Introduction to Statics

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Unit 24
Introduction to Friction

Helen Margaret Lester Plants
Late Professor Emerita
Wallace Starr Venable
Emeritus Associate Professor
West Virginia University, Morgantown, West Virginia

Unit 24
Introduction to Friction

Frame 24-1

Introduction

It is quite probable that you feel that the work you have done so far on equilibrium has been more than a little tinged with unreality. One reason for this is that we have studiously avoided all reference to friction. Since friction undeniably exists -- and it's a good thing that it does -- the time has come to come to grips with it.

Friction is both necessary -- as may be seen on an icy morning -- and detrimental. It is a non-mathematical quantity. Information about it must be observed rather than deduced. Consequently this unit will help you to organize your own observations and will inform you of the observations of others to enable you to understand, and perhaps eventually to control, the effects of friction on rigid body equilibrium.

Go to the next frame.

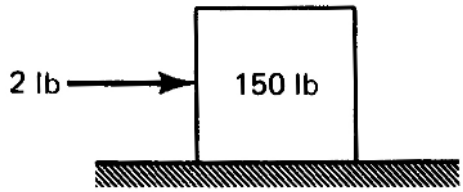
Correct response to preceding frame

No response

Frame 24-2

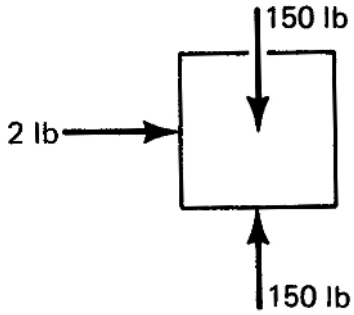
Observations on Friction

First, let's look at some everyday phenomena and see how much you already know about friction.



1. A block of concrete is at rest on a wooden floor when a 2 lb horizontal force is applied as shown. Do you think the block will move?

☐ Yes ☐ No



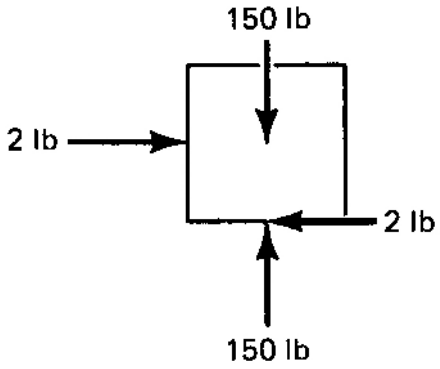
2. The free body diagram shows all the forces that we have studied so far acting on the block. Is the free body in equilibrium?

☐ Yes ☐ No

3. If your answers to the preceding questions are inconsistent, change the free body to make it agree with your observation.

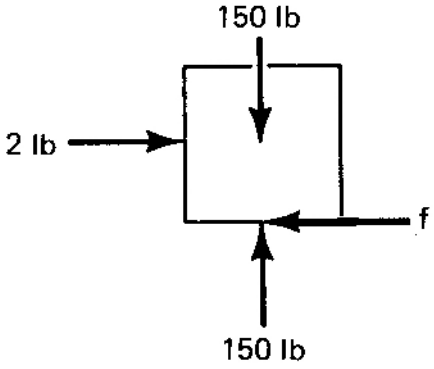
Correct response to preceding frame

1. I don't think it will.
2. No, the free body is not in equilibrium.
3.



Frame 24-3

Observations on Friction



The force f is called friction.

Friction is always in the (***same, opposite***) direction as the impending motion of the body.

Correct response to preceding frame opposite

opposite

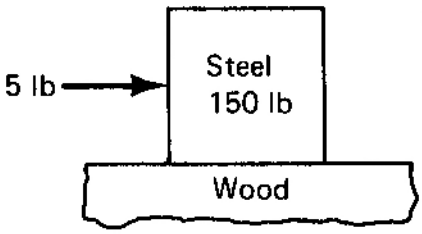
Frame 24-4

Observations on Friction

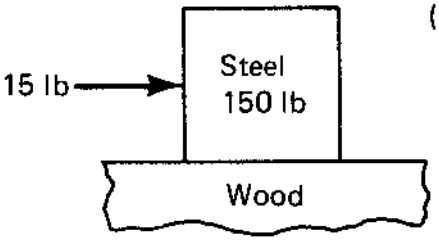
Answer the questions according to what you think most probable.

1. Does the block move?

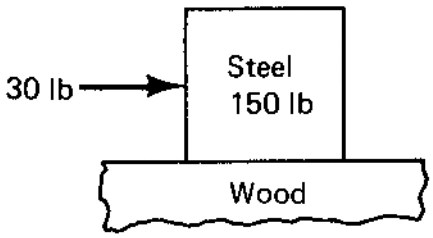
2.



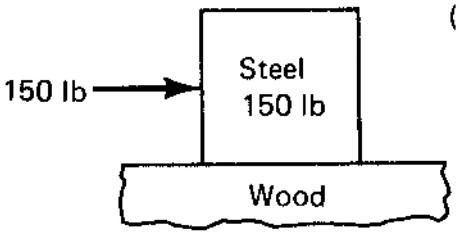
(a) ☐ Yes ☐ No ☐ Maybe



(b) ☐ Yes ☐ No ☐ Maybe



(c) ☐ Yes ☐ No ☐ Maybe



(d) ☐ Yes ☐ No ☐ Maybe

2. Can sufficient friction be developed to counteract any horizontal force, no matter how large?

☐ Yes ☐ No

Correct response to preceding frame

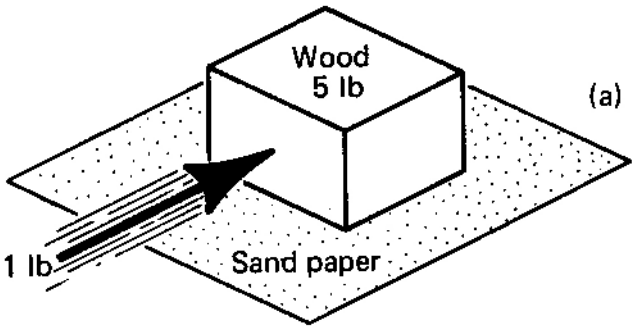
1. My answers:
    a) No
    b) No
    c) Maybe
    d) Yes
2. No. The amount of friction that can be developed at any given surface is limited.

Frame 24-5

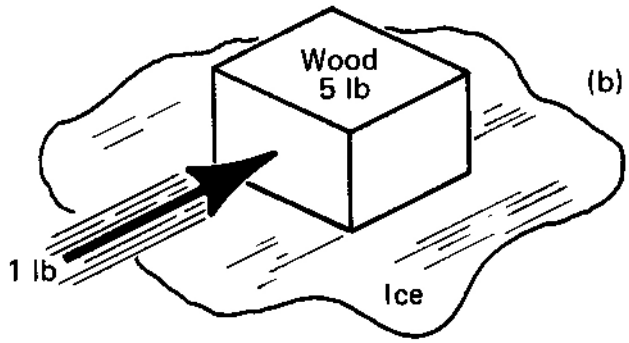
Observations on Friction

Check the answer that most nearly expresses your opinion.

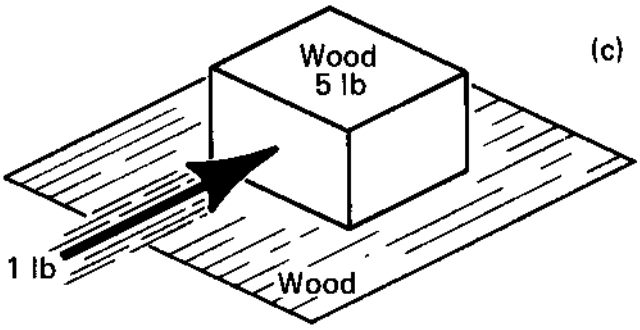
Will the block move?



☐ Yes ☐ Probably
☐ Not likely ☐ No



☐ Yes ☐ Probably
☐ Not likely ☐ No



☐ Yes ☐ Probably
☐ Not likely ☐ No

Correct response to preceding frame

My answers:

a) No
b) Yes
c) Not likely

Frame 24-6

Observations on Friction

The amount of friction which can be developed at a given surface depends upon

_____ .

Correct response to preceding frame

the materials in contact (Or equivalent response)

Frame 24-7

Definition

Friction is defined as the force between two bodies which _____ the motion of one body relative to another.

It is always (***normal, tangent***) to the contact surface between bodies.

Correct response to preceding frame

Friction is defined as the force which ***opposes*** the motion of one body relative to another.

It is always ***tangent*** to the contact surface between bodies.

Frame 24-8

Observations

For a summary of your observations and for some additional information read the portion of your notebook beginning on Page 24-1.

Correct response to preceding frame

No response

Frame 24-9

Transition

The preceding frames have reminded you of a few things you already knew. The next group of frames will help you organize your knowledge into a useful form. In order to do so we will introduce the "coefficient of friction."

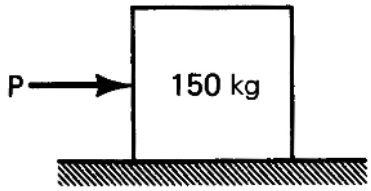
This is not a good place to stop. Keep working if you possibly can. Go to the next frame.

Correct response to preceding frame

No response

Frame 24-10

Observation



The force P acts on the block as shown. P is not constant but increases gradually.

Which of the following statements most closely agrees with your opinion of what will happen?

☐ (a) Since the force P increases gradually, the friction force will be able to increase at the same rate so that the block will not move.

☐ (b) The block will remain stationary for a while but when P becomes large enough the block will move.

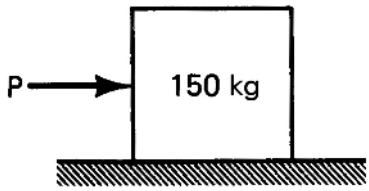
Correct response to preceding frame

Statement (b) is correct.
If you chose (a) you imply that P can become infinitely large without causing motion.
Reconsider.

Frame 24-11

Limiting Friction

The largest friction force that can be developed on a given surface is called the ***limiting friction***.



An experiment is conducted and in repeated trials it is observed that the block begins to move just as P exceeds 300 Newtons.

What is the limiting friction between the block and the plane?

$P_{limiting}$ = _____

Correct response to preceding frame

$P_{limiting} = 300\ N$

Frame 24-12

Limiting Friction

When motion is just about to begin we say it is impending.

Limiting friction is the value attained by the friction when _____ .

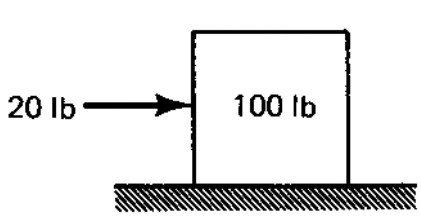
Correct response to preceding frame

motion impends (Or equivalent response)

Frame 24-13

Limiting Friction

The symbol we will use for friction is f. If the friction is limiting we will call it f′.



For the block shown we know that the block will move if the horizontal force exceeds 25 lb.

Draw a free body of the block loaded as shown.

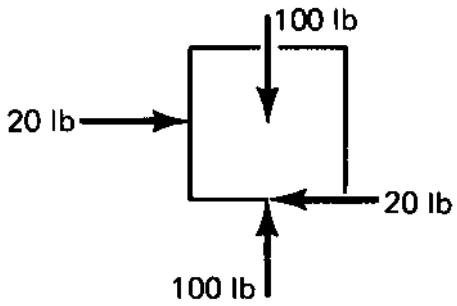
Now compute the following:

f = _____

f′ = _____

Does motion impend? ☐ Yes ☐ No

Correct response to preceding frame



f = 20 lb
f′ = 25 lb
Motion does not impend since f < f′

Frame 24-14

Limiting Friction

1. Can static friction at a given surface ever exceed the limiting friction at that surface?

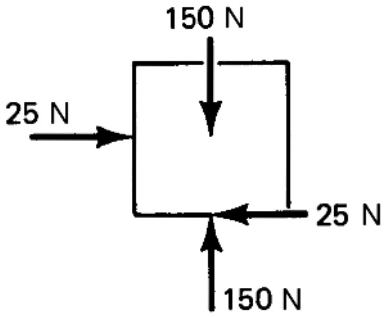
2. Can **f** ever exceed **f′** ? ☐ Yes ☐ No

Correct response to preceding frame

1. No. Limiting friction is the maximum attainable static friction for a given surface.
2. No. $f \leq f'$

Frame 24-15

Limiting Friction



The body is in static equilibrium.

Can you determine whether or not the 25 Newton force is the limiting friction?

☐ Yes ☐ No

If your answer is "Yes", answer the following question.

Does motion impend? ☐ Yes ☐ No

If your answer is "No", answer the following question.

What additional information do you need? _____

_____

Correct response to preceding frame

No. Additional information is needed about the characters of the surfaces in contact.

or

No. Additional information is needed about whether or not motion impends.

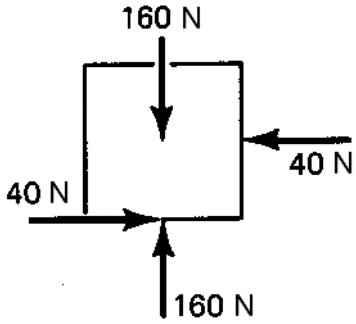
(Or equivalent response)

Frame 24-16

Coefficient of Friction

Information about contact surfaces is "packaged" in the coefficient of friction.

The coefficient of friction is found by dividing the limiting friction by the normal force.



Assuming it is known that motion impends for the block, determine the coefficient of friction.

Coefficient of friction = _____

Correct response to preceding frame

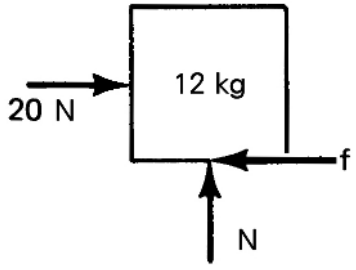
$$.25 = \frac{40\ N}{160\ N}$$

Frame 24-17

Coefficient of Friction

It is common practice in America to denote the normal force in friction problems as N. This was started long before we started using SI where the force unit is the Newton. (Life just ain't fair.)

What is the value of N for the block below?



N = _____

Correct response to preceding frame

N = 118 Newtons
(or N = 118 N) Some professors like to say, "Well, you'll know what I mean by the context," when they write things like this.

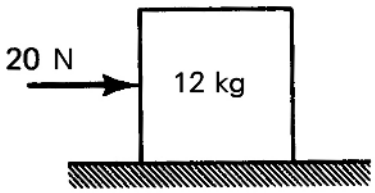
Frame 24-18

Coefficient of Friction

The symbol for coefficient of static friction is μ. (The coefficient of kinetic friction is μ_k. We'll come to it shortly.)

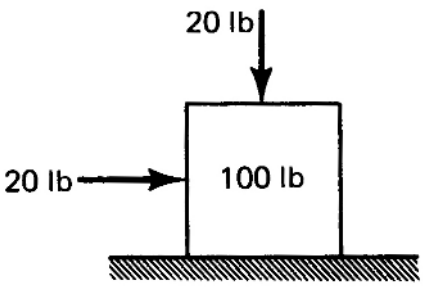
Draw the FBD and find μ for the following, assuming that motion impends with the forces shown.

1.



μ = _____

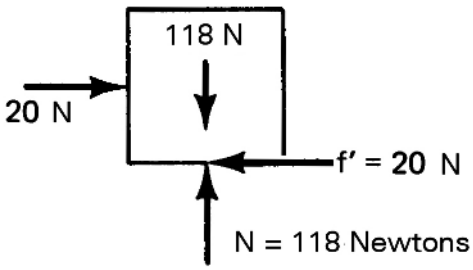
2.



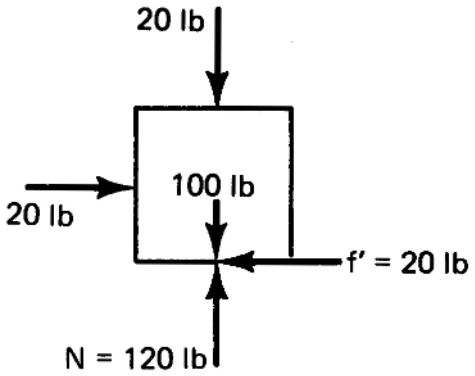
μ = _____

Correct response to preceding frame

1. $\mu = \frac{20}{118} = 0.170$



2. $\mu = \frac{20}{120} = .167$



Frame 24-19

Coefficient of Friction

Write the equation for the coefficient of friction

$f' =$ _____

and tell what each symbol means.

Correct response to preceding frame

$f' = \mu N$
μ = coefficient of friction
f' = limiting friction on a given plane
N = normal force on the plane

Frame 24-20

Review

Friction is _____

Limiting friction is _____

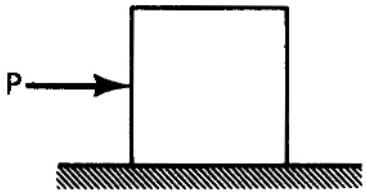
Limiting friction implies _____

Correct response to preceding frame

Friction is the force between two bodies which opposes the motion of one body relative to the other.
Limiting friction is the maximum value of friction obtainable at a given surface.
Limiting friction implies impending motion.

Frame 24-21

Observation



P is increased gradually until the block begins to move.

Check the best description of what happens.

☐ (a) The block moves off smoothly requiring the same force to keep it moving as it took to get it started.

☐ (b) The block starts with a jerk, but once started can be kept moving with a smaller force than was required to initiate motion.

Correct response to preceding frame

(b) is correct
(If your choice was (a), experiment with a book or some such thing until you are convinced.)

Frame 24-22

Kinetic Friction

Kinetic friction is the friction between surfaces which are actually moving with respect to one another.

For a given pair of surfaces, will the kinetic friction exceed the limiting static friction?

☐ Yes ☐ No

Correct response to preceding frame

No. Kinetic friction is always less than limiting static friction on the same surface. This may be verified experimentally by methods similar to those described in the last frame.

Frame 24-23

Coefficient of Kinetic Friction

The coefficient of kinetic friction, μ_k, is determined by dividing the friction force on the plane between sliding bodies by the normal force on that plane.

For a given plane which will be larger?

- ☐ μ
- ☐ μ_k

Correct response to preceding frame

$\mu > \mu_k$

Frame 24-24

Summary

Complete the next section of your notebook.

Correct response to preceding frame

No response

Frame 24-25

Transition

Now you know about all you can be told about friction. Further information about it can be obtained only by the study of experimental data.

The remainder of the unit will be devoted to some problems dealing with friction, limiting friction, and determination of whether or not motion impends.

This is a good place to stop for a moment to let the information you have just received sink in. The remainder of the unit will take 30 to 45 minutes.

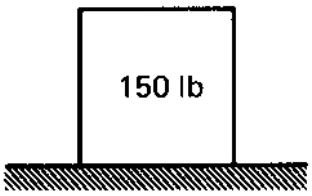
Since they will deal with friction, the coming problems will necessarily be rough. When you have recovered from that pun, go to the next frame.

Correct response to preceding frame

No response

Frame 24-26

Finding Friction

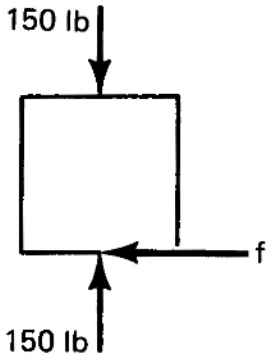


The box is at rest on the plane. The coefficient of friction between the box and the horizontal plane is 0.20.

1. Draw a FBD of the box.

2. From the FBD evaluate the friction.

Correct response to preceding frame

1. 

2. f = 0

Frame 24-27

Finding Friction

1. Does the inclusion of a coefficient of friction in the problem imply that there is a friction force?

☐ Yes ☐ No

2. Does friction exist in a situation where there is no tendency for a body to move?

☐ Yes ☐ No

3. Does the inclusion of a coefficient of friction imply impending motion?

☐ Yes ☐ No

Correct response to preceding frame

1. No, merely that one could exist if equilibrium required it.
2. No, friction resists the tendency to motion. No tendency, no friction.
3. No, the coefficient of friction is only a piece of information about the nature of the materials you are dealing with.

Frame 24-28

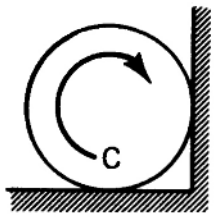
Impending Motion

Several problems are stated below. Do not work them but check those in which impending motion is stated or implied.

☐ (a)

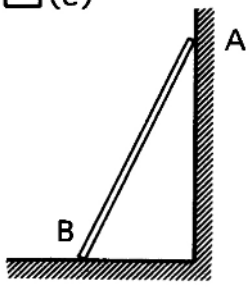
A man starts to climb the ladder shown. Determine how far he can climb before the ladder starts to slip.

☐ (b)



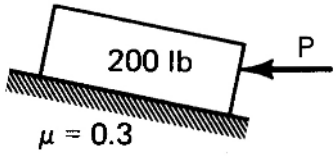
Find the maximum couple that can be applied without causing slippage if the coefficient of friction at the contact surfaces is 0.3.

☐ (c)



Determine the friction at A if the ladder weighs 20 pounds and the coefficient of friction at all contact surfaces is 0.2.

☐ (d)



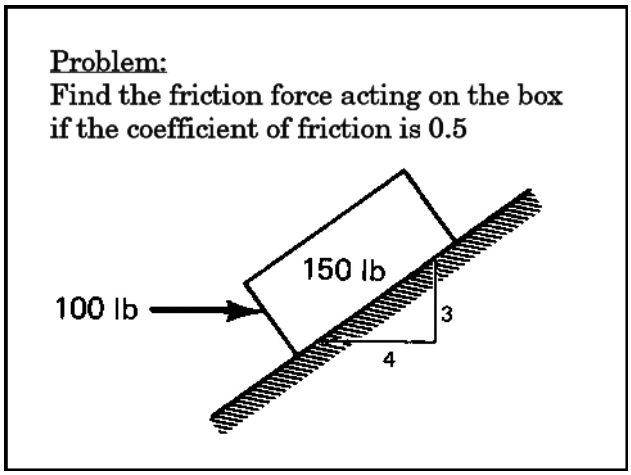
Find the minimum force P to keep the box in equilibrium.

Correct response to preceding frame

Impending motion is implied in (a), (b) and (d).
Any problem dealing with maximum or minimum values implies impending motion.

Frame 24-29

Finding Friction



Does the above problem imply impending motion?

☐ Yes ☐ No

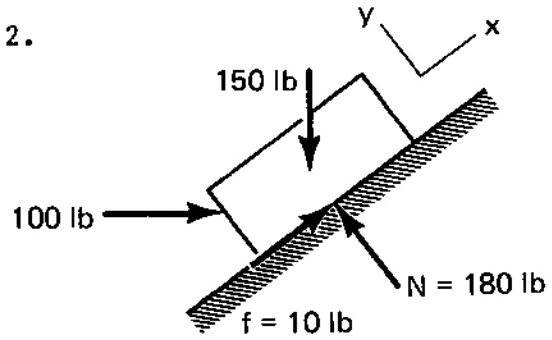
Draw a free body diagram of the box assuming equilibrium.

Solve for the unknown forces.

Correct response to preceding frame

1. No, motion may impend but we do not know that it does.

2.



f = 10 lb

N = 180 lb

Solution:

For convenience tilt axes as shown.

$100\,[.8\bar{i} - .6\bar{j}] + 150\,[-6\bar{i} - .8\bar{j}]$

$+ N\bar{j} + f\bar{i} = 0$

i coefficients

$80 - 90 + f = 0$

$f = 10$

j coefficients

$-60 - 120 + N = 0$

$N = 180$

Frame 24-30

Finding Friction

Using the information from the preceding frame, answer the following.

1. What friction force actually acts on the box? f = _____

2. What is the limiting friction? ($\mu = 0.5$) f′ = _____

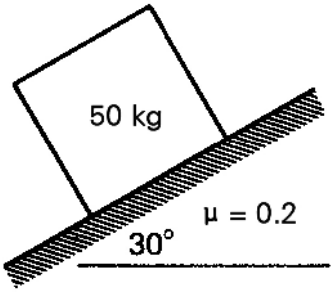
3. Does motion impend? ☐ Yes ☐ No

Correct response to preceding frame

1. 10 lb = f
2. 90 lb = μN = f'
3. No f < f'

Frame 24-31

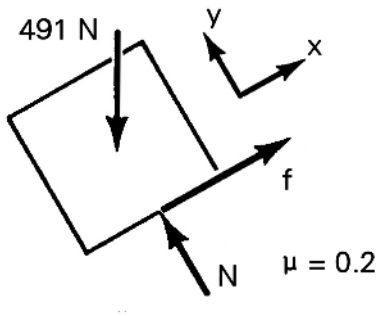
Does Motion Impend?



The block weighs 50 kilograms. The coefficient of friction between the block and the plane is 0.2.

Assuming the block to be in equilibrium, draw a free body diagram of it and solve for all forces.

Correct response to preceding frame



Solution:

$$491\,(-\sin 30\,\bar{i} - \cos 30\,\bar{j}) + f\,\bar{i} + N\,\bar{j} = 0$$

$$-491 \sin 30 + f = 0$$

$$f = 245$$

$$-491 \cos 30 + N = 0$$

$$N = 425$$

Frame 24-32

Does Motion Impend?

Using the free body and data from the last problem answer the following questions.

1. What is the value of the limiting friction? $f' =$ _____

2. What friction is necessary to keep the block from slipping? $f =$ _____

3. Can the block be in equilibrium? ☐ Yes ☐ No

Correct response to preceding frame

1. F′ = μN = 85 Newtons
2. f = 245 lb
3. No. 245 > 85 so the block slips and this problem is not a problem in statics.

Frame 24-33

Does Motion Impend?

1. If f is less than f′, motion (***does, does not***) impend and the body (***is, is not***) in static equilibrium.

2. If f equals f′ motion (***does, does not***) impend and the body (***is, is not***) in static equilibrium.

3. If f is greater than f′ the body _____ .

Correct response to preceding frame

1. f < f′ motion does not impend body in equilibrium
2. f = f′ motion impends but the body is still in equilibrium
3. f > f′ body moves and is not in equilibrium (Or equivalent response)

Frame 24-34

Problem Solution

The steps in solving a statics problem involving friction are:

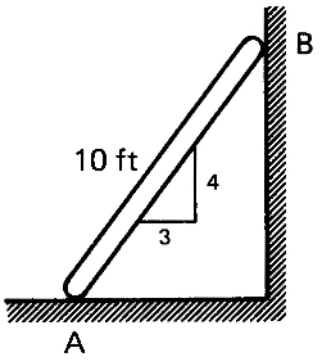
1. Determine whether the problem states or implies that motion impends.

2. If motion does not impend* assume the body to be in equilibrium and draw a free body diagram.

3. Solve for unknown forces.

4. By comparing f to f′ determine whether
   a) motion impends
   b) the body is really in equilibrium

Use these steps to work the following problem:

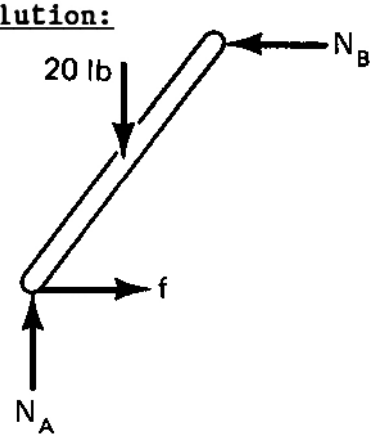


The bar weighs 20 lb and the coefficient of friction at A is 0.4. The wall is smooth. Find the forces acting at A.

*We will deal with the case when it does a little later.

Correct response to preceding frame

Solution:



$\sum \overline{F} = 0$ gives $N_A = 20$ and $N_B = f$

$\sum \overline{M}_B = 0$ gives $8f + 20(3) - 20(6) = 0$

$$f = \frac{60}{8} = 7.5 \text{ lb}$$

$f' = 8$ lb

$f < f'$ so body is in equilibrium and motion does not impend.

Note that the "real" answer to this problem is a decision - "the ladder will not slip" - not a number.

Frame 24-35

Problem Solution

The steps you took in solving the last problem were

1. Read the problem to determine _____ .

2. Draw the FBD assuming _____ .

3. Solve for all unknowns.

4. Compare f to f′ to see if _____ and if _____ .

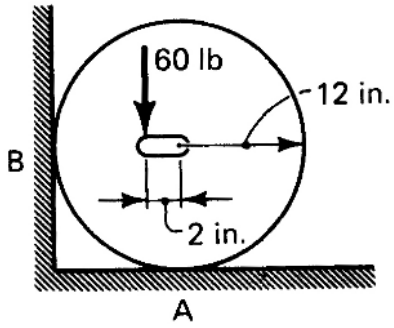
Correct response to preceding frame

1. Read the problem to determine if motion impends.
2. Draw the FBD assuming equilibrium.
3. Solve.
4. Compare `f` to `f'` to see if motion impends and if the body is in equilibrium.

Frame 24-36

Problem Solving

The wheel shown weighs 100 lb. The vertical wall is smooth and the coefficient of friction between the wall and the floor is 0.3.



1. Find the friction force exerted by the floor on the wheel.

2. Suppose the vertical wall were on the other side of the wheel. What would the result be?

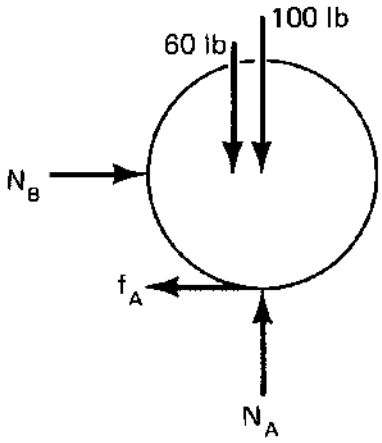
_____

Correct response to preceding frame

1. f = 10
   body in equilibrium

2. The wheel would roll away since to hold it N_B would have to have an impossible direction.

Solution:



$\sum \overline{F} = 0$ gives

$N_A = 160$

$f_A = N_B$

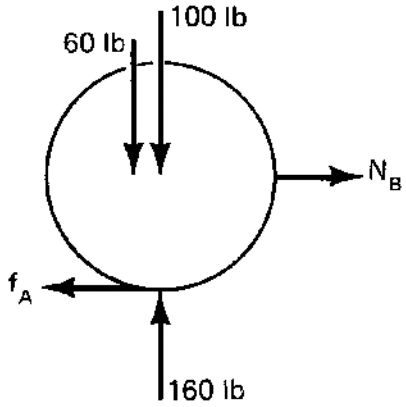
$\sum \overline{M}_A = 0$

gives

$60(2) - N_B(12) = 0$

$N_B = 10 = f$

$f' = 48$ $\quad f < f'$



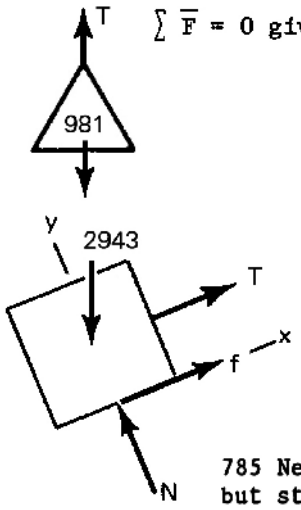
Frame 24-37

Summary

Complete your notebook.

Correct response to preceding frame

No. The body slides. Solution:



$\sum \overline{F} = 0$ gives $T = 981$

$\sum \overline{F} = 0$ gives

$N = 0.8\ (2943) = 2354$

$f = 0.6\ (2943) - T = 785$

The friction equation gives

$f' = 0.30\ (2354) = 706$

$f > f'$

785 Newtons are required for equilibrium, but static friction will provide only 706 Newtons.

Frame 24-38

Conclusion

This unit has helped you organize the observations you and others have made about friction. It has emphasized that information about friction must be observed rather than derived.

You have looked at friction, limiting friction, coefficient of friction and impending motion. You have worked a number of problems involving these concepts in problems where motion is not known to impend. (The units which follow will apply these concepts to problems with motion known to be impending.)

Since friction is an observable phenomenon, the most important single thing to remember in any problem involving friction is, "Keep a firm grasp on reality; use common sense." It should put you halfway to a successful conclusion of your labors.