

# Introduction to Statics

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## Unit 26

# Friction of Machine Elements

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# Unit 26

## Friction of Machine Elements

Frame 26-1

### **Introduction**

This unit will give you more experience with problems which involve friction.

The problems in the preceding unit were selected so that you could solve them with a free body diagram, a two dimensional force equation, and one or more friction equations for each body. This unit will require you to use a moment equation for at least one of the bodies in addition to the information which you have been using.

Go to the next frame.

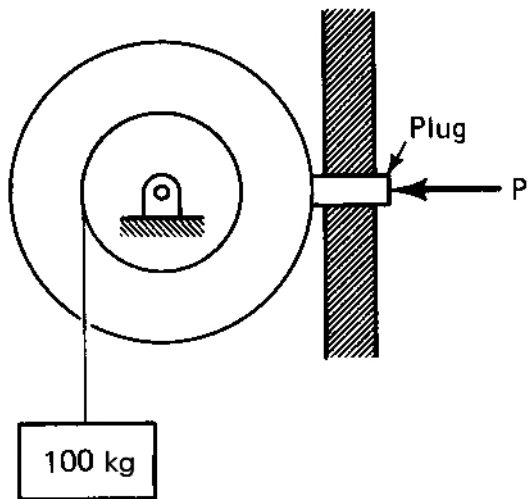
Correct response to preceding frame

No response

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Frame 26-2

**Friction on Cylinders**



The drum on a hoist is to be held in equilibrium by a wooden plug pushed against the drum by a force  $P$ .

If motion impends, the drum will

- (a) move to the left
- (b) fall down
- (c) rotate counterclockwise
- (d) turn clockwise

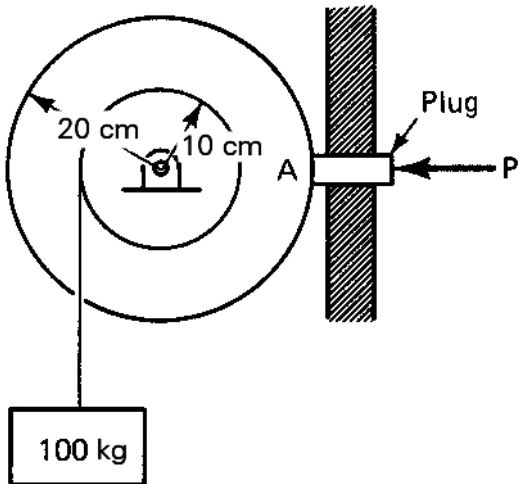
Correct response to preceding frame

(c) rotate counterclockwise

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Frame 26-3

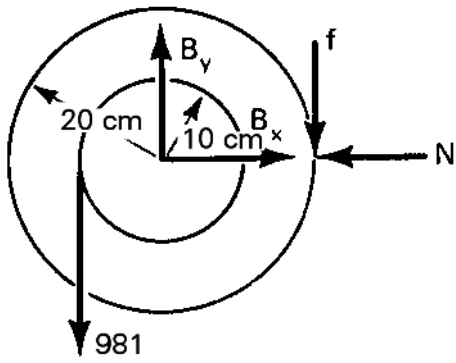
### Friction on Cylinders



In order to prevent rotation, the friction force at **A** must be tangent to the circumference of the drum and oppose the rotation.

Draw a free body diagram of the drum. The drum itself is of negligible weight.

Correct response to preceding frame



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Frame 26-4

### Friction on Cylinders

Using your free body from the preceding frame, write an equilibrium equation and determine the value of  $f$  which will hold the drum in equilibrium.

$f =$  \_\_\_\_\_

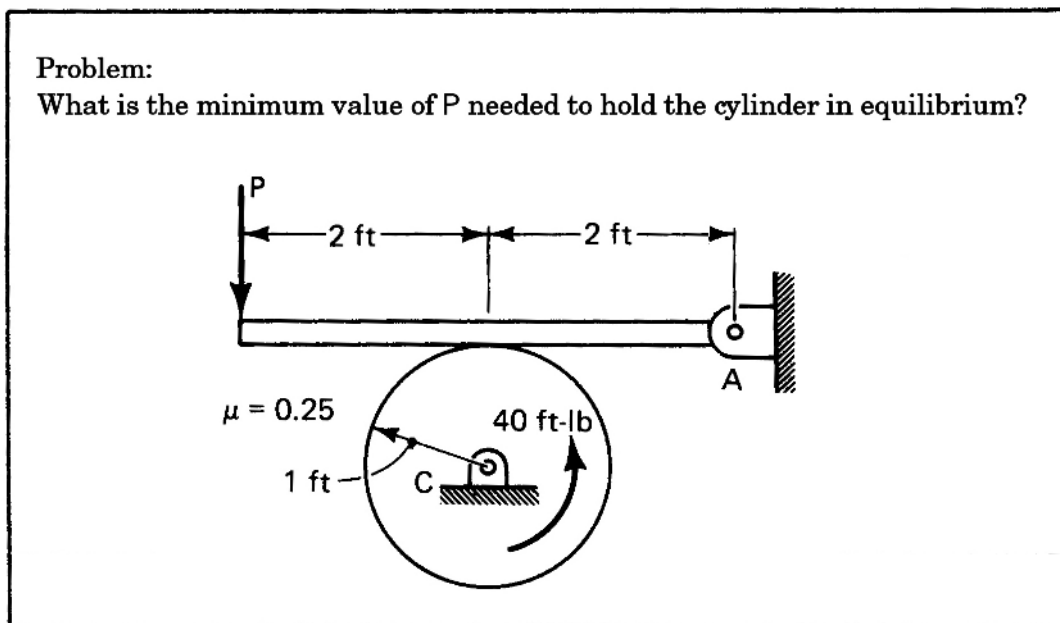
Correct response to preceding frame

$$\begin{aligned}\Sigma \bar{M}_B &= 0 \\ &= (-10\bar{i} \times -981\bar{j}) + (20\bar{i} \times -f\bar{j}) \\ \bar{f} &= -491\bar{j} \text{ N}\end{aligned}$$

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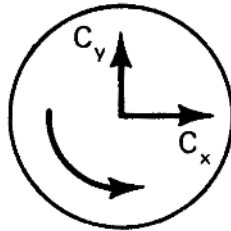
Frame 26-5

### Friction on Cylinders



Complete the free body diagrams of the cylinder and the bar.

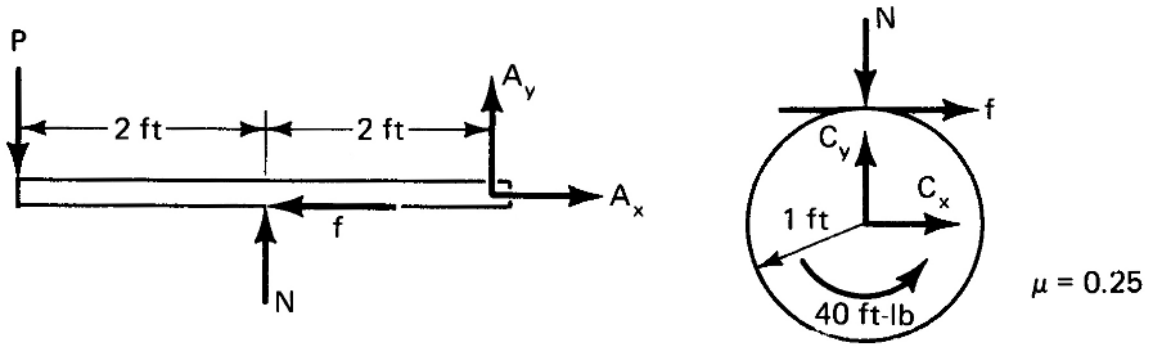




40 ft-lb



Correct response to preceding frame



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Frame 26-6

### Friction on Cylinders

Using the free bodies above, write a moment equation and find the friction force needed to hold the cylinder in equilibrium.

Use a moment equation for the bar and  $f \leq \mu N$  to find the minimum value of  $P$  for equilibrium.

Correct response to preceding frame

$$f = 40 \text{ lb}$$

$$P = 80 \text{ lb}$$

Solution:

$$\Sigma \bar{M}_C = 0$$

$$(40 - f)\bar{k} = 0$$

$$f = 40 \text{ lb}$$

$$\Sigma \bar{M}_A = 0$$

$$(4P - 2N)\bar{k} = 0$$

$$f = .25N$$

$$N = 160 \text{ lb}$$

$$P = \frac{N}{2}$$

---

Frame 26-7

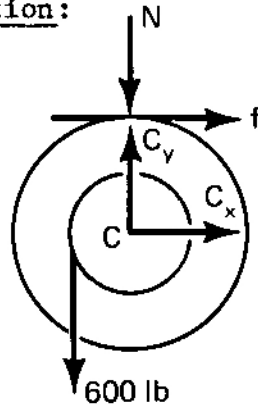
**Notebook**

Work Problem 26-1 in your notebook.

Correct response to preceding frame

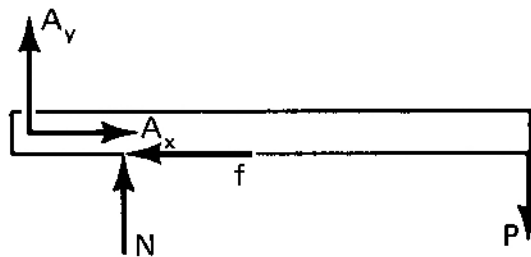
$$P = 160 \text{ lb}$$

Solution:



$$\Sigma \bar{M}_C = (600 - 3f)\bar{k} \text{ ft/lb} = 0$$

$$f = 200 \text{ lb}$$



$$\Sigma \bar{M}_A = (N - 5P)\bar{k} = 0$$

$$P = \frac{1}{5} N$$

$$f = \mu N$$

$$N = 800$$

---

Frame 26-8

### Transition

The first two problems which you worked in the unit were selected so that you could set up independent equations for  $f$  and  $N$  and then relate them with  $f \leq \mu N$ . This is not always possible. The next section will introduce situations in which the friction force increases or decreases the normal force.

Go to the next frame.

Correct response to preceding frame

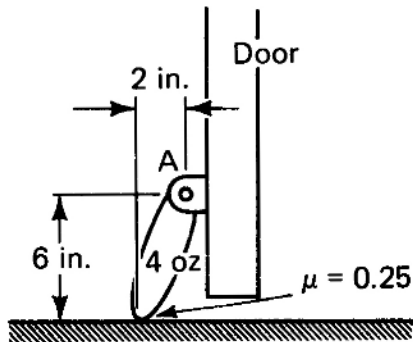
No response

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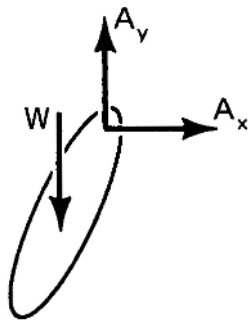
Frame 26-9

### Variations in Normal Force

Let's start with a mechanical device you may have used -- a door stop.



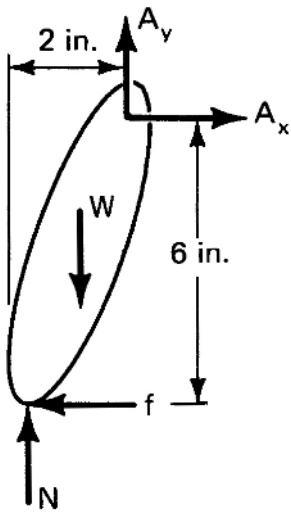
Complete the free body diagram of the stop when the door is being pushed toward the right.



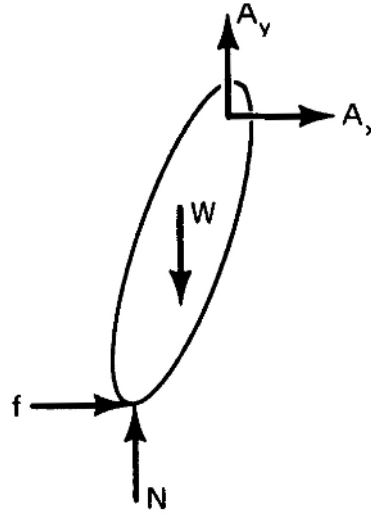
Draw a free body diagram of the stop when the door is being pushed toward the left.

Correct response to preceding frame

**Pushed toward the right**



**Pushed toward the left**



---

Frame 26-10

**Variations in Normal Force**

Use your free bodies from the preceding frame. Assume that motion impends, write an equation for the moment of the force system about point **A**, and solve for **N** in each case.

$W = 4$  ounces (0.25 pound)

$\mu = 0.25$

Correct response to preceding frame

**Pushed toward the right**

$$\Sigma \bar{M}_A = \frac{1}{4} (1) - 2N - 6(0.25)N = 0$$

$$N = \frac{1}{14} \text{ lb}$$

**Pushed toward the left**

$$\Sigma \bar{M}_A = \frac{1}{4} (1) - 2N + 6(0.25)N = 0$$

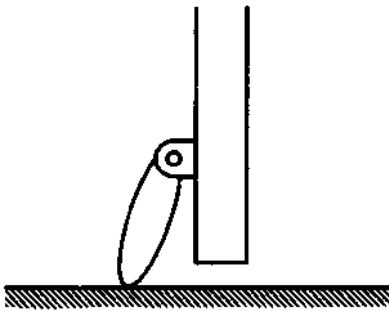
$$N = \frac{1}{2} \text{ lb}$$

---

Frame 26-11

### Variations in Normal Force

The magnitude of the normal force on the door stop was affected by the friction force.



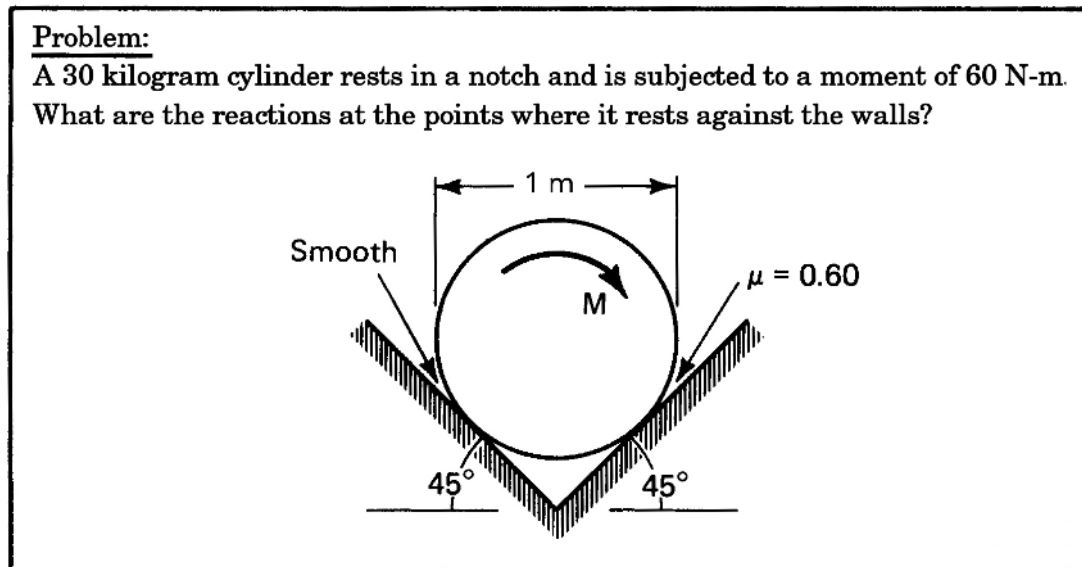
1. When the door was pushed toward the left, the moments of the friction and normal were of the (*same, opposite*) sense. The normal force was (*increased, decreased*).
2. When the door was pushed toward the right the moments of the friction and the normal were of the (*same, opposite*) sense. The normal force was (*increased, decreased*).

Correct response to preceding frame

1. opposite, increased
  2. same, decreased
- 

Frame 26-12

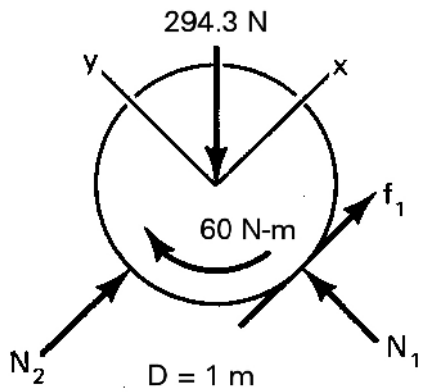
### Variations in Normal Force



Draw the free body diagram.

Does the problem statement imply impending motion?  Yes  No

Correct response to preceding frame



No

---

Frame 26-13

### Variations in Normal Force

Write the equilibrium equations for the cylinder in the preceding frame and determine the reactions.



Correct response to preceding frame

$$f_1 = 120 \text{ N}$$

$$N_1 = 208 \text{ N}$$

$$N_2 = 88.1 \text{ N}$$

Solution:

$$\Sigma \bar{M}_o = 0$$

$$\left[ \frac{1}{2} f_1 - 60 \right] \bar{k} = 0$$

$$\Sigma \bar{F} = 0$$

$$\left[ N_2 + f_1 - \frac{294.3}{\sqrt{2}} \right] \bar{i} + \left[ N_1 - \frac{294.3}{\sqrt{2}} \right] \bar{j} = 0$$

---

Frame 26-14

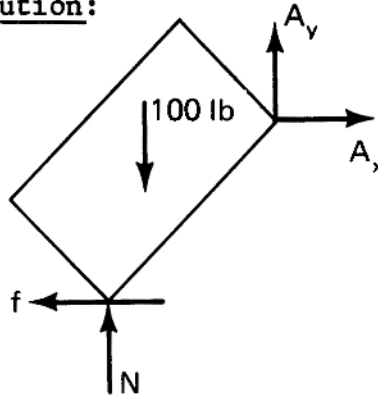
**Notebook**

Work Problem 26-2 in your notebook.

Correct response to preceding frame

$$\bar{\mathbf{A}} = 11.8\bar{\mathbf{i}} + 41.1\bar{\mathbf{j}} \text{ lb}$$

Solution:



$$\Sigma \bar{\mathbf{M}}_A = [2(100) - 2f - 3N]\bar{\mathbf{k}} = 0$$

$$f = .2N = \frac{N}{5}$$

$$N = 58.9$$

$$f = 11.8$$

$$\Sigma \bar{\mathbf{F}} = 0$$

$$(-11.8 + A_x)\bar{\mathbf{i}} + (58.9 - 100 + A_y)\bar{\mathbf{j}} = 0$$

Frame 26-15

### Transition

In each of the three preceding problems you encountered a situation in which the tendency of the frictional force to rotate the body resulted in a "change" in a normal force from what it would have been in a frictionless situation. This will not always be the case, but you should be on the lookout for such situations.

On to the next frame.

Correct response to preceding frame

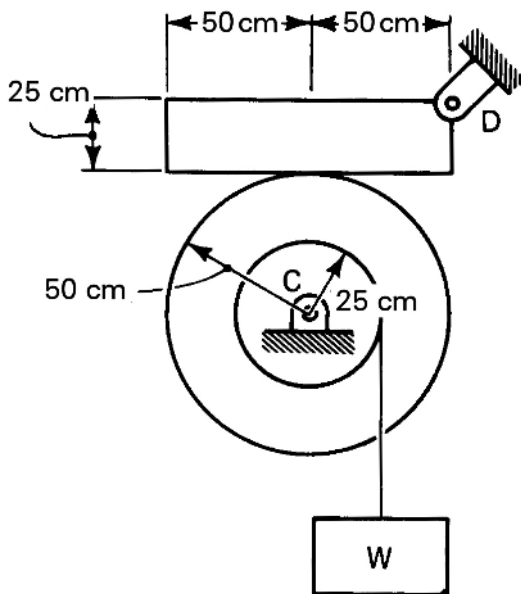
No response

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Frame 26-16

**Brakes**

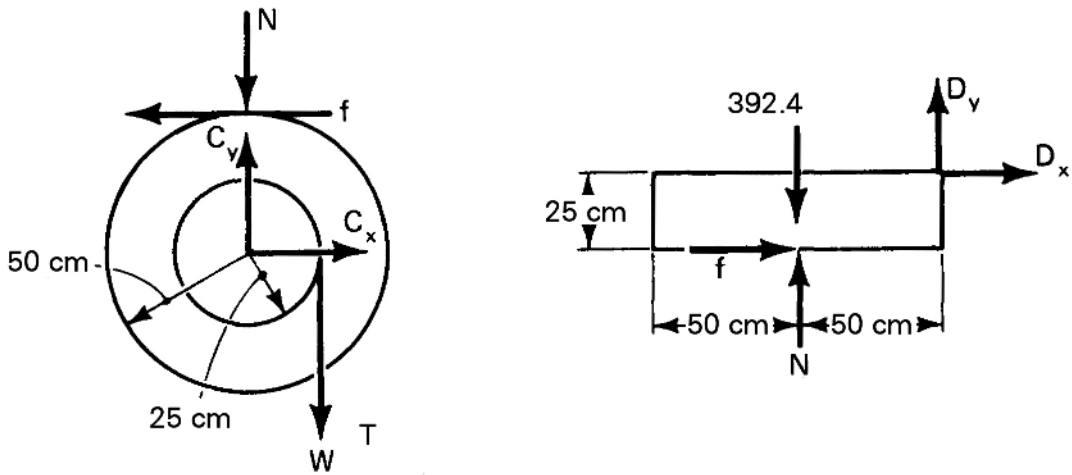
The device shown is used to hold up a weight.



The block weighs 40 kilograms.

Draw a free body diagram of the pulley and one of the block.

Correct response to preceding frame



---

Frame 26-17

### Brakes

Using the free body diagrams given above, determine the maximum weight which may be held. The coefficient of friction between the block and the pulley is 0.45.

Correct response to preceding frame

$$W = 46.5 \text{ kg}$$

Solution: on pulley

$$\Sigma \bar{M}_C = \left[ \frac{1}{2} f - \frac{1}{4} T \right] = 0$$

$$T = 2f$$

on block

$$\Sigma \bar{M}_D = \left[ \frac{1}{4} f + \frac{1}{2} (392.4) - \frac{1}{2} N \right] \bar{k} = 0$$

$$2N - f = 2 (392.4)$$

When motion impends,

$$f = .45N$$

$$1.55 N = 784.8$$

$$N = 506 \text{ Newtons}$$

$$f = .45N = 227.8$$

$$T = 2f = 456$$

$$W = T / 9.81$$

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Frame 26-18

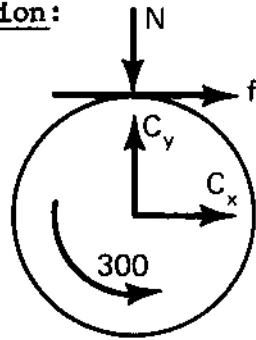
### Brakes

Work Problem 26-3 in your notebook.

Correct response to preceding frame

$P = 67 \text{ N}$

Solution:



$$\sum M_C = 0$$

$$300 - \left(\frac{3}{2}\right) f = 0$$

$$f = 200$$

since motion impends

$$f = f'$$

$$200 = \mu N = .6N$$

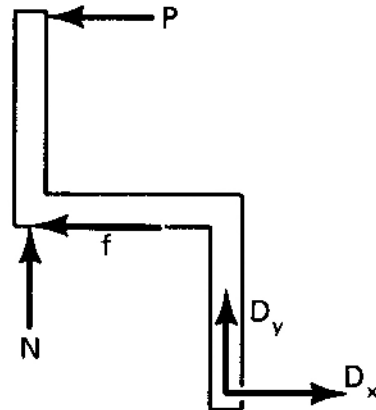
$$N = 334$$

$$\sum M_D = 0$$

$$2f + 4P - 2N = 0$$

$$2(200) + 4P - 2(334) = 0$$

$$P = 67$$



Frame 26-19

### Transition

The last section in this unit will deal with problems in which several different motions of a body are possible and in which you must make several calculations in order to determine which condition actually applies.

This section is rather short. When you're settled, turn to the next frame.

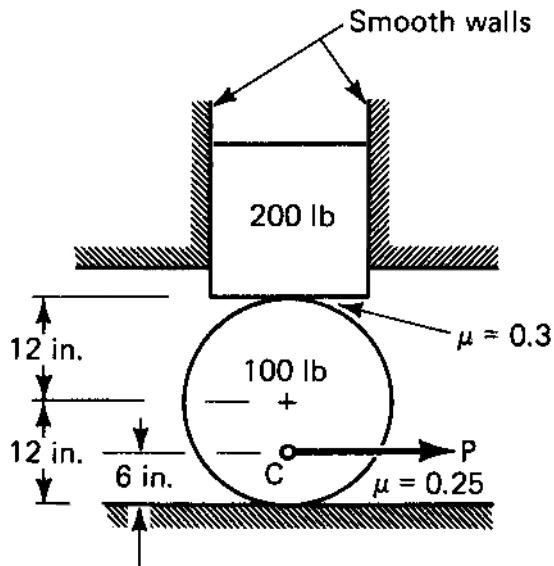
Correct response to preceding frame

No response

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Frame 26-20

### Identification of Impending Motion

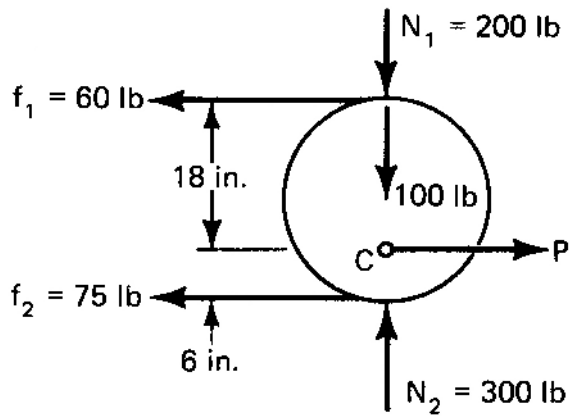


The block weighs 200 lb and the cylinder weighs 100 lb.

Assume that the cylinder is about to slide at both the top and the bottom.

Draw a free body diagram of the cylinder.

Correct response to preceding frame



---

Frame 26-21

### Identification of Impending Motion

Use the free body diagram from the preceding frame. Use  $\sum \bar{F} = 0$  to determine the value of  $P$  which will cause the cylinder to have impending motion.

Is the cylinder in equilibrium?

Yes

No



Correct response to preceding frame

$$P = 135 \text{ lb}$$

No, not in equilibrium since  $\Sigma \bar{M}_C \neq 0$

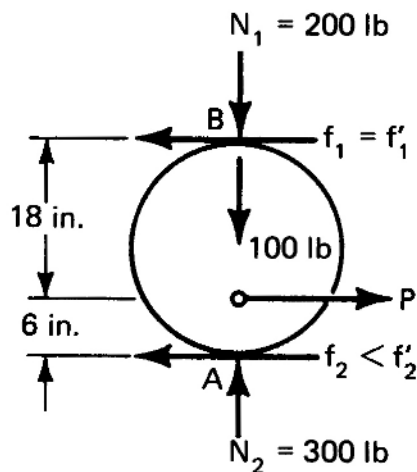
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Frame 26-22

### Identification of Impending Motion

Since the cylinder would not be in equilibrium if it slipped at the top *and* bottom, it will slip *either* at the top or bottom.

I have drawn the free body diagram and determined **P** for slip at the top. Draw the diagram for slip at the bottom and determine the value of **P** for that case.



Slip at top

$$f_1' = (0.3)N_1$$

$$\Sigma \bar{M}_A = 24(60) - 6P = 0$$

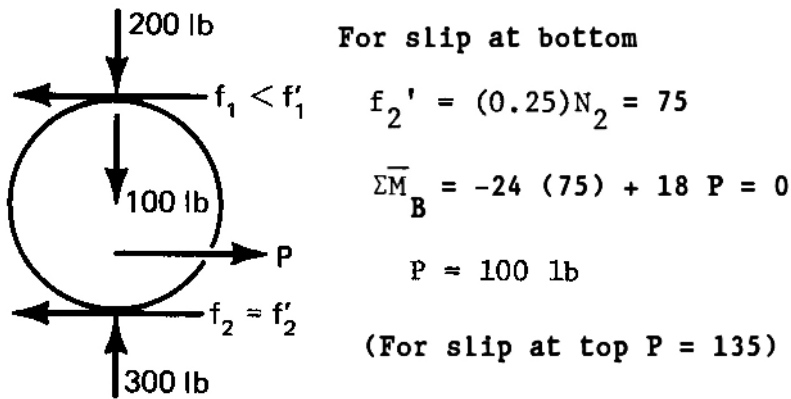
$$P = 240 \text{ lb}$$

Slip at bottom

$$f_2' = (0.25)N_2$$

The smaller value of **P** indicates where it will slip. Where does it slip? \_\_\_\_\_

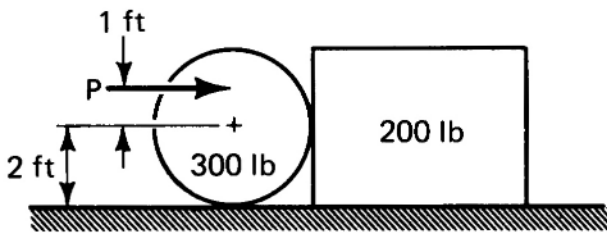
Correct response to preceding frame



$P = 100 \text{ lb}$   
Slips at bottom

Frame 26-23

**Identification of Impending Motion**



The coefficient of friction is 0.15 at the horizontal contact surface. The contact surface between the block and the wheel is smooth.

There are three ways in which this system could move if  $P$  is great enough. Complete the following summary of the motions.



Wheel	Block
	stationary
skids to the right	
	slides to the right

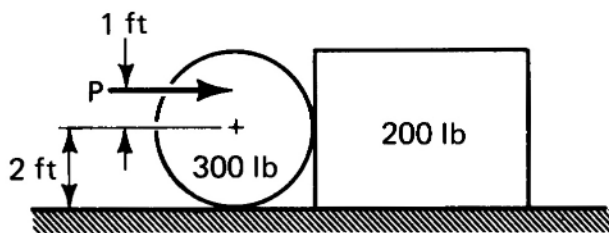
Correct response to preceding frame

Wheel	Block
turns in place	stationary
skids to the right	slides to the right
rolls to the right	slides to the right

---

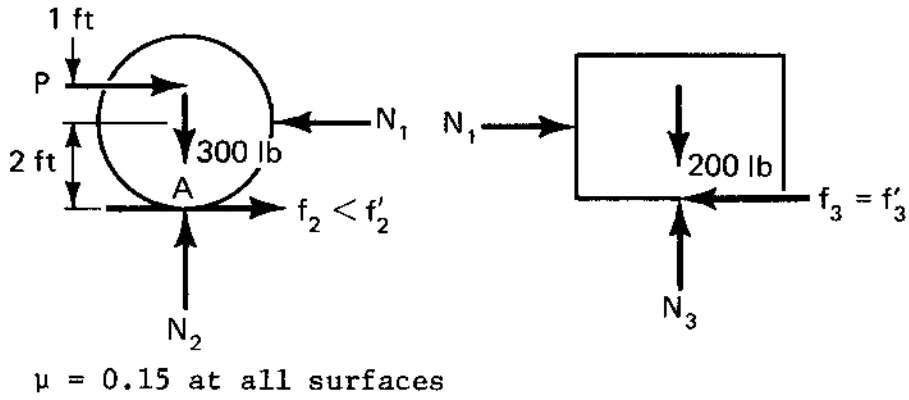
Frame 26-24

**Identification of Impending Motion**



Draw the free body diagrams for the case of impending motion in which the wheel will roll to the right and the block will slide. Identify the places where  $f = f'$  and those where  $f < f'$ .

Correct response to preceding frame



---

Frame 26-25

### Identification of Impending Motion

Using the free body diagrams shown above, determine the value of  $P$  necessary to cause impending motion for this situation.

Correct response to preceding frame

$P = 20 \text{ lb}$     Solution:    on block

$$f_3 = \mu N_3 = 30 \text{ lb}$$

on wheel

$$\Sigma M_A = 0$$

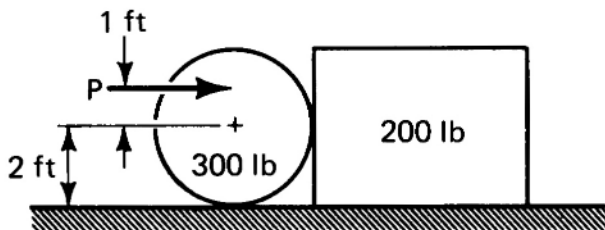
$$30(2) - P(3) = 0$$

$$P = 20 \text{ lb}$$

---

Frame 26-26

### Identification of Impending Motion

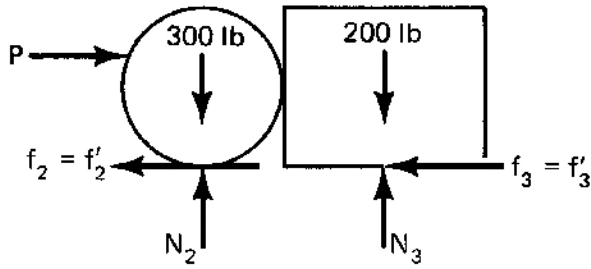


Draw the free bodies and solve for the other two cases of impending motion.  
(Both slip and wheel spins)

Correct response to preceding frame

Both slide

Solution:



$P = 75 \text{ lb}$

$$\Sigma \bar{F} = 0$$

$$(N_2 + N_3)\bar{j} - 500\bar{j} + P\bar{i} - (f_2 + f_3)\bar{i} = 0$$

$$N_2 + N_3 = 500$$

$$P = f_2 + f_3$$

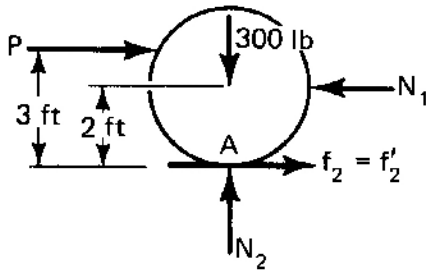
$$f_2 = f_2' = .15N_2 \quad f_3 = f_3' = .15N_3$$

$$P = f_2 + f_3 = .15N_2 + .15N_3$$

$$P = .15(N_2 + N_3) = .15(500) = 75 \text{ lb}$$

Wheel spins

Solution:



$P = 90 \text{ lb}$

$$\Sigma \bar{F} = 0$$

$$N_2\bar{j} - 300\bar{j} + P\bar{i} + f_2\bar{i} - N_1\bar{i} = 0$$

$$N_2 = 300 \quad f_2 = f_2' = 45 \text{ lb}$$

$$P + f_2 = N_1$$

$$\Sigma \bar{M}_A = 0$$

$$2N_1 - 3P = 0 \quad N_1 = \frac{3}{2}P$$

$$P + 45 = \frac{3}{2}P$$

$P = 90 \text{ lb}$

(For wheel rolling and block sliding, we got  $P = 20 \text{ lb}$ .)

Frame 26-27

### Identification of Impending Motion

What is the maximum value of P that can be applied without causing motion to impend?

$P_{\max} = \underline{\hspace{2cm}}$



Correct response to preceding frame

$$P_{\max} = 20 \text{ lb}$$

---

Frame 26-28

### **Identification of Impending Motion**

Study Problem 26-4 in your notebook to see how tough these things can get. If you enjoy a challenge and have the time, try your own solution before looking at ours.

Correct response to preceding frame

Complete solution is in your notebook.

---

Frame 26-29

### **Closure**

Happily, you won't run into very many of these messes where you have to solve four problems in order to get one answer, but it does happen.

That's all there is to this unit. The purpose of the unit was to give you practice at applying a wide range of problem solving skills to a rather complex group of problems. Now that you have finished it you should be more adept at problem solving even though you haven't acquired much new information.