Introduction to Statics

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Unit 25
Friction

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Unit 25
Friction

Frame 25-1

Introduction

In this unit you will gain experience with problems involving friction. As you learned in the preceding unit, these problems are more involved than frictionless ones in that you will have to make inferences about the friction even before drawing your free body diagram, and then close by rechecking your assumption.

Turn to the next frame.

Correct response to preceding frame

No response

Frame 25-2

Review

In the last unit you learned that the coefficient of friction places a limit on the friction force, and the equation $f = \mu N$ applies if and only if ____________________________ .

In all other cases, when equilibrium exists the inequality which applies is ________ .

Correct response to preceding frame

motion impends
$f < \mu N$ or $f < f'$ (Or equivalent response)

Frame 25-3

Review

In cases where f is not less than, or equal to, μN, can you work the problem?

☐ Yes ☐ No

Correct response to preceding frame

not with statics

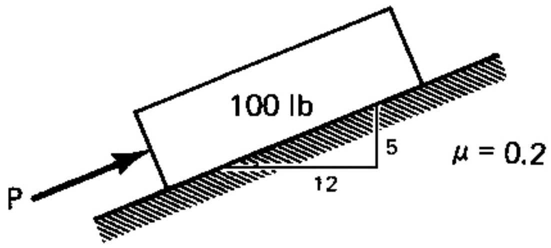
Frame 25-4

Impending Motion

Read the problem and answer the questions about it.

Problem.

A force P = 10 pounds acts on the 100 pound block.
Determine the friction force on the block



Does the picture tell you that motion impends? ☐ Yes ☐ No

Does the problem statement tell you that motion impends? ☐ Yes ☐ No

Correct response to preceding frame

No
No

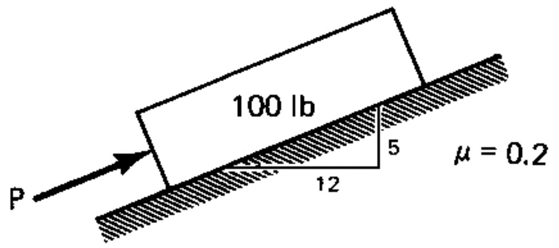
Frame 25-5

Impending Motion

The problem below has the same picture but a different statement. Read it and answer the questions below it.

Problem.

Problem. What is the value of the force P which will move the 100 pound block up the slope?



Does the picture tell you motion impends? ☐ Yes ☐ No

Does the statement tell you motion impends? ☐ Yes ☐ No

Correct response to preceding frame

No
Yes

Frame 25-6

Impending Motion

In general, we infer that motion impends from the

- ☐ Picture
- ☐ Problem statement
- ☐ Both

Correct response to preceding frame

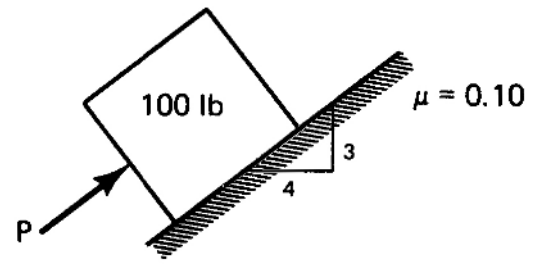
problem statement

Frame 25-7

Single Bodies

Sometimes friction problems require more than one answer, read the following problem and answer the questions below.

Problem.
For what range of values of P is the 100 pound block in equilibrium?



If P is too large which way will the block move? ☐ upward ☐ downward

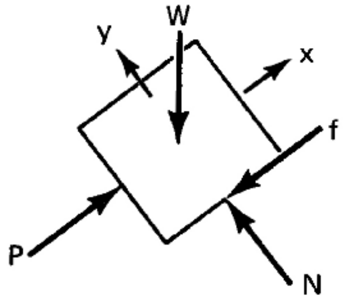
Draw the FBD and find the maximum value of P.

Correct response to preceding frame

Upward

P_{max} = 68 lb

<u>Solution:</u>



for P_{max} motion impends

$f = f' = \mu N$

$\sum \bar{F} = P\bar{i} - f\bar{i} + N\bar{j} + W(-\frac{3}{5}\bar{i} - \frac{4}{5}\bar{j}) = 0$

N = 80 lb

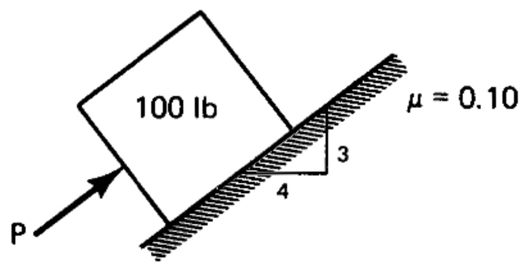
f' = 8 lb

$P_{max} - 8 - 60 = 0$

Frame 25-8

Single Bodies

<u>Problem.</u>
For what range of values of P is the 100 pound block in equilibrium?



If P is too small which way will the block move? ☐ upward ☐ downward

Draw a new FBD and find P .

What is the range of stability?

__________ < P < __________

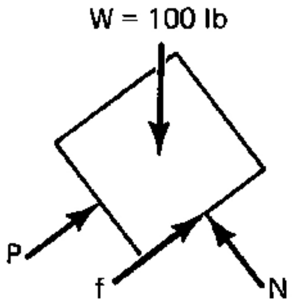
Correct response to preceding frame

downward

$P_{min} = 52$ lb

$52 < P < 68$ lb

Solution:



For P_{min} motion impends

$N = 80$ lb

$f' = 8$ lb

$P_{min} + 8 - 60 = 0$

Frame 25-9

Comment

The preceding material should have convinced you that "friction problems" may be stated many different ways. We will examine some more of these as applied to single bodies before getting involved in systems of bodies.

Go to the next frame.

Correct response to preceding frame

No response

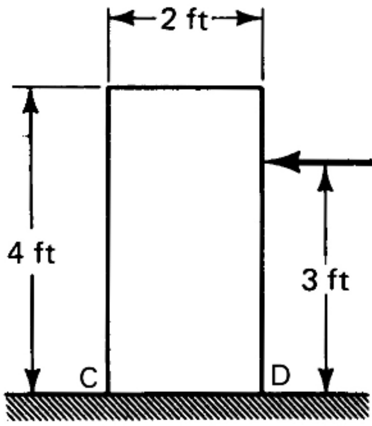
Frame 25-10

Single Bodies

Read the following problem and follow the instruction below.

Problem.

The coefficient of friction between homogeneous a 300 pound block and the floor is 0.4. Will the block tip or slip if P is large enough to move the block?

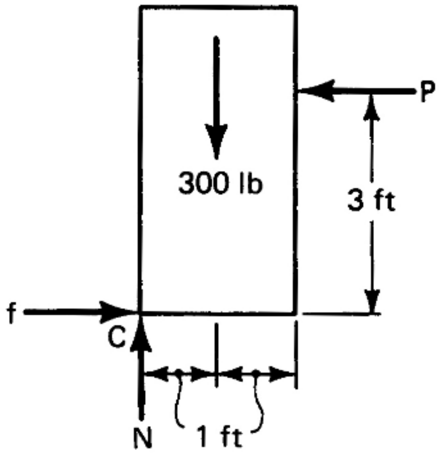


Ask yourself "If the block tips, will it rotate about C or D ?" As it begins to tip, both f and N will act at the point of rotation.

Draw a free body diagram for the block and write the equilibrium equations, assuming that tipping impends.

Correct response to preceding frame

C will be the center of rotation.



$$\sum \overline{F} = (f - P)\overline{i} + (N - 300)\overline{j} = 0$$

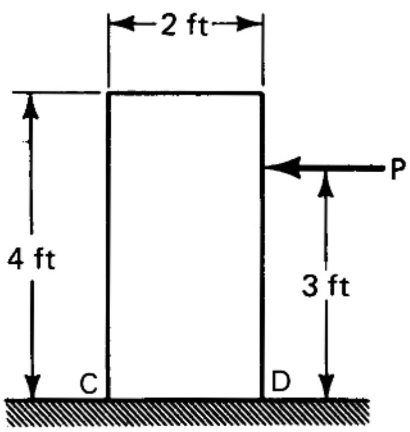
$$\sum \overline{M}_C = [3P - 1(300)]\overline{k} = 0$$

Frame 25-11

Single Bodies

Problem.

The coefficient of friction between homogeneous a 300 pound block and the floor is 0.4. Will the block tip or slip if P is large enough to move the block?



Same problem.

1. Find P, f and N and determine whether the block will slip.

2. Under what conditions will the block tip?

Correct response to preceding frame

1. P = 100 lb

   f = 100 lb

   N = 300 lb

   Non-slip since 100 = f < f' = 120 lb

2. Block will begin to tip if P is just a little more than 100 lb.

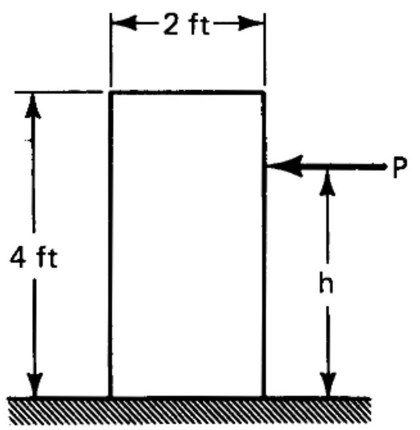
Frame 25-12

Single Bodies

Let's rephrase our last problem a little:

Problem:

The coefficient of friction between the 300 pound block and the floor is 0.4. Determine the maximum height above the floor at which the block will slip and not tip.



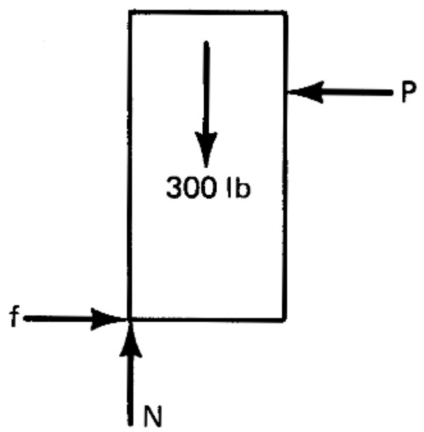
1. Does motion impend? ☐ Yes ☐ No

2. Draw a FBD and write and solve the equilibrium equations. (Work this out on Page 25-1 in your notebook.)

Correct response to preceding frame

1. Yes, so $f = \mu N$

2.



$$\sum \overline{F} = (f - P)\overline{i} + (N - 300)\overline{j} = 0$$

$$N = 300$$

$$f = 120$$

$$P = 120$$

$$\sum \overline{M}_c = [hP - 1\ (300)]\overline{k} = 0$$

$$h = \frac{300}{120} = 2.5 \text{ ft}$$

Frame 25-13

Transition

The following problem is a demonstration of a major experimental approach to friction studies so we will record most of it in the notebook. Open your notebook to 25-2 and read down to the STOP sign; then turn to Frame 25-14.

Correct response to preceding frame

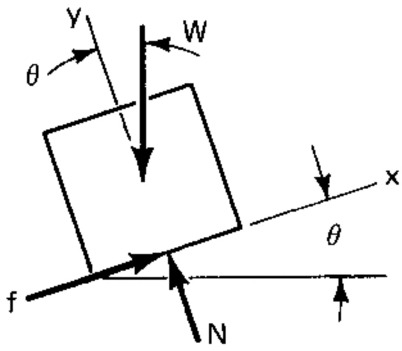
No response

Frame 25-14

Experimental Determination of μ

Draw a FBD of the block. Using a coordinate system with y normal to the surface and x tangent to the surface, write the equilibrium equation for the block.

Correct response to preceding frame



$$\sum \overline{F} = (f - W \sin\theta)\overline{i} + (N - W \cos\theta)\overline{j} = 0$$

After checking your answers, enter the FBD and equation in the spaces in your notebook.

Frame 25-15

Experimental Determination of μ

Next step:

What is the angle at which motion impends, called? ______________________________

Call that angle θ_R, and using θ_R in your equations calculate the relationship between μ and θ_R.

Correct response to preceding frame

angle of repose

$\mu = \tan \theta_R$

<u>Solution:</u>

$f' = W \sin \theta_R$

$N = W \cos \theta_R$

$$\mu = \frac{f'}{N} = \frac{W \sin \theta_R}{W \cos \theta_R}$$

After checking your answers, enter them in the spaces in your notebook.

Frame 25-16

Transition

We will now direct your attention to some systems consisting of more than one body and generally containing more than one friction force.

Turn to the next frame.

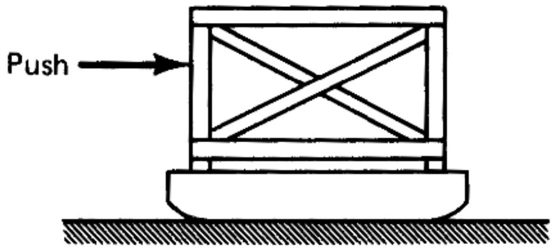
Correct response to preceding frame

No response

Frame 25-17

Systems of "Blocks"

A popular friction problem involves stacks of "blocks" with various coefficients of friction between the surfaces. For example, suppose a wooden crate rests on a steel skid.



Give your opinion on the following questions:

1. If the floor is covered with ice,

(a) will the crate slide off the skid?	☐ Yes	☐ No
(b) will the skid slide over the floor?	☐ Yes	☐ No
(c) how will the skid and crate move?	☐ As a unit	☐ Separately

2. If the floor is rough concrete:

(a) will the crate slide off the skid?	☐ Yes	☐ No
(b) will the skid slide over the floor?	☐ Yes	☐ No
(c) how will the skid and crate move?	☐ As a unit	☐ Separately

Correct response to preceding frame

My opinions are:

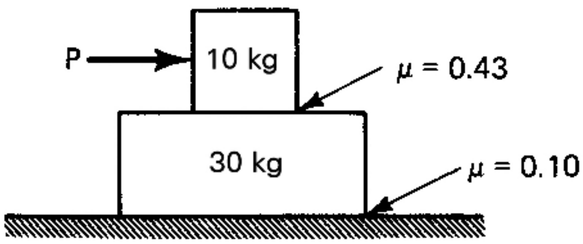
1. (a) No
   (b) Yes
   (c) As a unit
2. (a) Yes
   (b) No
   (c) Separately

Frame 25-18

Systems of Blocks

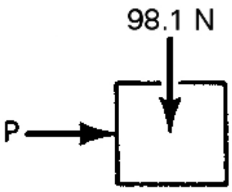
Read the following problem and answer the question below.

Problem:
What is the minimum force necessary to move the top block?



What is the minimum force P necessary to move the top block?

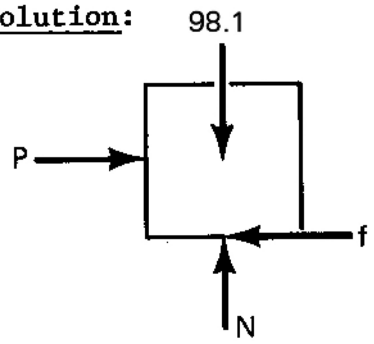
Assuming the top block moves separately, complete the FBD and find the force P for this case.



Correct response to preceding frame

P = 42.2 Newtons

<u>**Solution:**</u>



$$\sum \overline{F} = (N - 98.1)\overline{j} + (P - f)\overline{i} = 0$$

We have *assumed* the motion impends, therefore

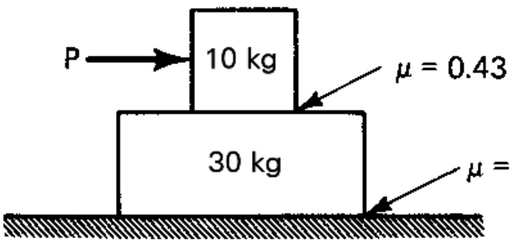
$$f = \mu N = 0.43\ (98.1)$$

Frame 25-19

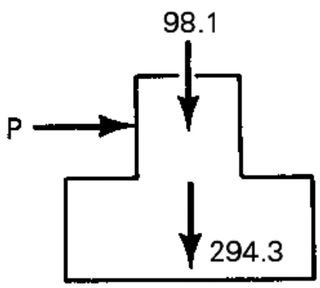
Systems of Blocks

(Problem continued)

Problem:
What is the minimum force necessary to move the top block?



Now assuming that the two blocks move as a unit, complete the FBD and solve for P.



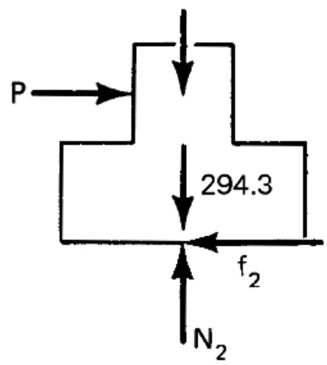
What is the minimum value of P for motion? P_{min} = ________________

Correct response to preceding frame

P_{unit} = 39.2 Newtons

P_{min} = 39.2 Newtons

<u>Solution:</u>



$\sum \overline{F} = 0 = (P - f_2)\overline{i} + (N_2 - 392.4)\overline{j}$

We have *assumed* the motion impends, therefore

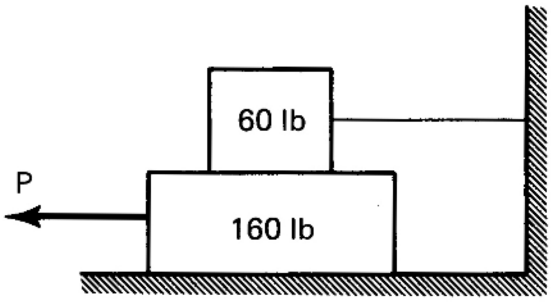
$F_2 = \mu N_2 = 0.1\ (392.4)$

Frame 25-20

Systems of Blocks

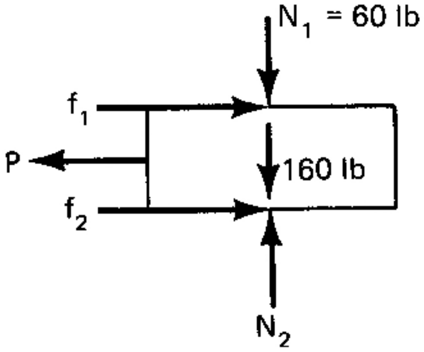
Sometimes the blocks form sandwiches with the filling being pulled out, in which case you have friction on two sides of a body. Read the following problem and follow the instruction below.

Problem:
If $\mu = 0.25$ between all surfaces, what must P be in order to remove the lower block?



Draw a FBD of the lower block.

Correct response to preceding frame



Frame 25-21

Systems of Blocks

Now write your equilibrium equations and find the value of P which will cause the block to move. ($\mu = 0.25$ on both surfaces.)

Correct response to preceding frame

P = 70 lb

Solution:

$$\sum \overline{F} = 0 = (f_1 + f_2 - P)\overline{i} + (N_2 - 60 - 160)\overline{j}$$

motion impends so

$$f_1 = \mu N_1 = 15$$

$$f_2 = \mu N_2 = 55$$

Frame 25-22

Systems of Blocks

Now work the block problem on Page 25-3 in your notebook.

Correct response to preceding frame

P_{min} = 168 Newtons

If B pulls out by itself,

$$P_1 = 0.6\ (88.29) + 0.6\ (235.44)$$
$$= 194.2 \text{ Newtons}$$

If B and C slide as a unit,

$$P_2 = 0.6\ (88.29) + 0.3\ (382.59)$$
$$= 168.4 \text{ Newtons}$$

Frame 25-23

Transition

The final section of this unit will give you an introduction to friction on wedges and inclined planes. Since the wedge and its close relatives, the square thread and the cam, occur in many machines, these few ideas may be useful to you in a wide range of situations.

The "secret" to working wedge problems is not new to you; it is "carefully draw your FBD and write your equilibrium equations". The trouble is the faces aren’t either perpendicular or parallel. If they were, they wouldn’t be wedges.

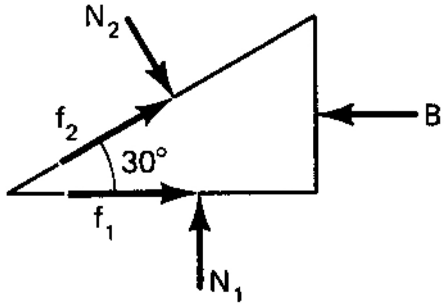
Onward! (Next frame.)

Correct response to preceding frame

No response

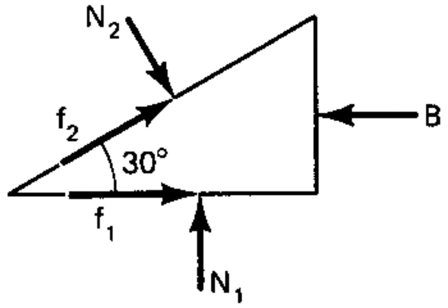
Frame 25-24

Wedges



Here is an example of a FBD of a wedge; write its equilibrium (force) equation.

Correct response to preceding frame



$$\sum \bar{F} = 0$$

$$= \left[f_1 + f_2\left(\frac{\sqrt{3}}{2}\right) + N_2\left(\frac{1}{2}\right) - B\right]\bar{i} + \left[N_1 - N_2\left(\frac{\sqrt{3}}{2}\right) + f_2\left(\frac{1}{2}\right)\right]\bar{j}$$

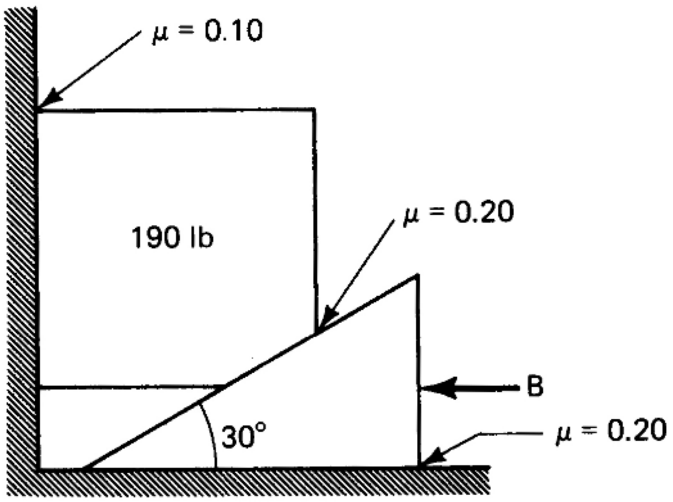
or

$$= \left[f_1 + f_2 \cos 30^\circ + N_2 \sin 30^\circ - B\right]\bar{i} + \left[N_1 - N_2 \cos 30^\circ + f_2 \sin 30^\circ\right]\bar{j}$$

Frame 25-25

Wedges

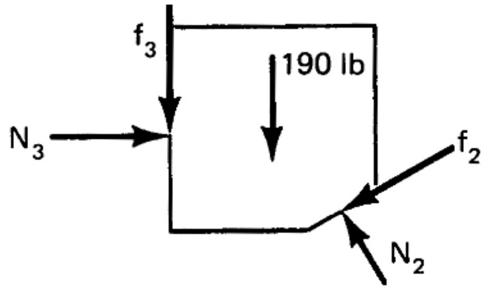
You've already got a messy equation and you're just starting!



The wedge is being used to lift a block.

Draw a FBD of the block and write the equilibrium equation. Make sure that your work is consistent with the problem statement and with the preceding free body.

Correct response to preceding frame



$$\sum \bar{F} = \left[N_3 - N_2\left(\frac{1}{2}\right) - f_2\left(\frac{\sqrt{3}}{2}\right)\right]\bar{i} + \left[N_2\left(\frac{\sqrt{3}}{2}\right) - f_2\left(\frac{1}{2}\right) - 190 - f_3\right]\bar{j} = 0$$

Frame 25-26

Wedges

Resume:
Assume B is large enough to lift block.

1. How many unknowns do you have? ____________

2. How many other equations? ____________

3. Have you enough information to solve the problem? ☐ Yes ☐ No

Correct response to preceding frame

1. 7 unknowns
2. total of 7 equations (4 equilibrium equations and 3 friction equations)
3. Yes, if you want to solve the equations, you can. I'd rather start a new problem.

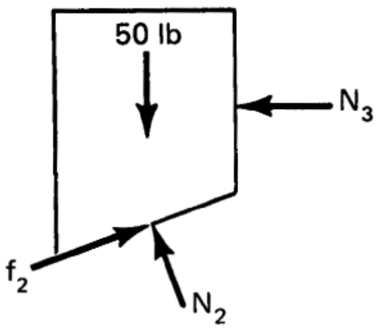
Frame 25-27

Wedges

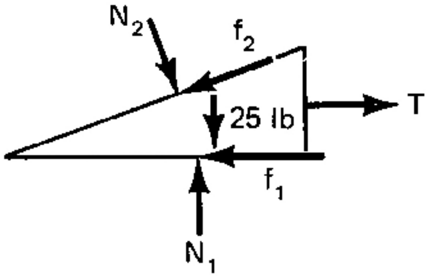
Since you may want to remind yourself how easy this concept is at a later date, work Problem 25-3 in your notebook.

Correct response to preceding frame

T = 24.3 pounds **Solution:**



$$\sum \bar{F} = 0 = \left(\frac{12}{13} f_2 - \frac{5}{13} N_2 - N_3 \right) \bar{i} + \left(\frac{5}{13} f_2 + \frac{12}{13} N_2 - 50 \right) \bar{j}$$



$$\sum \bar{F} = 0 = \left(-f_1 - \frac{12}{13} f_2 + T + \frac{5}{13} N_2 \right) \bar{i} + \left(N_1 - \frac{12}{13} N_2 - \frac{5}{13} f_2 - 25 \right) \bar{j}$$

$$f_2 = .4N_2$$

$$f_1 = \frac{1}{3} N_1$$

Frame 25-28

Closure

The end at last!

You may have realized on your way through that this unit has dealt with the equilibrium of bodies with flat surfaces, and that most of the bodies have been considered as particles (that is, subject to concurrent forces). The next unit in this series will give you some work with friction on bodies with curved surfaces and others for which you must write both force and moment equations.