Introduction to Statics

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Unit 27
Belt Friction

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Unit 27
Belt Friction

Frame 27-1

Introduction

The material covered in this unit is generally referred to as "friction on flat belts".

You will see how the principles of statics which you have learned can be combined with the techniques of calculus to derive formulas for systems which involve distributed forces, and you will learn how to work problems which involve friction between flat belts or ropes and cylindrical pulleys or drums.

Turn to the next frame and begin.

Correct response to preceding frame

No response

Frame 27-2

Derivation

The first section of this unit involves reading a derivation. Most of the derivation is in your notebook but you will be asked to respond to several questions in the unit and to fill in several key steps in the notebook.

Open your notebook to page 27-1 and read until you come to the first 

. Then turn to the next frame.

Correct response to preceding frame

No response

Frame 27-3

Discussion

From the information in the first paragraph determine the direction in which belt slippage impends.

- ☐ The belt moves from a toward b
- ☐ The belt moves from b toward a

Correct response to preceding frame

from a toward b since T_L is the larger

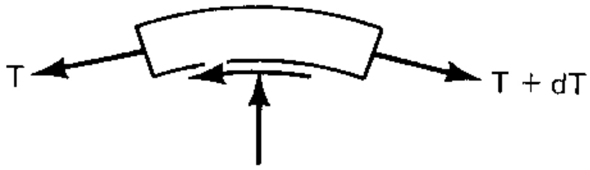
Frame 27-4

Discussion of Derivation

The sketch of the element of the belt is not a free body diagram. In addition to the tension T on the left end and the tension T + dT on the right end, the contact between the belt and the drum will result in a normal force and a friction force on the element.

Complete the diagram by showing the friction force in the proper direction to put the element in equilibrium.

Correct response to preceding frame



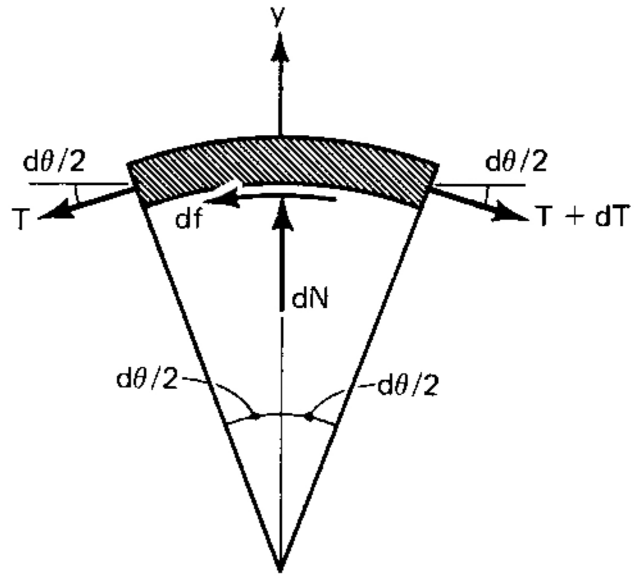
Frame 27-5

Discussion of Derivation

The tensions T and T + dT are perpendicular to the radial lines which form the ends of the element, hence the angle between the tensile forces and x axis is $\frac{d\theta}{2}$

Label the forces df and dN and the angles between the tensile forces and the x axis on the element on notebook page 27-1.

Correct response to preceding frame



Frame 27-6

Discussion of Derivation

Write equilibrium equations for the x and y direction in terms of T, T + dT, dN, and df, on the element on the spaces in the notebook just below the first STOP.

(Work them out on this frame first if you wish.)

Correct response to preceding frame

$$(T + dT)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} - df = 0$$

$$dN - T\sin\frac{d\theta}{2} - (T + dT)\sin\frac{d\theta}{2} = 0$$

Frame 27-7

Discussion of Derivation

Use the small angle approximations in the notebook and reduce your equations from the preceding frame to a simpler form.

Fill in the appropriate blanks in your notebook and continue to the next STOP sign.

Correct response to preceding frame

df = dT
dN = T dθ

Frame 27-8

Discussion of Derivation

Our next step is to separate the variables in the equation, that is, to rearrange the equation like this

$$\frac{dT}{T} = \mu d\theta$$

Now we can integrate the equation from end a to end b of the contact between the rope and the drum.

First let's establish the limits of integration. Look at the first picture in this derivation.

At a

T = ______________ and θ = _____________

At b

T = ______________ and θ = _____________

Correct response to preceding frame

At a $T = T_L$ and $\theta = 0$
At b $T = T_S$ and $\theta = \beta$

Frame 27-9

Discussion of Derivation

Using the information you figured out in the preceding frame, put the proper limits on the integral in your notebook, then turn to frame 27-10.

Correct response to preceding frame

$$\int_{T_S}^{T_L} \frac{dT}{T} = \mu \int_{0}^{\beta} d\theta$$

Frame 27-10

Discussion of Derivation

We have now written the equation as a definite integral.

$$\int_{T_S}^{T_L} \frac{dT}{T} = \mu \int_{0}^{\beta} d\theta$$

Evaluate the integrals. (You may consult your calculus book, if necessary.)

$$\mu \int_{0}^{\beta} d\theta = \underline{\hspace{6cm}}$$

$$\int_{T_S}^{T_L} \frac{dT}{T} = \underline{\hspace{6cm}}$$

Correct response to preceding frame

$$\mu \int_0^{\beta} d\theta = \mu\beta$$

$$\int_{T_S}^{T_L} \frac{dT}{T} = \ln T_L - \ln T_S = \ln \left(\frac{T_L}{T_S}\right)$$

Frame 27-11

Discussion of Derivation

Using the results of the integration, write an equation for T_L and T_S in terms of μ and β.

____________________ = __________________

Correct response to preceding frame

$$\ln T_L - \ln T_S = \mu\beta$$

or

$$\ln \left(\frac{T_L}{T_S}\right) = \mu\beta$$

Record your results as Form I in your notebook.

Frame 27-12

Discussion of Derivation

The definition of the natural logorithm allows us to express the equation below in another form.

$$\ln \left(\frac{T_L}{T_S}\right) = \mu\beta$$

Try doing it from memory, confirm your results on the next page, then write the correct form in your notebook as Form II.

$$\frac{T_L}{T_S} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Correct response to preceding frame

$$\frac{T_L}{T_S} = e^{\mu\beta}$$

Frame 27-13

Transition

You have now completed the derivation section of this unit. The remaining portion of the unit will give you opportunity to apply the equations which have been derived to several problems involving belt and rope friction.

This is a logical place for a break.

When you're ready to "have at it" again, close your notebook and turn to the next frame.

Correct response to preceding frame

No response

Frame 27-14

Identifying Tension Values

In the belt equation

$$\frac{T_L}{T_S} = e^{\mu\beta}$$

the quantities μ and β are both positive values therefore $e^{\mu\beta}$ is always greater than one.

Hence, T_L is the (***larger , smaller***) of the two tensions, and T_S is the (***larger , smaller***) of the two tensions.

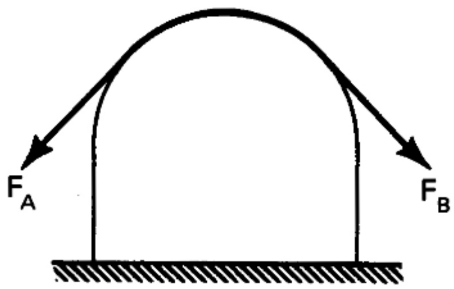
Correct response to preceding frame

T_L is the larger
T_S is the smaller

Frame 27-15

Identifying Tension Values

In this situation the rope is about to slide toward the left.



$$\frac{T_L}{T_S} = e^{\mu\beta}$$

Match F_A and F_B to the proper terms.

T_L = __________

T_S = __________

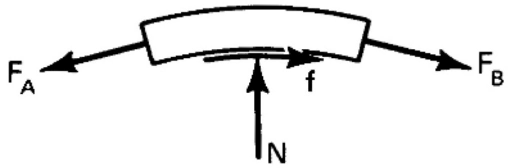
Correct response to preceding frame

$T_L = F_A$

$T_S = F_B$

Solution:

Motion impends toward the left, so our FBD is

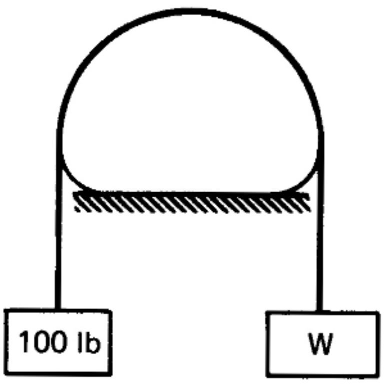


Since f is as shown, $F_A = F_B + f$ and F_A is the larger tension.

Frame 27-16

Identifying Tension Values

In the situation shown the 100 pound block has impending motion upward.



$$\frac{T_L}{T_S} = e^{\mu\beta}$$

T_L = ________

T_S = ________

Substitute these values into the flat belt equation

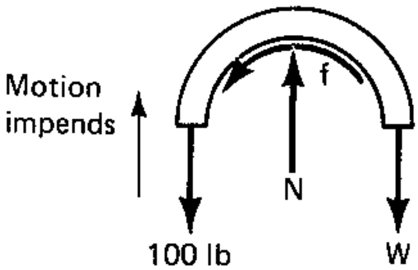
$$e^{\mu\beta} = \frac{\_\_\_}{\_\_\_\_\_\_}$$

Correct response to preceding frame

$T_L = W$

$T_S = 100$ lb

$e^{\mu\beta} = \frac{W}{100}$



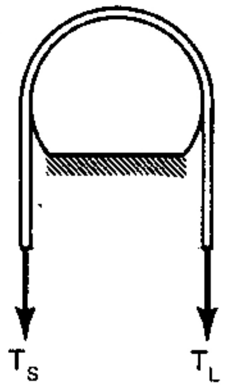
Frame 27-17

Determination of β

In belt problems, as in all problems which involve the integration or differentiation of angles, the angles must be expressed in radians.

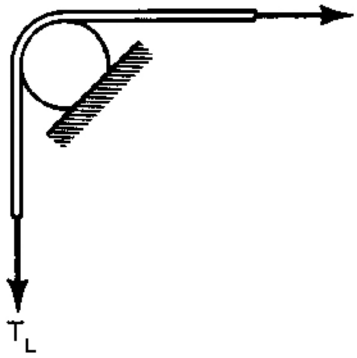
State the value of **β** for each of the situations shown.

(a)



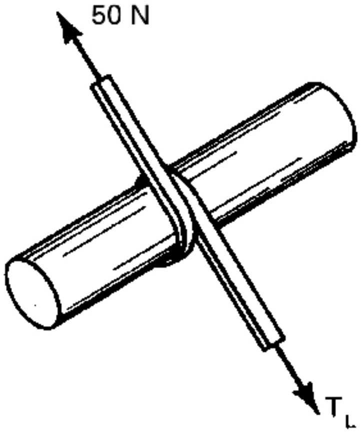
β = ____________ radians

(b)



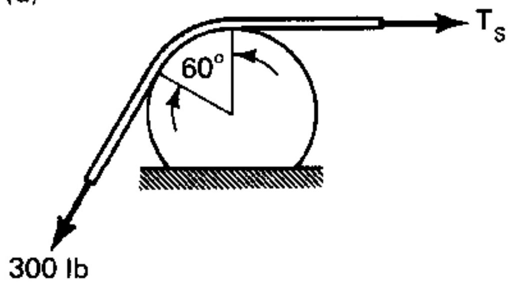
β = ____________ radians

(c)



β = ____________ radians

(d)



β = ____________ radians

Correct response to preceding frame

a) π radians

b) $\frac{\pi}{2}$ radians

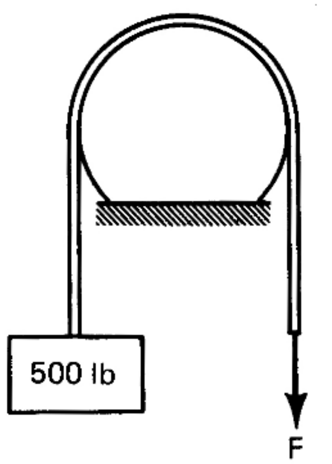
c) 2π radians

d) $\frac{\pi}{3}$ radians

Frame 27-18

Application of Flat Belt Equation

Suppose we were to find the minimum force F which is needed in order to hold up the 500 pound weight, and we were told that the coefficient of friction between the drum and the belt is 1/10.



Draw a FBD of the belt.

In the equation $\frac{T_L}{T_S} = e^{\mu\beta}$ what are the following values?

T_L = ______________

T_S = ______________

μ = ______________

β = ______________

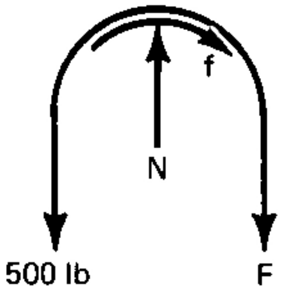
Correct response to preceding frame

T_L = 500 lb

T_S = F

μ = 0.1

β = π radians



Frame 27-19

Application of Flat Belt Equation

Use the list of values in the response to the previous frame and evaluate F using the equation

$$\frac{T_L}{T_S} = e^{\mu\beta}$$

F = ______________

Correct response to preceding frame

F = 365 lb

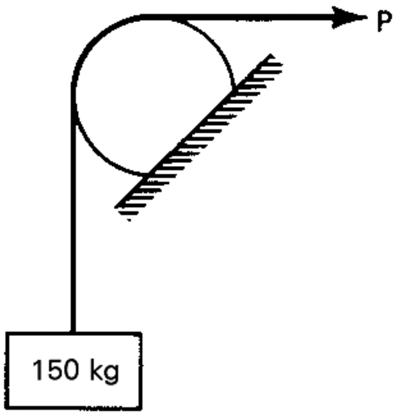
Solution:

$$F = \frac{500}{e^{\mu\beta}} = \frac{500}{e^{\pi/10}} = \frac{500}{1.37}$$

Frame 27-20

Application of Flat Belt Equation

The force P on the belt is just enough to raise the block.



What are the values of the following?

β = ____________

T_L = ____________

T_S = ____________

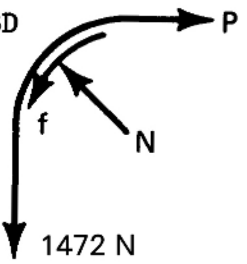
Correct response to preceding frame

$\beta = \frac{\pi}{2}$

$T_L = P$

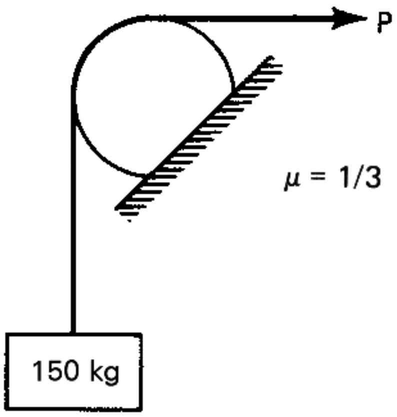
$T_S = 1472$ N

from the FBD



Frame 27-21

Application of Flat Belt Equation



Set up the belt equation and solve for P, the force needed to raise the block.

P= ____________________

Correct response to preceding frame

P = 2480 N

Solution:

$$\frac{T_L}{T_S} = e^{\mu\beta}$$

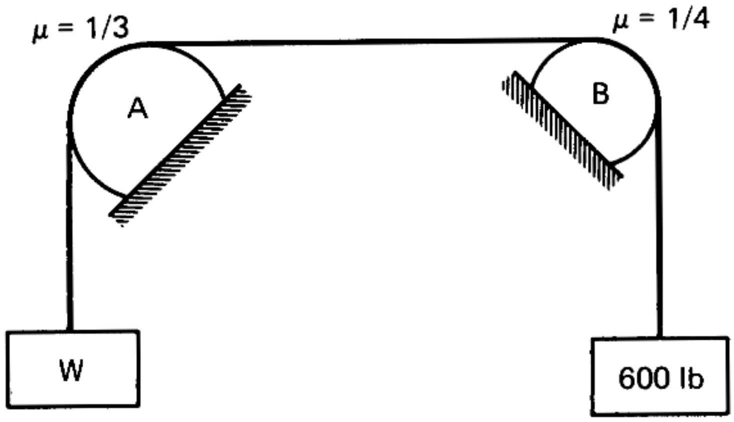
$$\frac{P}{1472} = e^{1/3(\frac{\pi}{2})}$$

$$P = 1472\ e^{\pi/6}$$

Frame 27-22

Application of Flat Belt Equation

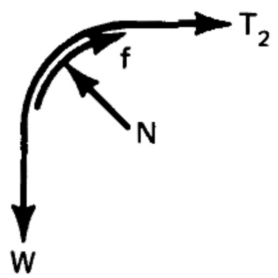
Here's a slightly more complex situation:



The unknown weight W is about to move downward.

This problem must be divided into two parts.

On cylinder A,

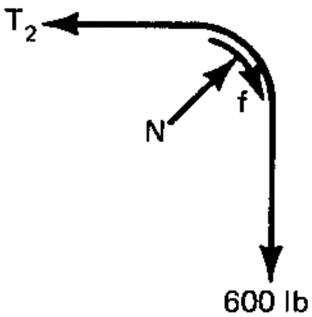


$T_{L_A} = W$ and $T_{S_A} = T_2$

so $\frac{T_L}{T_S} = \frac{W}{T_2} = e^{\mu\beta} = e^{\left[\frac{1}{3}\right]\left[\frac{\pi}{2}\right]} = e^{\frac{\pi}{6}}$

Draw a FBD of the belt as it passes around B. Identify the tensions and set up the belt equation for B.

Correct response to preceding frame



$T_{L_B} = T_2$

$T_{S_B} = 600\ \text{lb}$

$\frac{T_2}{600} = e^{\frac{\pi}{8}}$

Frame 27-23

Application of Flat Belt Equation

You now have two equations:

$\frac{W}{T_2} = e^{\frac{\pi}{6}}$ on A, $\frac{T_2}{600} = e^{\frac{\pi}{8}}$ and on B.

Solve these equations and find the value of W which causes the system to begin to move.

W = ________________

Correct response to preceding frame

W = 1500 lb

Solution:

$$\frac{W}{T_2}\left[\frac{T_2}{600}\right] = e^{\frac{\pi}{6}}\left(e^{\frac{\pi}{8}}\right)$$

$$\frac{W}{600} = e^{\pi\left[\frac{1}{6}+\frac{1}{8}\right]}$$

$$W = 600e^{7\pi/24} = 1500$$

Frame 27-24

Problem 27-1

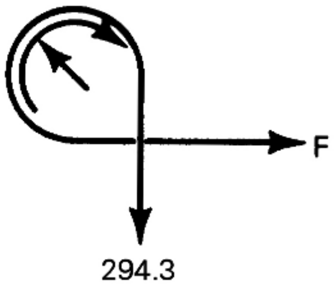
Work Problem 27-1 in your notebook.

Correct response to preceding frame

Answer to problem 27-1 in notebook

F = 956 N

Solution:



$F > 294.3$

$$\frac{F}{294.3} = e^{(0.25)\frac{3\pi}{2}}$$

Frame 27-25

Transition

At this point you should be able to work relatively simple problems which involve belt friction.

Often belt friction is part of a problem which also includes other friction surfaces or machine elements. The last part of this unit will give you some experience with these combinations.

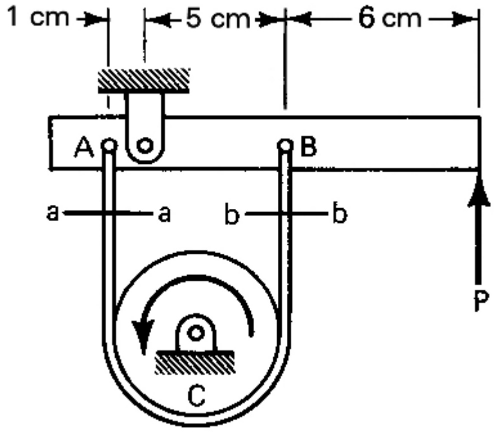
If you want a rest take it now. When you're ready to proceed, turn to the next frame.

Correct response to preceding frame

No response

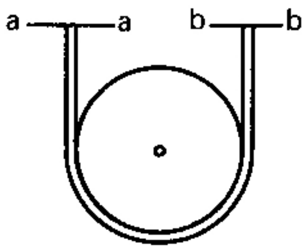
Frame 27-26

Brake



This system is called a band brake. In the course of the next few frames you will determine how large force P must be in order to keep the drum from rotating due to the 9 N-m couple C.

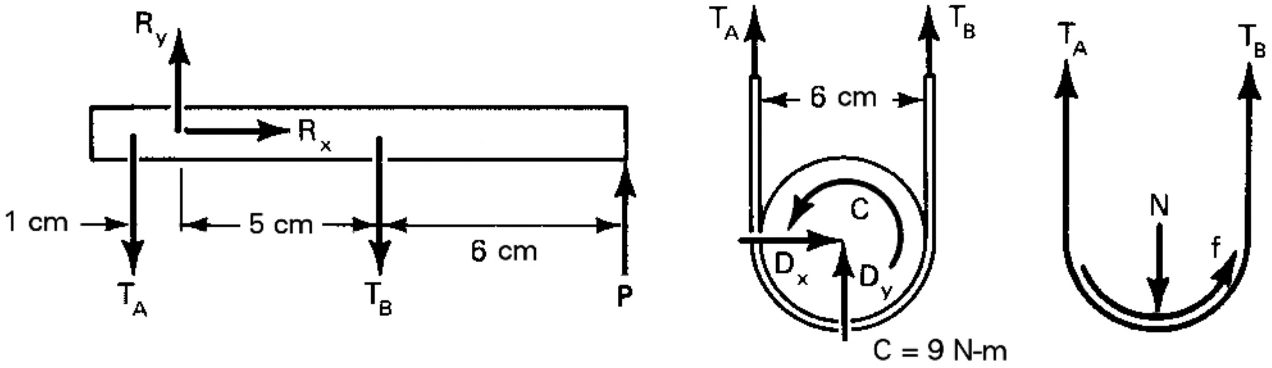
Draw a free body diagram of the beam.



Complete the free body diagram of the system consisting of the drum and the rope from a-a to b-b.

Draw a FBD of the rope.

Correct response to preceding frame



Frame 27-27

Brake

Refer to the answer to the FBDs above as necessary.

1. Write equilibrium equations for the beam.

2. Write equilibrium equations for the drum and rope system.

3. Now assume that $\mu = 0.25$, and that the rope is about to slip on the drum.

Write a friction equation.

Correct response to preceding frame

1. $R_x = 0$

   $R_y - T_A - T_B + P = 0$

   $0.01 T_A - 0.05 T_B + 0.11 P = 0$

2. $D_x = 0$

   $T_A + T_B + D_Y = 0$

   $0.03 T_B - 0.03 T_A + 9 = 0$

3. $\frac{T_A}{T_B} = e^{\pi/4}$

Frame 27-28

Brake

Select an appropriate group of equations from the response above and solve for the value of P.

P = ____________________

Correct response to preceding frame

P = 64 Newtons

Solution:

$$0.03T_B - 0.03T_A + 9 = 0$$

$$T_A - T_B = 300$$

$$\frac{T_A}{T_B} = e^{\pi/4}$$

So $T_B\,(e^{\pi/4} - 1) = 300$

$$T_B = 251$$

and $T_A = 551$

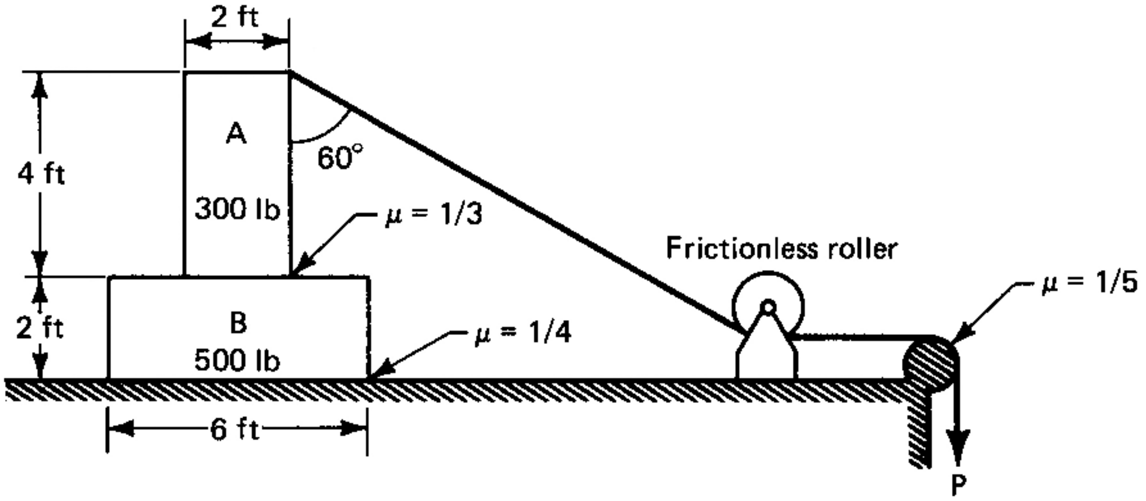
$$0.01T_A - 0.05\,T_B + 0.11P = 0$$

$$P = \frac{5T_B - T_A}{11} = \frac{704}{11} = 64$$

Frame 27-29

Alternative Motions

Problem:
Find the minimum value of P needed to move block A.



Before you begin to write equations, list the ways in which A may move.

____

____

____

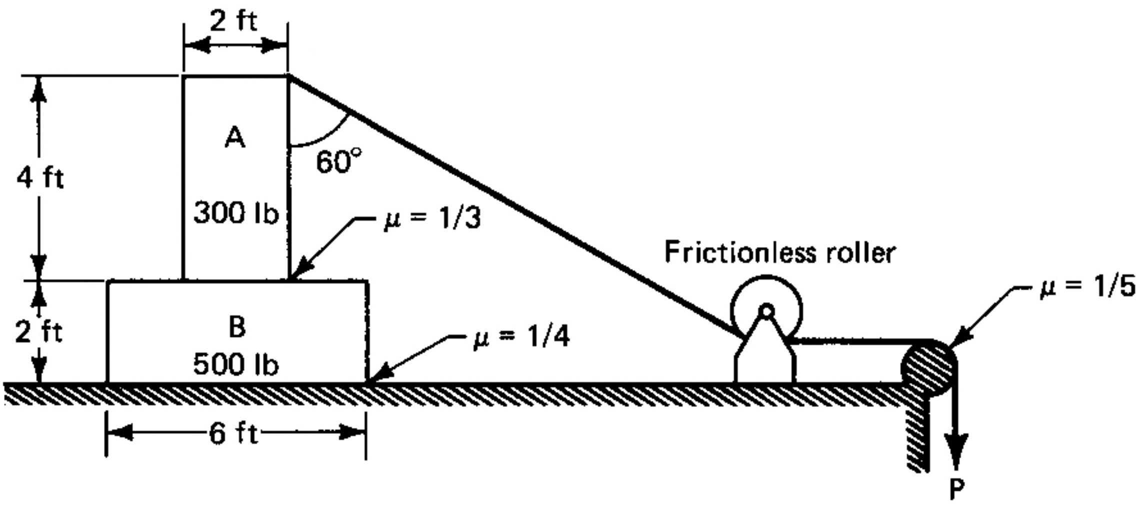
Correct response to preceding frame

(a) A may tip
(b) A may slide on B
(c) A and B may slide on ground as a unit

Frame 27-30

Alternative Motions

Problem:
Find the minimum value of P needed to move block A.



Solve for the value of P which will cause block A to move and tell how A will move.

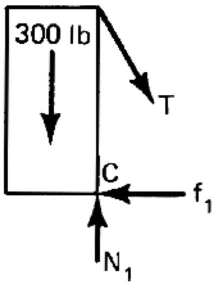
Correct response to preceding frame

P = 118.5 pounds

block will tip

<u>Solution:</u>

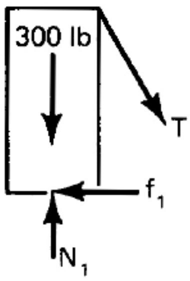
(a) For tipping



$$\bar{M}_c = 300\bar{k} - 4\left[\frac{\sqrt{3}}{2}\right]T\bar{k} = 0$$

$$T_{(a)} = 86.6 \text{ lb}$$

(b) For A slipping on B

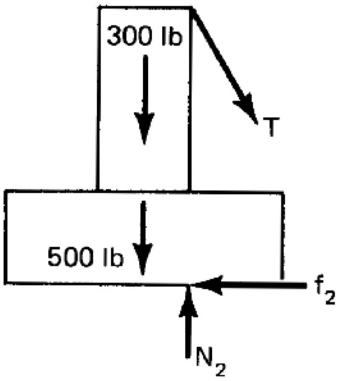


$$\Sigma\bar{F} = \left[-f_1 + \frac{\sqrt{3}}{2}T\right]\bar{i} + \left[N_1 - 300 - \frac{1}{2}T\right]\bar{j} = 0$$

and $f_1 = \frac{1}{3}N_1$

so that $T_{(b)} = 143$ lb

(c) For B slipping on ground



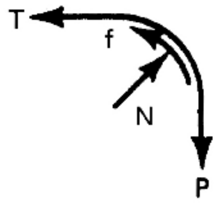
$$\Sigma\bar{F} = \left[-f_2 + \frac{\sqrt{3}}{2}T\right]\bar{i} + \left[N_2 - 800 - \frac{1}{2}T\right]\bar{j} = 0$$

and $f_2 = \frac{1}{4}N_2$

so $T_{(c)} = 270$ lb

$T_{(a)}$ is smallest, so block will tip when T = 86.6 pounds.

For rope



$$\frac{P}{86.6} = e^{\frac{1}{5}\left[\frac{\pi}{2}\right]}$$

Frame 27-31

Problem 27-2

Work Problem 27-2 in your notebook

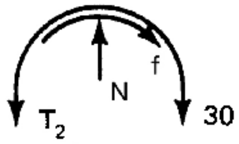
Correct response to preceding frame

Answer to problem 27-2

P = 198 pounds

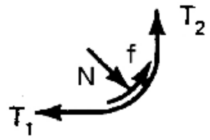
Solution:

For part "A" of rope



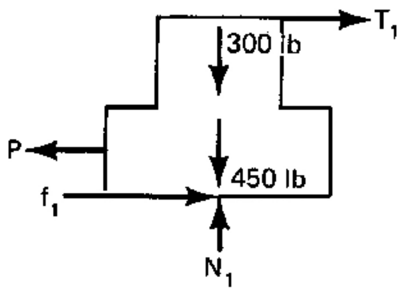
$$\frac{T_2}{30} = e^{\frac{1}{10}\pi} \quad \text{so} \quad T_2 = 41.1 \text{ lb}$$

For part "B" of rope



$$\frac{T_1}{\pi} = e^{\frac{1}{10}\left[\frac{\pi}{2}\right]} \quad \text{so} \quad T_1 = 48.1 \text{ lb}$$

For block system

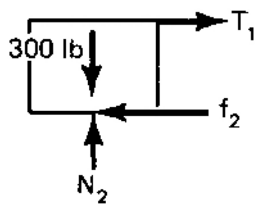


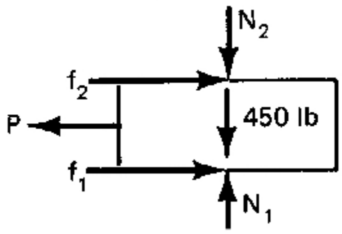
Case 1: Assume both blocks slip together and rope slips to left.

$$\Sigma\vec{F} = \left[-P + f_1 + T_1\right]\vec{i} + \left[N_1 - 450 - 300\right]\vec{j} = 0$$

$N_1 = 750$ and $f_1 = \mu_1 N_1 = 150$

Therefore $P = 198.1$





Case 2: Assume 450 lb block slips to left but 300 lb block stays in place.

$f_2 = \mu_2 N_2 = 105$

$f_1 = 150$ as before

P = 255, which is more than for Case 1

Therefore blocks slip together at P = 198.1

Frame 27-32

Closure

Here we are at the end at last.

In this unit you have seen how one of the more complicated friction equations can be derived and you have learned to use the "flat belt friction" equation for situations involving ropes and belts, both alone and in combination with other elements.