x = 17.33$

$y_G = 1.66$ m

A_p	y_{Gp}	$A_p y_{Gp}$
12	1.5	18
$-\frac{\pi}{2}$	$\frac{4\,(1)}{3\pi}$	-.67

Frame 12-34

Negative Areas



Find the centroid of the area shown.

(Practice drawing your own table from memory.)

Correct response to preceding frame

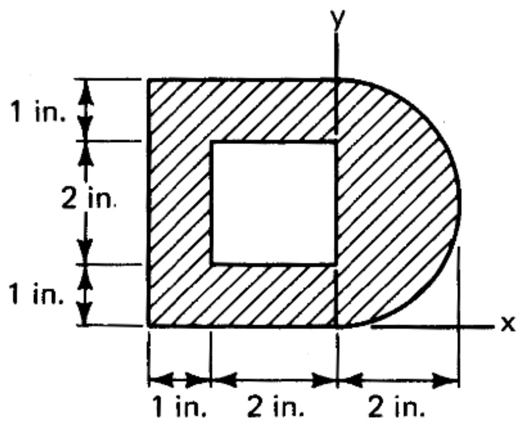
x_G = 2.39 in.

y_G = 2.00 in. by symmetry

Part	A_p	x_{Gp}	$A_p x_{Gp}$
□	12	$\frac{3}{2}$	18
D	6.28	3.85	24.2
⊟	−4	2	−8
Totals	14.28		34.2

Frame 12-35

Negative Areas

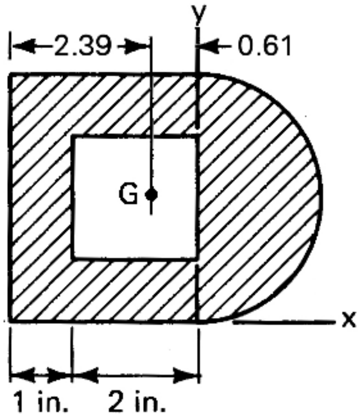


Same area as last frame. Different reference axes. Find x_G using the axes shown. Make sure you get the same location (not coordinate) you got before.

Correct response to preceding frame

$x_G = -.610$ in.

Part	A_p	x_{Gp}	$A_p x_{Gp}$
□	12	$-\frac{3}{2}$	−18
D	6.28	$\frac{8}{3\pi}$	5.33
⊟	−4	− 1	+4
Totals	14.28	X	−8.67



Frame 12-36

Selection of Reference Axes

There is no one right way to select reference axes. One selection may give you all positive numbers, but bad arithmetic. Another may clean up the arithmetic at the cost of negative signs. Each time you work a problem you must make a choice. The thing to remember is: ***choose*** and get to work. Think about the advantages and disadvantages of each axis location before you begin work.

The centroid isn’t "just math," it’s a physical location. I usually provide the answer on a sketch so you can quickly check your work. I would suggest that you create a sketch with your work so that you can check whether your looks right, and to make it easier for a grader to do so.

This last set of problems will require you to select your own reference axes, but whatever your choice, the actual location of the centroid should agree with the answer given.

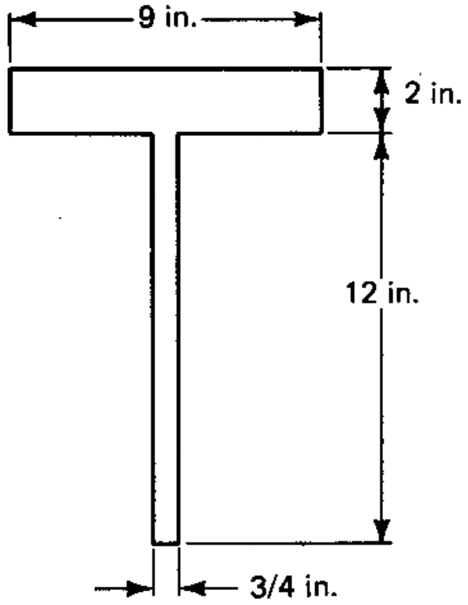
Go to the next frame.

Correct response to preceding frame

No response

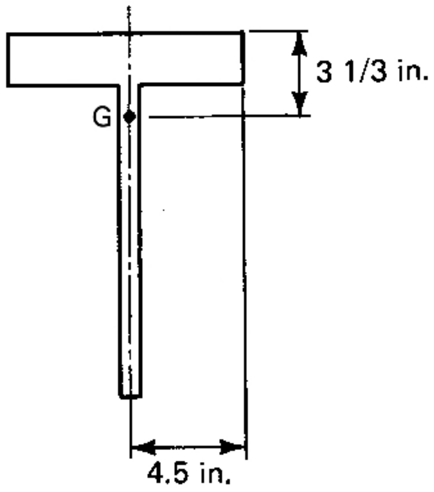
Frame 12-37

Centroid of a Composite Area



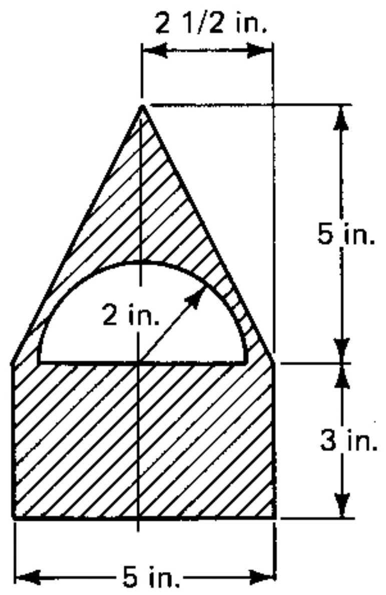
Find the centroid of the area shown and locate it on a sketch.

Correct response to preceding frame



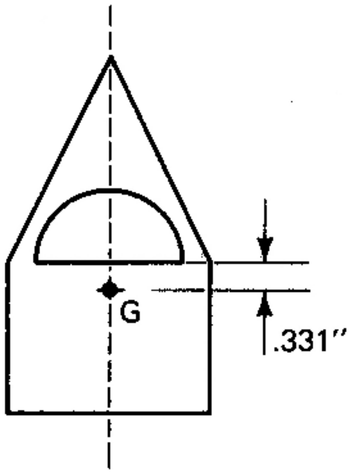
Frame 12-38

Centroid of a Composite Area



Locate on a sketch the centroid of the area shown.

Correct response to preceding frame

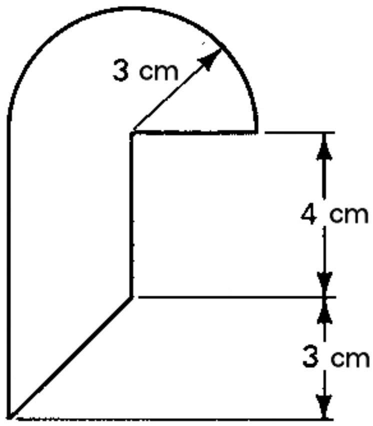


Part	A_p	y_{Gp}	$A_p x_{Gp}$
⌓	-6.28	$\frac{8}{3\pi}$	-5.33
□	15	$-\frac{3}{2}$	-22.5
△	12.5	$\frac{5}{3}$	20.8
Totals	21.22		-7.03

$y_G = -.331$

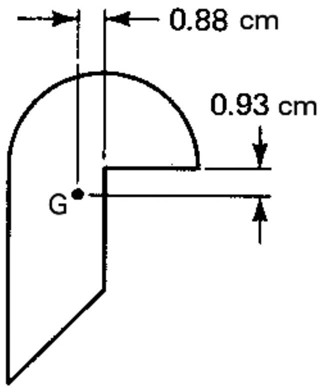
Frame 12-39

Centroid of a Composite Area



Find the centroid of the area shown and locate it on a sketch.

Correct response to preceding frame

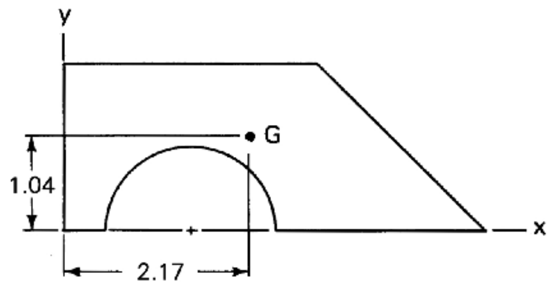


Frame 12-40

Summary

You will be happy to know that the end is near. Fill in Page 12-2 in the notebook and read the closure, and you're done!

Correct response to preceding frame



Shape	A_p	x_{Gp}	$A_p x_{Gp}$	y_{Gp}	$A_p y_{Gp}$
▭	6	$1\frac{1}{2}$	9.00	1	6.00
◺	2	$3\frac{2}{3}$	7.34	1	2.00
⌓	$-\frac{\pi}{2}$	$1\frac{1}{2}$	−2.36	$\frac{4}{3\pi}$	−0.67
Totals	6.43		13.98		6.67

$$y_G = \frac{6.67}{6.43} = 1.04$$

$$x_G = \frac{13.98}{6.43} = 2.17$$

Frame 12-41

Closure

If you have been moderately successful in calculating the centroids in this unit you should be able to locate the centroid of any area. If you can break it down into simple geometric shapes you can use composite methods directly. If some parts require integration, you must do that first then use composite methods. In mechanics of materials you will learn to combine areas of cross-sections for composite areas made up of "steel beams" and other shapes you will look up in tables.

Given sufficient time and patience, you will always win through to a right answer eventually Well, almost always.