

Introduction to Statics

.PDF Edition – Version 0.95

Unit 23

Equilibrium with Couples

Helen Margaret Lester Plants

Late Professor Emerita

Wallace Starr Venable

Emeritus Associate Professor

West Virginia University, Morgantown, West Virginia

© Copyright 2010 by Wallace Venable

Conditions of Use

This book, and related support materials, may be downloaded without charge for personal use from www.SecretsOfEngineering.net

You may print one copy of this document for personal use. You may install a copy of this material on a computer or other electronic reader for personal use.

Redistribution in any form is expressly prohibited.

Unit 23

Equilibrium with Couples

Frame 23-1

Introduction

In this unit you will gain further experience with problems in which the applied force systems involve couples. Situations of this sort are particularly common in machines involving motors, shafts, gears and pulleys, so this type of problem is sometimes referred to as "equilibrium of machine elements".

Turn to the next frame.

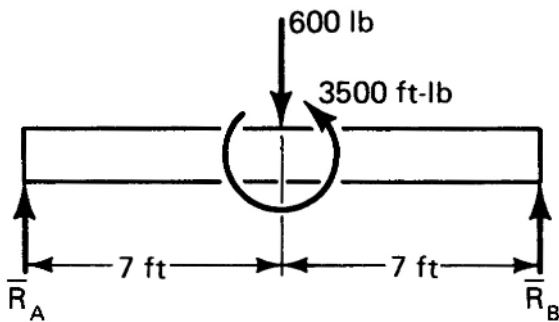
Correct response to preceding frame

No response

Frame 23-2

Couple Loadings

Let's look at the end reactions on a beam which is subjected to a single concentrated force plus a couple.



When we include the couple, the equations $\sum \bar{F} = 0$ and $\sum \bar{M} = 0$ must still be satisfied.

$\sum \bar{F} = 0$ gives

$\sum \bar{M}_A = 0$ gives

$$(+7\bar{i}) \times (-600\bar{j}) + (14\bar{i}) \times (R_B\bar{j}) + 3500\bar{k} = 0$$

Solve the simultaneous equations for R_A and R_B .

Correct response to preceding frame

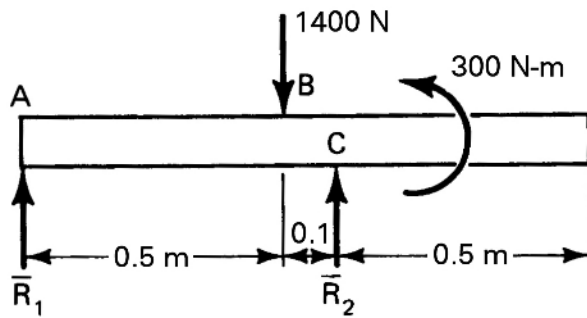
$$\bar{R}_A + \bar{R}_B - 600\bar{j} = 0$$

$$R_B = 50 \text{ lb}$$

$$R_A = 550 \text{ lb}$$

Frame 23-3

Couple Loadings



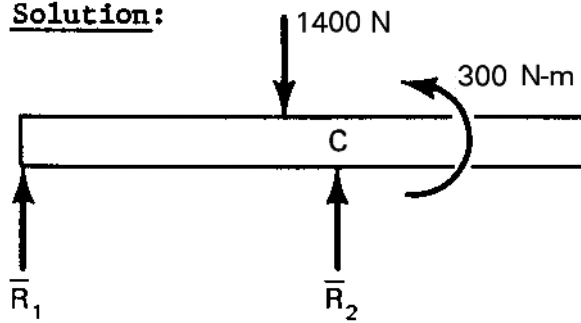
Find \bar{R}_1 and \bar{R}_2 on the beam shown.

Correct response to preceding frame

$$\bar{R}_1 = 733 \bar{j} \text{ N}$$

$$\bar{R}_2 = 667 \bar{j} \text{ N}$$

Solution:



$$\sum F_y = R_1 + R_2 - 1400 = 0$$

$$R_1 + R_2 = 1400$$

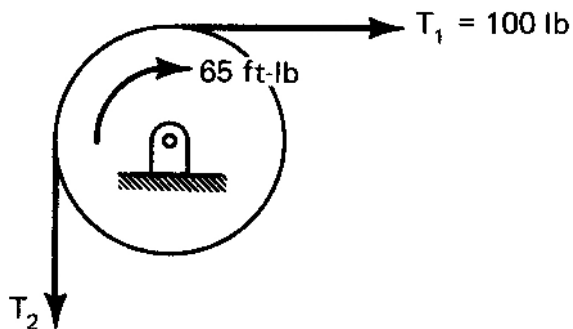
$$\sum \bar{M}_C = 300\bar{k} + (-0.6\bar{i} \times R_1\bar{j}) + (-0.1\bar{i} \times -1400\bar{j}) = 0$$

$$0.6R_1 = 300 + 140$$

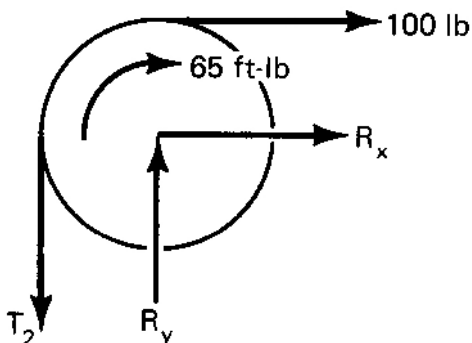
Frame 23-4

Couple Loadings

Couples are often applied to pulleys and drums.



The drum shown is 2 feet in diameter and has a couple and two cables acting on it.



Find the tension T_2 and the reactions on the pin following the steps given below.

1. Sum forces on the drum.
2. Sum moments about the pin.
3. Solve for the unknowns.

Correct response to preceding frame

$$\begin{aligned}\bar{T}_2 &= 165 \text{ lb} \\ \bar{R}_x &= -100 \text{ lb} \\ \bar{R}_y &= 165 \text{ lb}\end{aligned}$$

Frame 23-5

Couple Loadings

The new concept which this unit has introduced into the computation of reactions with the equations $\sum \bar{F} = 0$ and $\sum \bar{M} = 0$ is that when couples are applied to the body we must include _____ in _____ .

Correct response to preceding frame

include *the couples* in *the moment equation*

Frame 23-6

Notebook

Work the problem on page 23-1 of your notebook.

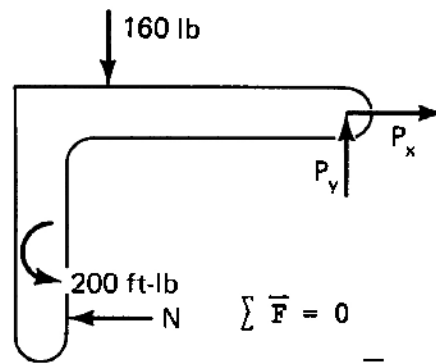
Correct response to preceding frame

$$P_x = 280 \text{ lb}$$

$$P_y = 160 \text{ lb}$$

$$N = 280 \text{ lb}$$

Solution:



$$\sum \bar{F} = 0$$

$$(P_x - N)\bar{i} + (P_y - 160)\bar{j} = 0$$

$$P_y = 160$$

$$P_x = N$$

$$\sum \bar{M}_p = 0$$

$$200\bar{k} + (-4\bar{i} \times -160\bar{j}) + (-3\bar{j} \times -N\bar{i}) = 0$$

$$200 + 640 - 3N = 0$$

Frame 23-7

Transition

In the problems you have studied so far you have found forces which resulted from applied couples. Next we will consider some cases in which the applied loads result in a reaction which includes a couple. You will find that these include some important engineering applications so let's go to the next section of this unit.

Correct response to preceding frame

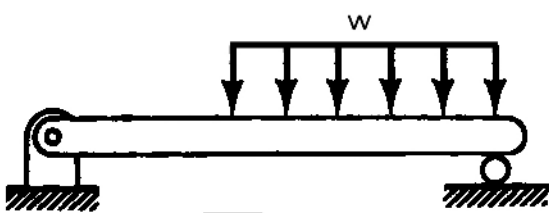
No response

Frame 23-8

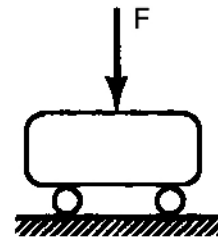
Beams

A beam is a relatively slender structural member subject to transverse (cross-wise) loads. We usually think of it as being horizontal, but that need not be the case.

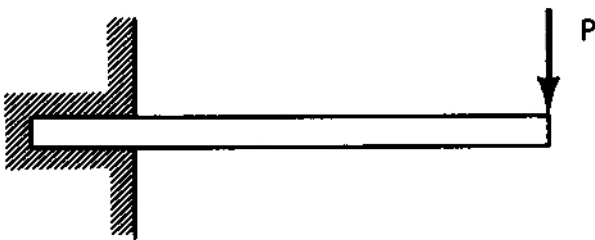
Which of the following would you think would be treated as beams?



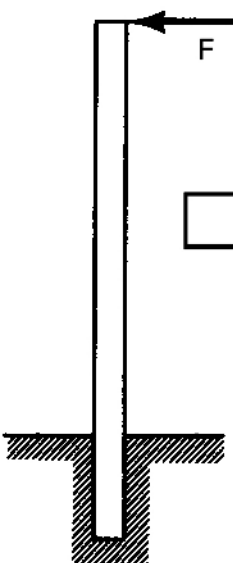
(a)



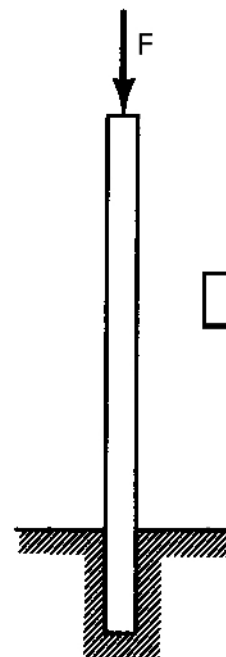
(b)



(c)



(d)



(e)

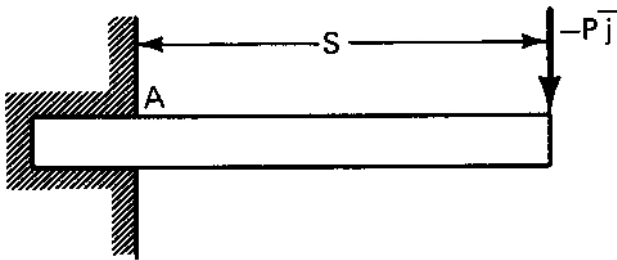
Correct response to preceding frame

- (a) (c) and (d) would be considered as beams
 - (b) is too short and (e) is an axially loaded member called a column
-

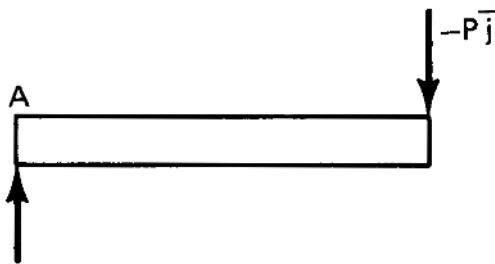
Frame 23-9

Couples As Reactions

A very common type of structure which we have been carefully avoiding is called the cantilever beam, a beam with one end built into its support.



Our free body diagram will show the beam as being cut off at A.



What force at A will be necessary to hold the beam in equilibrium?

Will the force alone give us a situation which satisfied the moment equation?

- Yes No

Correct response to preceding frame

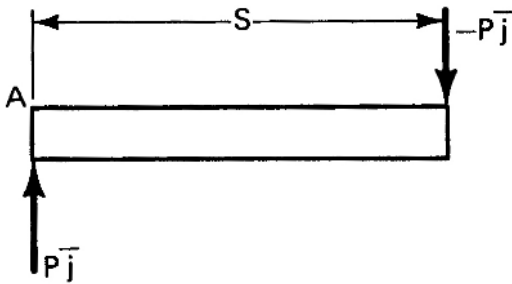
$$\bar{A}_y = +13\bar{j}$$

No

Frame 23-10

Couples as Reactions

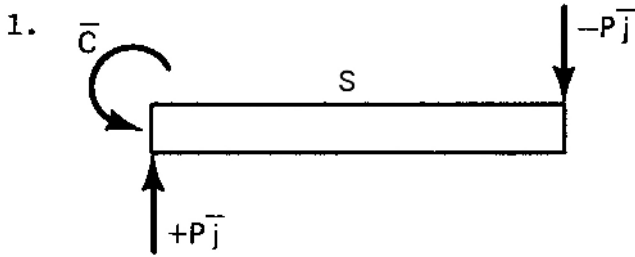
Our cantilever beam now looks like this.



We must now add a couple to the FBD at A to satisfy the equation $\sum \bar{M}_A = 0$

1. Draw a couple \bar{C} on the FBD at A.
2. Write and solve the moment equation to find \bar{C} .

Correct response to preceding frame



2. $\bar{C} = PS\bar{k}$

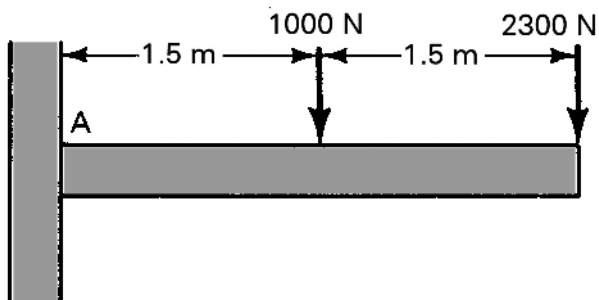
Solution:

$$\sum \bar{M}_A = C\bar{k} + (S\bar{i} \times -P\bar{j}) = (C - PS)\bar{k}$$

Frame 23-11

Couples as Reactions

The beam below is welded to the column at one end and is loaded as shown.

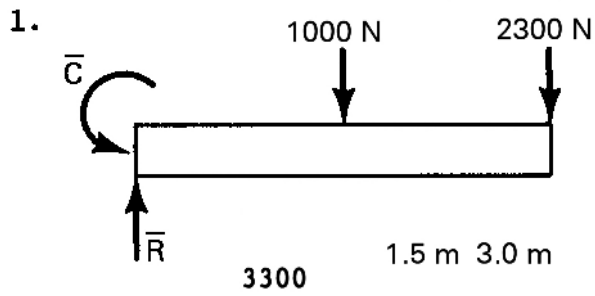


1. "Cut" the beam off at A and draw its FBD.

2. Find the force at the cut.

3. Write a moment equation and find the couple at the "built-in" end.

Correct response to preceding frame



2. $\sum \bar{F} = (R - 1000 - 2300) \bar{j} = 0$

$$\bar{R} = 3300 \bar{j} \text{ N}$$

3. $\sum \bar{M}_A = [C - 1.5(1000) - 3.0(2300)] \bar{k}$

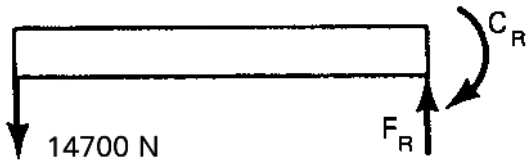
$$\bar{C} = 8400 \bar{k} \text{ N-m}$$

Frame 23-12

Problem

Work Problem 23-2 in your notebook.

Correct response to preceding frame



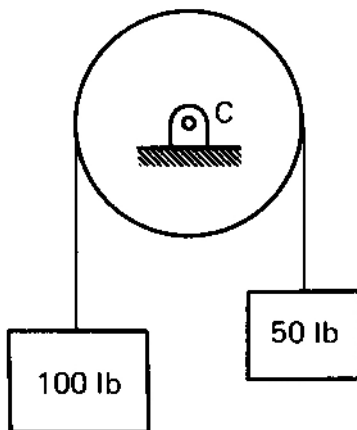
$$\vec{F}_R = 14700 \vec{j} \text{ N}$$

$$\vec{C}_R = -29400 \vec{k} \text{ N-m}$$

Frame 23-13

Pulleys on Shafts

Here is a pulley which is fixed to a shaft.



Will there be a vertical reaction at C?

Yes No

Will the pulley be in equilibrium?

Yes No

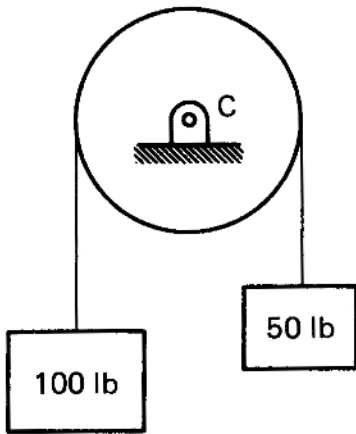
Correct response to preceding frame

Yes

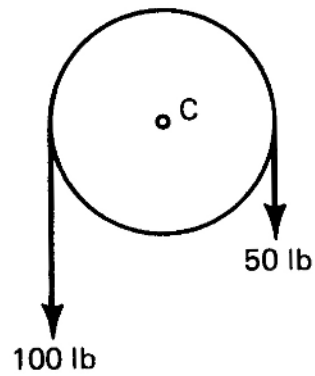
No, it will turn.

Frame 23-14

Pulleys on Shafts



A



B

The pulley is back.

If the pulley is in equilibrium and you find the reaction at C, which of the following will you get? (circle your answer)

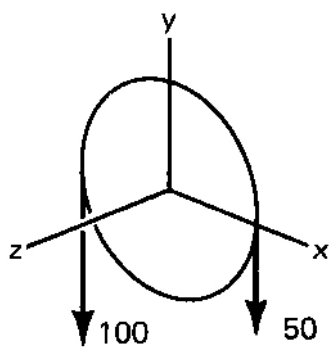
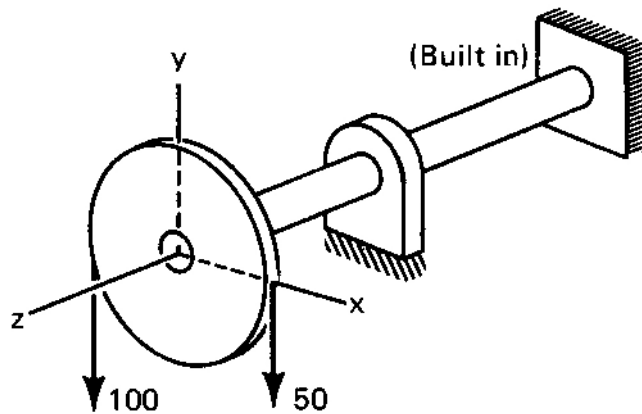
- (a) a force
- (b) two forces
- (c) a couple
- (d) a force and a couple

Correct response to preceding frame

(d) a force and a couple

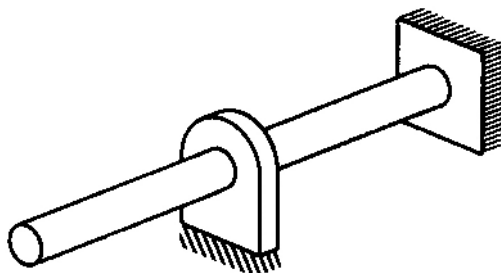
Frame 23-15

Pulleys on Shafts

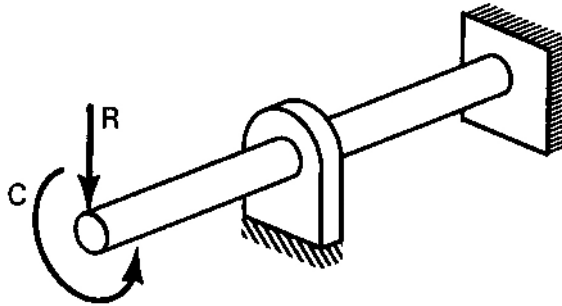
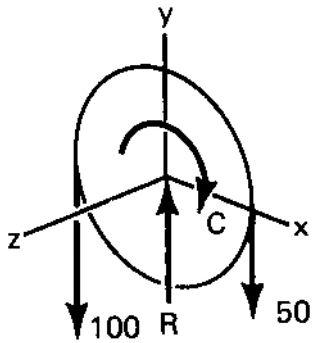


1. Complete the free body diagram of the pulley. (Assume equilibrium.)

2. Show the reaction on the left end of the shaft.



Correct response to preceding frame



Frame 23-16

Pulleys on Shafts

The couple C must be taken into account in our equilibrium equations in addition to the forces. Since a couple has a moment but no net force it enters the moment equation but not the force equation.

Using this and the additional information that the diameter of the pulley is 3, solve for \bar{R} and \bar{C} .

$$\bar{R} = \underline{\hspace{4cm}}$$

$$\bar{C} = \underline{\hspace{4cm}}$$

Correct response to preceding frame

$$\bar{R} = 150\bar{j}$$

$$\bar{C} = -75\bar{k}$$

Solution:

$$\sum \bar{F} = 0$$

$$(-100 - 50 + R)\bar{j} = 0$$

$$R = 150$$

$$\sum \bar{M} = 0$$

$$\left(-\frac{3}{2}\bar{i} \times -100\bar{j}\right) + \left(\frac{3}{2}\bar{i} \times -50\bar{j}\right) - c\bar{k} = 0$$

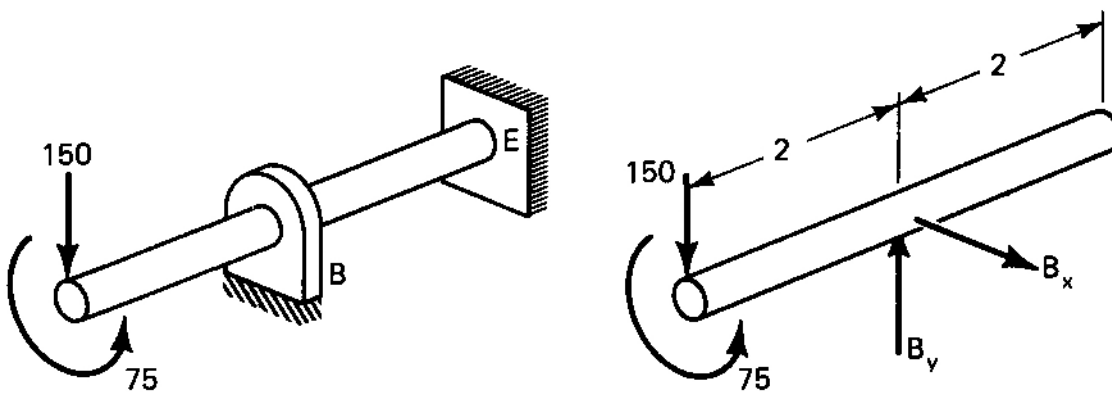
$$(150 - 75 - c)\bar{k} = 0$$

$$c = 75$$

Frame 23-17

Pulleys on Shafts

If we now turn our attention to the shaft,



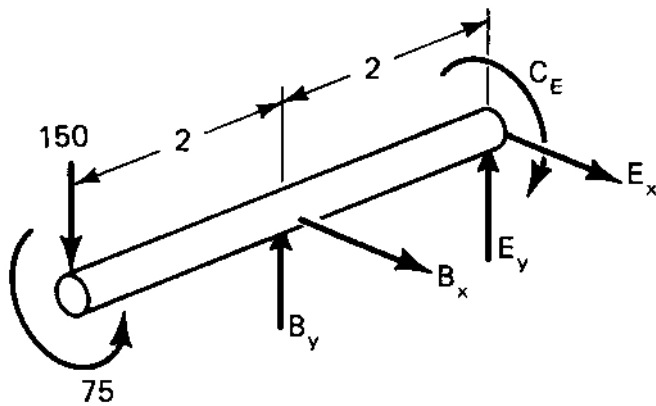
we see that there must be some reaction at E in addition to the support of the smooth bearing at B.

What forces and/or moments are necessary at E for equilibrium? _____

Complete the free body diagram.

Correct response to preceding frame

two forces and a couple



We see at once that $B_x = E_x = 0$

Frame 23-18

Pulleys on Shafts

We can solve for B_y , E_y and C_E on the shaft by taking moments about E.

$$\sum \bar{M}_E = (4\bar{k} \times -150\bar{j}) + (2\bar{k} \times B_y\bar{j}) + 75\bar{k} - C_E\bar{k} = 0$$

Write the force equation, work out the products and solve for the unknowns.

Correct response to preceding frame

$$\bar{B}_y = 300\bar{j}$$

$$\bar{E}_y = -150\bar{j}$$

$$\bar{C}_E = -75\bar{k}$$

Solution:

$$\sum \bar{F} = 0$$

$$-150\bar{j} + B_y\bar{j} + E_y\bar{j} = 0$$

$$B_y + E_y = 150$$

$$\sum \bar{M}_E = 0$$

$$600\bar{i} - 2B_y\bar{i} + 75\bar{k} - C_E\bar{k} = 0$$

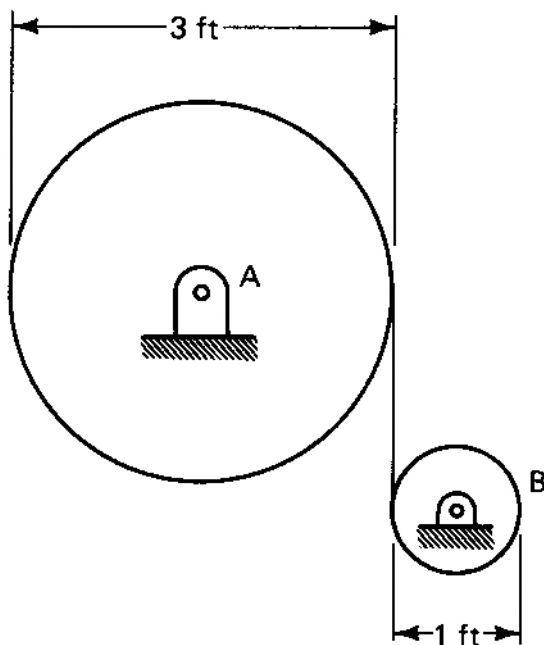
$$600 - 2B_y = 0$$

$$75 - C_E = 0$$

Frame 23-19

Pulleys on Shafts

When a couple is applied about the axis of a shaft the moment of the couple is frequently called the "torque" on the shaft.



If a counter-clockwise torque of 60 foot-pounds is applied to the shaft connected to the upper pulley what torque must be applied to the lower pulley for equilibrium?

$$\bar{C}_B = \underline{\hspace{2cm}}$$

What is the reaction force at shaft B?

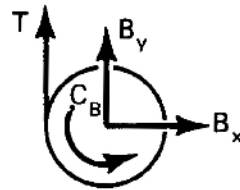
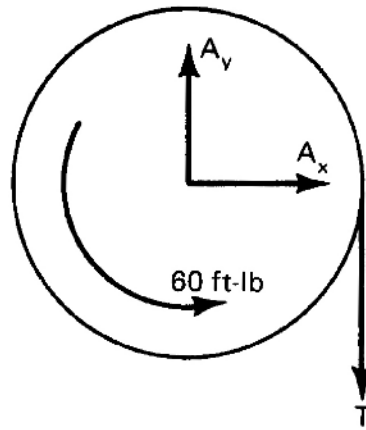
$$\bar{R}_B = \underline{\hspace{2cm}}$$

Correct response to preceding frame

$$\bar{C}_B = 20\bar{k} \text{ ft-lb}$$

$$\bar{R}_B = -40\bar{j} \text{ lb}$$

Solution:



$$\sum \bar{M}_A = 0$$

$$60\bar{k} + \left[\frac{3}{2} \bar{i} \times -T\bar{j} \right] = 0$$

$$T = 40 \text{ lb}$$

$$\sum \bar{M}_B = 0$$

$$C_B\bar{k} + \left[-\frac{1}{2} \bar{i} \times 40\bar{j} \right] = 0$$

$$C_B = 20 \text{ ft-lb}$$

$$\sum \bar{F} = 0$$

$$40\bar{j} + B_y\bar{j} + B_x\bar{i} = 0$$

$$B_x = 0$$

$$B_y = -40 \text{ lb}$$

Frame 23-20

Notebook

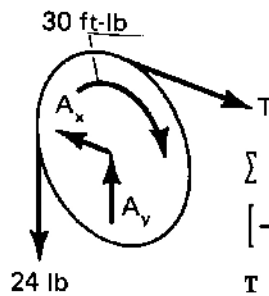
Work the Problem 23-3 in your notebook so you will have something to refer to later.

Correct response to preceding frame

$$\bar{C}_B = -12\bar{k} \text{ ft-lb on shaft}$$

$$\bar{R}_A = 12\bar{i} - 24\bar{j} \text{ lb on shaft}$$

Solution:



$$\sum \bar{M}_A = 0$$

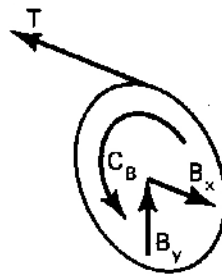
$$\left[-\frac{5\bar{i}}{2} \times -24\bar{j} \right] - 30\bar{k} + \left[\frac{5}{2}\bar{j} \times T\bar{i} \right] = 0$$

$$T = \frac{2}{5} (30) = 12 \text{ lb}$$

$$\sum \bar{F} = 0$$

$$(-A_x + 12)\bar{i} + (A_y - 24)\bar{j} = 0$$

$$A_x = 12 \text{ lb} \quad A_y = 24 \text{ lb}$$



$$\sum \bar{M}_B = 0$$

$$(1\bar{j} \times -T\bar{i}) + C_B\bar{k} = 0$$

$$(12 + C_B)\bar{k} = 0$$

$$C_B = -12 \text{ ft-lb}$$

Frame 23-21

Transition

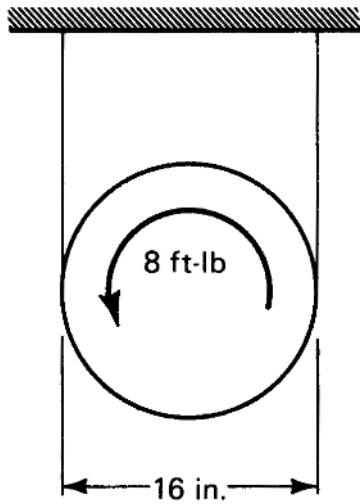
In addition to giving you some problem solving information, the preceding problem should have given you some idea as to how couples may be applied to a body. Often a couple will be shown as a given load without telling you where it comes from, just as forces are often arbitrarily specified.

Correct response to preceding frame

No response

Frame 23-22

Applied Couples



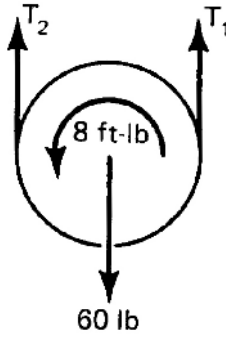
A 60 pound disc is supported by two cables and has a 8 foot-pound couple applied to it. Find the tension in the cables.

Correct response to preceding frame

$$T_1 = 24 \text{ lb}$$

Solution:

$$T_2 = 36 \text{ lb}$$



$$\sum \bar{F} = (-60 + T_1 + T_2)\bar{j} = 0$$

$$T_1 + T_2 = 60$$

$$\sum \bar{M}_O = \left[8 + \frac{8}{12} T_1 - \frac{8}{12} T_2 \right] \bar{k} = 0$$

$$T_1 - T_2 = -12$$

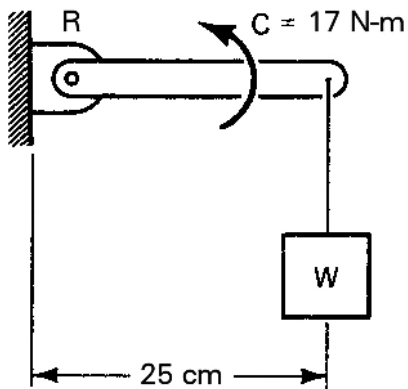
$$T_1 + T_2 = 60$$

$$2 T_1 = 48$$

Frame 23-23

Applied Couples

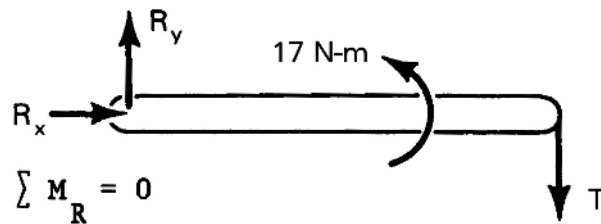
A couple of $17\bar{k}$ Newton-meters acts as shown on a weightless bar. Find the weight W which will be held in equilibrium.



Correct response to preceding frame

$$W = 9.93 \text{ kg}$$

Solution:



$$\sum M_R = 0$$

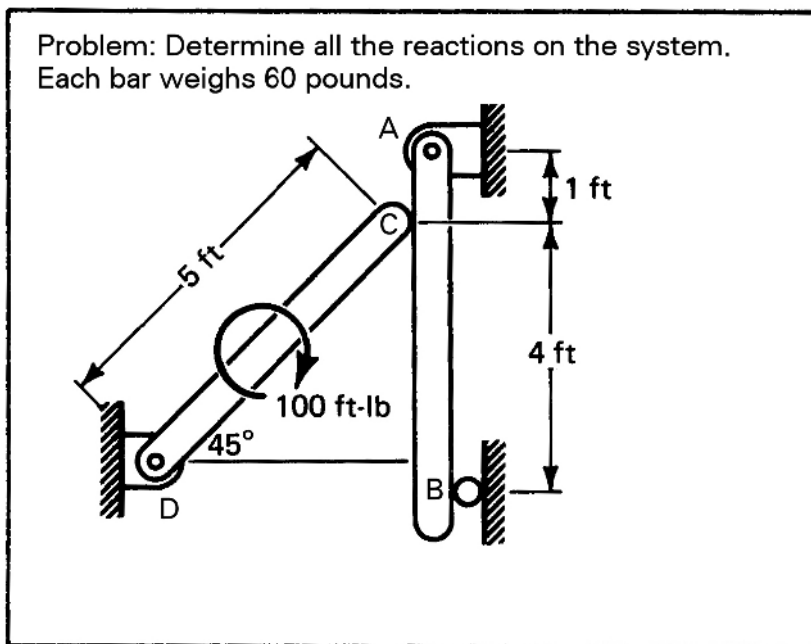
$$17\bar{k} - 0.25T\bar{k} = 0$$

$$T = \frac{17}{0.25} = 68 \text{ N}$$

$$W = \frac{T}{9.81} = 9.93 \text{ kg}$$

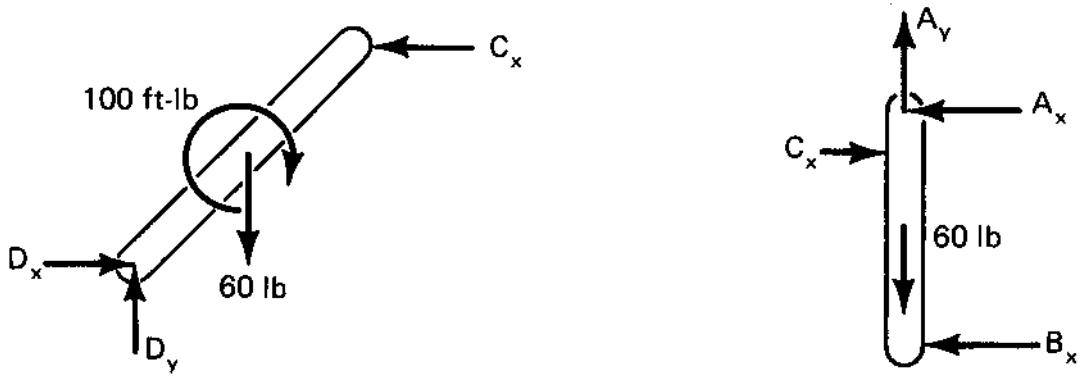
Frame 23-24

Applied Couples--Systems



Draw the free body diagrams of the members. Do not write the equations yet.

Correct response to preceding frame



Frame 23-25

Applied Couples--Systems

Write the equilibrium equations for the system given in the preceding frame. The FBDs are shown above.

Correct response to preceding frame

on AB

$$\sum \bar{F} = 0$$

$$(-A_x - B_x + C_x)\bar{i} + (A_y - 60)\bar{j} = 0$$

$$\sum \bar{M}_A = (-\bar{j} \times C_x\bar{i}) + (-5\bar{j} \times -B_x\bar{i}) = 0$$

on CD

$$\sum \bar{F} = (D_x - C_x)\bar{i} + (D_y - 60)\bar{j} = 0$$

$$\sum \bar{M}_D = 0$$

$$\left[\left(\frac{5\bar{i}}{\sqrt{2}} + \frac{5\bar{j}}{\sqrt{2}} \right) \times -C_x\bar{i} \right] - 100\bar{k} + \left[\left(\frac{5\bar{i}}{2\sqrt{2}} + \frac{5\bar{j}}{2\sqrt{2}} \right) \times -60\bar{j} \right] = 0$$

Frame 23-26

Applied Couples--Systems

Complete the problem by solving your equations for the reactions.

Correct response to preceding frame

$$C_x = 58.3 \text{ lb}$$

$$D_x = 58.3 \text{ lb}$$

$$D_y = 60 \text{ lb}$$

$$A_y = 60 \text{ lb}$$

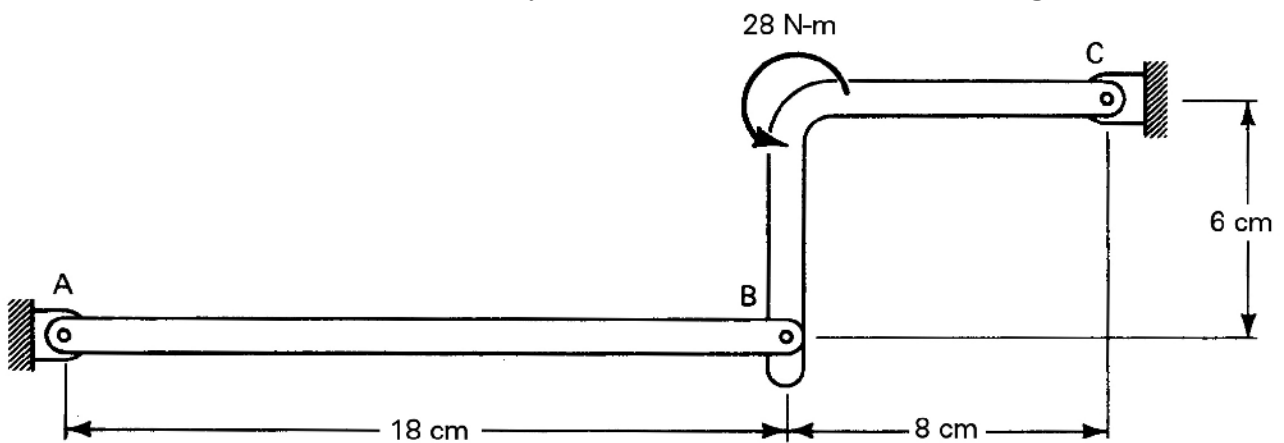
$$A_x = 46.6 \text{ lb}$$

$$B_x = 11.7 \text{ lb}$$

Frame 23-27

Applied Couples--Systems

Determine all the reactions on the system, (consider the members weightless.)



Correct response to preceding frame

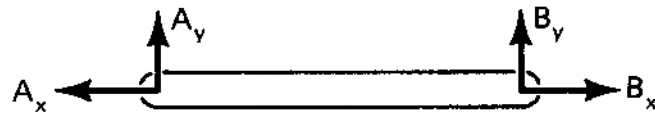
$$A_y = 0 \quad A_x = 467 \text{ N}$$

$$B_y = 0 \quad B_x = 467 \text{ N}$$

$$C_y = 0 \quad C_x = 467 \text{ N}$$

directions as shown

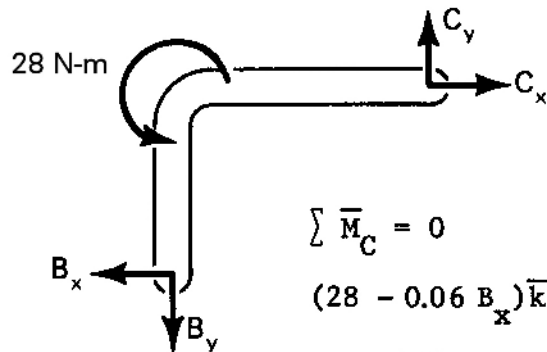
Solution:



$$\sum \bar{M}_A = 0 \quad \sum \bar{F} = 0$$

$$B_y = 0 \text{ and } A_y = 0$$

$$-A_x + B_x = 0$$



$$\sum \bar{M}_C = 0$$

$$(28 - 0.06 B_x) \bar{k} = 0$$

$$B_x = 467$$

$$\sum \bar{F} = 0$$

$$\therefore C_y = 0$$

$$C_x = 467$$

Frame 23-28

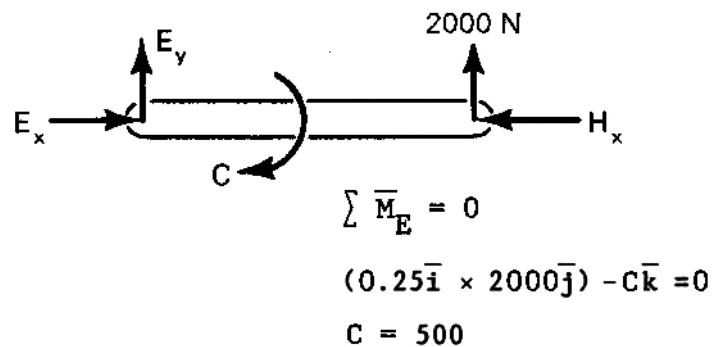
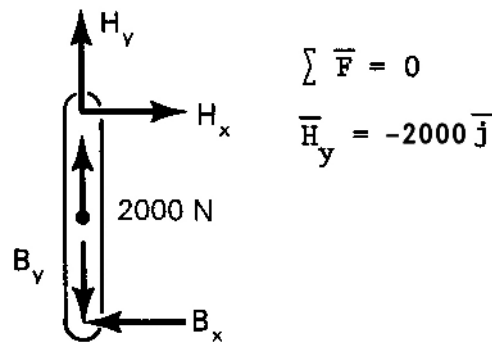
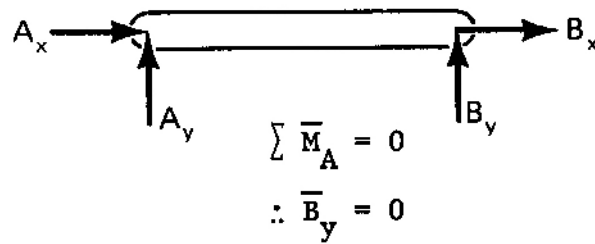
Notebook

Now work Problem 23-4 in your notebook.

Correct response to preceding frame

$$\bar{C} = 500\bar{k} \text{ N-m}$$

Solution:



Frame 23-29

Transition

Up to this point you have been considering couples which are specifically shown as being applied to the bodies.

In the next section you will deal with some situations in which you must recognize where a couple will exist as part of a reaction and include it on your free body diagram.

On to the next frame.

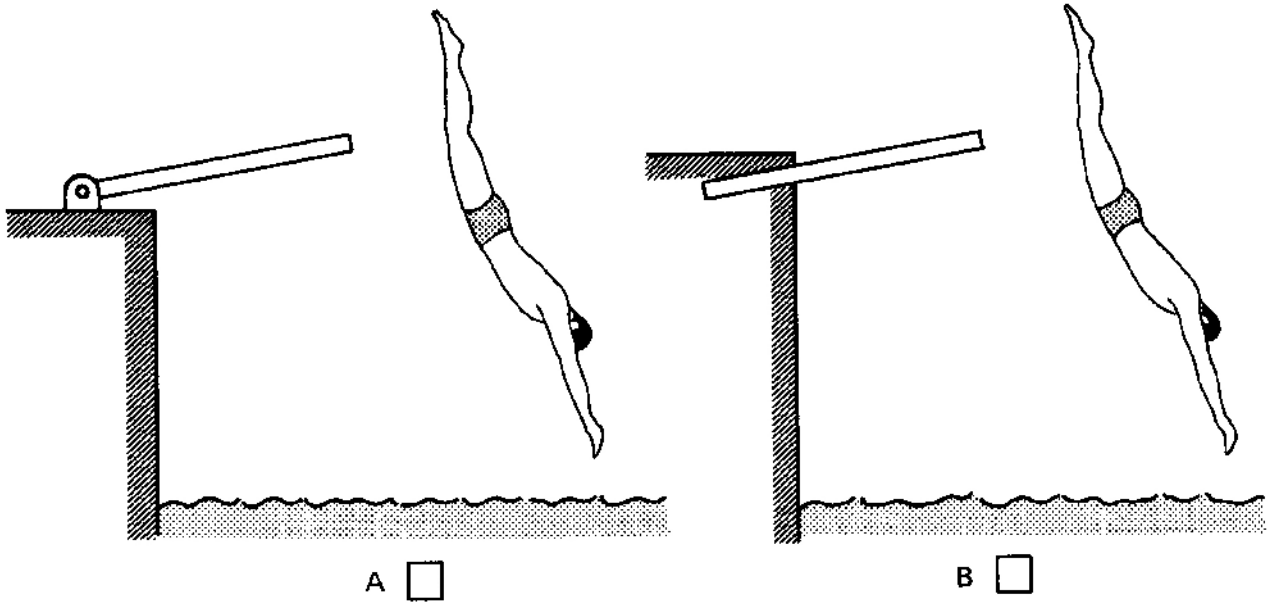
Correct response to preceding frame

No response

Frame 23-30

Couples as Reactions

Often design situations call for members which are firmly anchored at one end, rather than being pinned or supported on rollers, cables or the like. For example, which of the structures below looks more useful as a diving board?



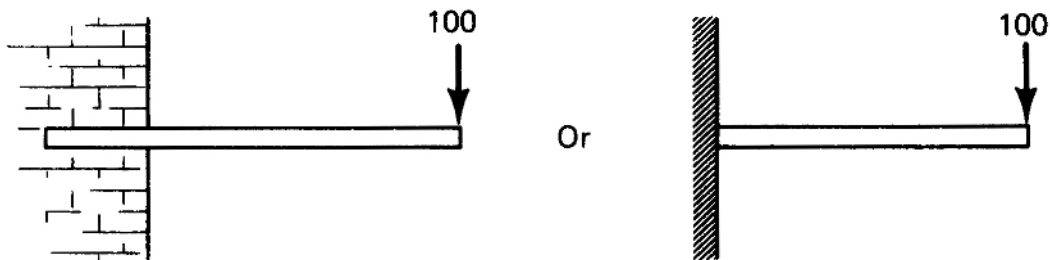
Correct response to preceding frame

B of course (A would act like a trap door rather than a diving board.)

Frame 23-31

Couples as Reactions--Review

Diving boards and other members with built in ends require you to add another trick to your free body diagrams repertoire. The beam below is built into a brick wall and we wish to determine the reaction at A.



If we make a cut across the beam at A we will have the sketch below.



Will the beam be in equilibrium with the force on it?

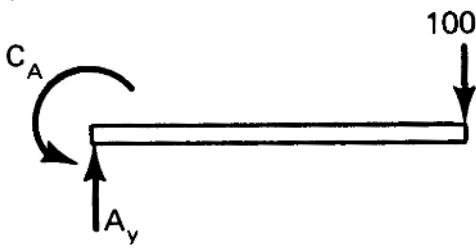
Yes

No

If not add an appropriate reaction at A.

Correct response to preceding frame

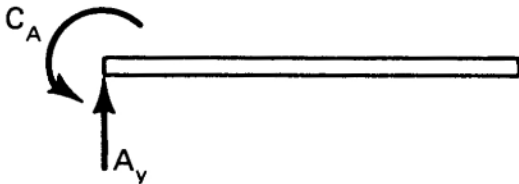
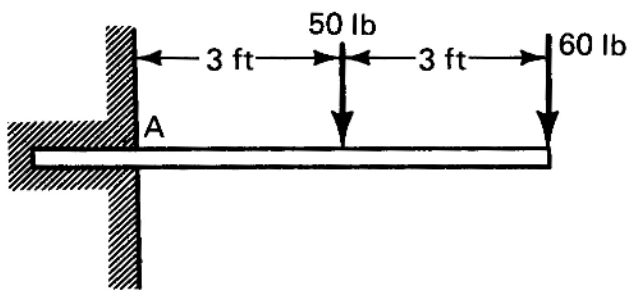
No



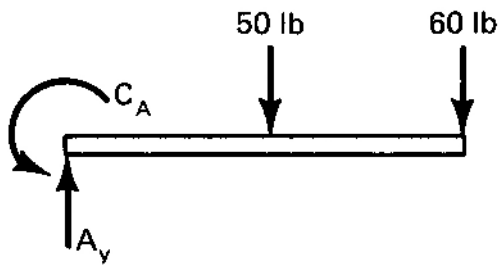
Frame 23-32

Couples as Reactions--Review

Complete the free body diagram of the cantilever beam and find the reactions at A.



Correct response to preceding frame



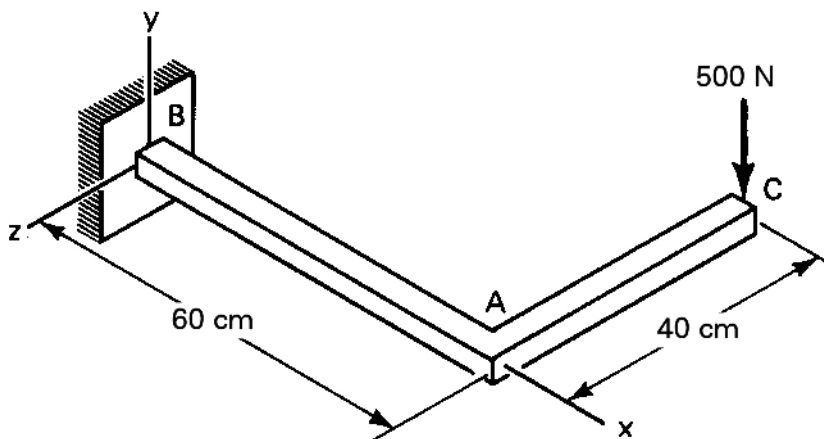
$$\bar{A}_y = 110\bar{j} \text{ lb}$$

$$\bar{C}_A = 510\bar{k} \text{ ft-lb}$$

Frame 23-33

Couples as Reactions

Let's look at a more involved one, a bracket.



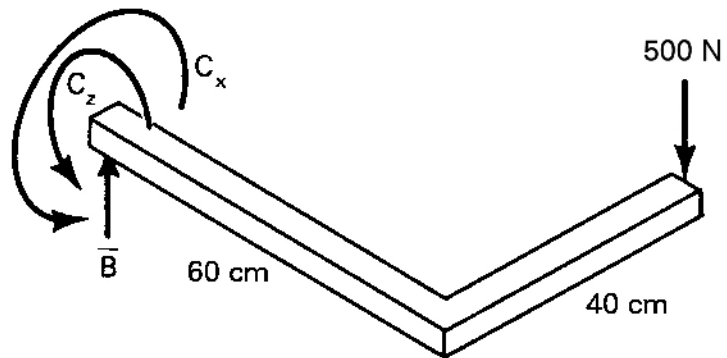
Find the reactions at B.

Correct response to preceding frame

$$\bar{\mathbf{B}} = 500\bar{\mathbf{j}} \text{ N}$$

Solution:

$$\bar{\mathbf{C}} = 300\bar{\mathbf{k}} + 200\bar{\mathbf{i}} \text{ N-m}$$



$$\sum \bar{\mathbf{F}} = 0$$

$$\bar{\mathbf{B}} - 500\bar{\mathbf{j}} = 0$$

$$\bar{\mathbf{B}} = 500\bar{\mathbf{j}}$$

$$\sum \bar{\mathbf{M}}_{\mathbf{B}} = 0$$

$$\bar{\mathbf{C}} + [(0.60\bar{\mathbf{i}} - 0.40\bar{\mathbf{k}}) \times -500\bar{\mathbf{j}}] = 0$$

$$\bar{\mathbf{C}} = 300\bar{\mathbf{k}} + 200\bar{\mathbf{i}}$$

Frame 23-34

Notebook

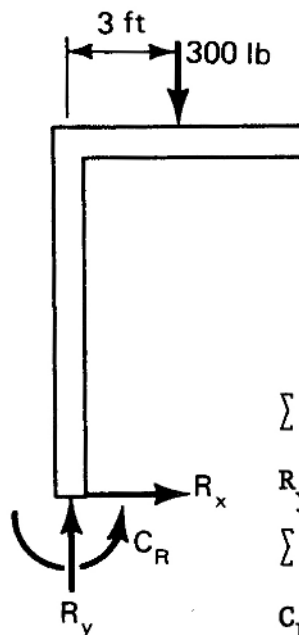
Work the Problem 23-5 in your notebook.

Correct response to preceding frame

$$\bar{R} = 300\bar{j} \text{ lb}$$

$$\bar{C}_R = 900\bar{k} \text{ ft-lb}$$

Solution:



$$\sum \bar{F} = (-300 + R_y)\bar{j} + R_x\bar{i} = 0$$

$$R_y = 300 \text{ lb}$$

$$\sum \bar{M}_B = (C_R - 900)\bar{k}$$

$$C_R = 900 \text{ ft-lb}$$

Frame 23-35

Transition

When we define **statics** in a narrow sense, we usually think of it as covering only the external forces acting on bodies and the interactions between bodies. On the other hand, the **principles of statics** may be applied to finding forces within a body as well, as long as we define where we want to cut the body.

We have already done that when we analyzed trusses using the method of sections.

In your Mechanics of Materials (aka Strength of Materials) course you will do this regularly. Some universities teach about a week's worth of material on Shear and Moment Diagrams and Shear and Moment Equations in their Statics course, but we generally do not. If your instructor intends to do so, you will receive directions to appropriate material. It would be appropriate to make that the next lesson.

As the last section of this unit we will provide something of an overview of the topics so that you may have a glimpse of your future.

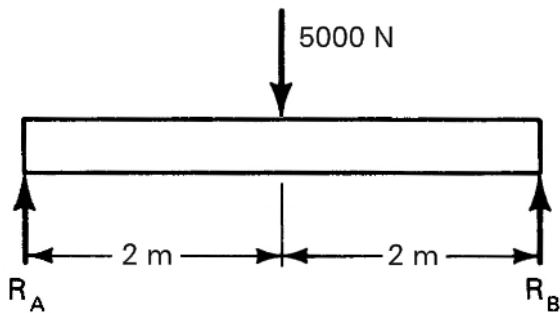
Turn to the next frame.

Correct response to preceding frame

Frame 23-36

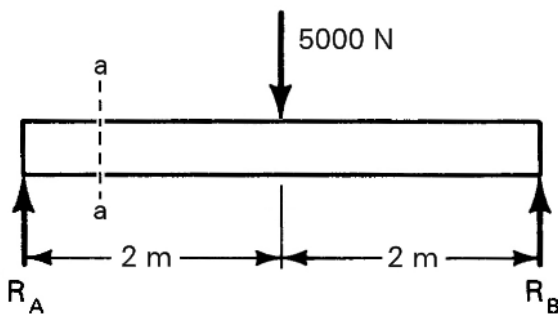
Shear and Moment in Beams

Let's consider a beam carrying a concentrated load at its midpoint.

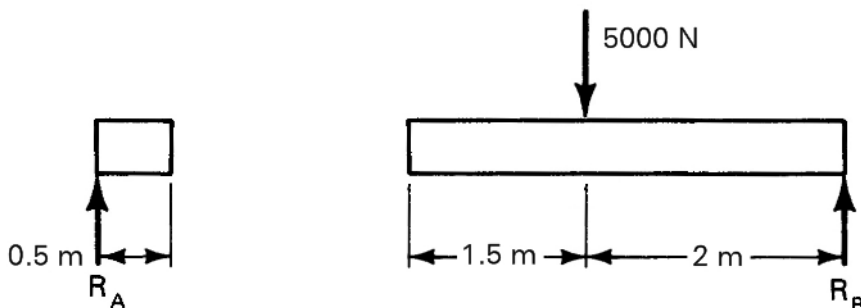


You should have no trouble finding the reactions by using whole body statics.

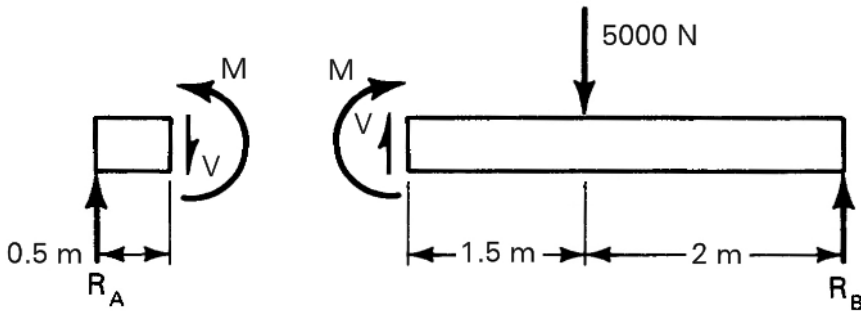
Now let us think about the beam as being cut into two sections 0.5 meters from the left end at a-a.



That, conceptually, results in two cantilever beams. Complete the free body diagrams below.



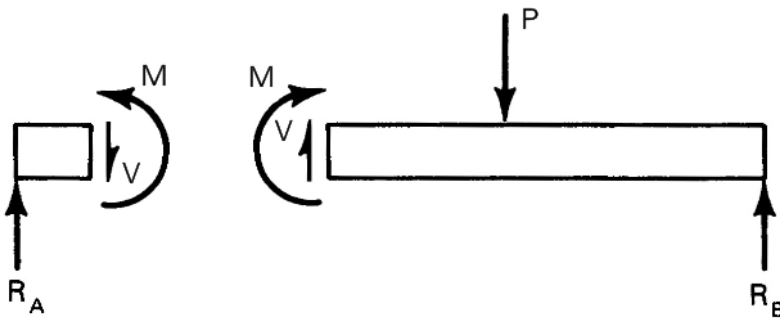
Correct response to preceding frame



Frame 23-37

Shear and Moment – Sign and Notation Conventions

The simply supported beam, supported at the ends and with loads in the middle, might be considered the "basic beam," or "a natural beam." Shown symbolically, the beam we have just considered has the FBDs below.



The couples shown on the sections shown will have (*positive, negative*) magnitudes.

The forces shown on the sections shown will have (*positive, negative*) magnitudes.

Describe the arrows I have used to show the forces on the sections. _____

Correct response to preceding frame

positive
positive
one sided arrow heads (or equivalent response)

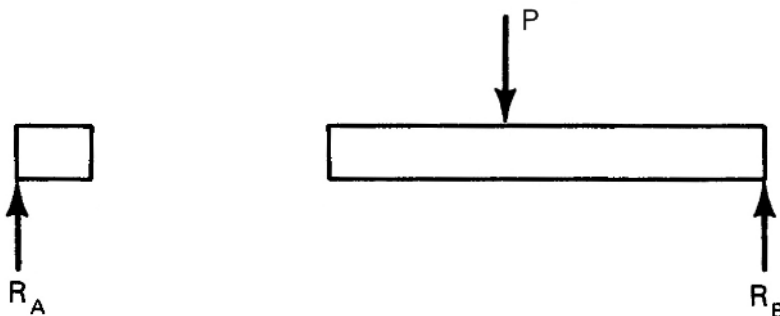
Frame 23-38

Shear and Moment – Sign and Notation Conventions

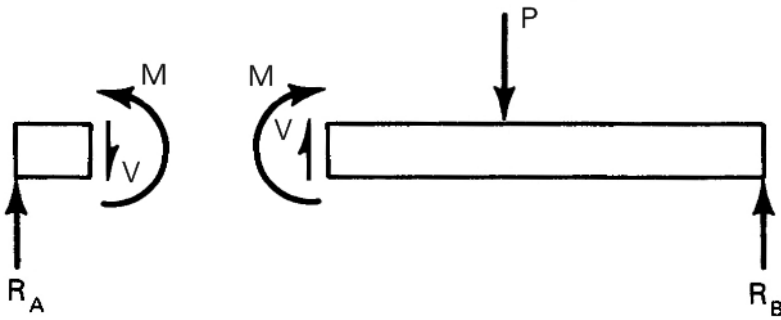
Earlier in this unit we used C as the notation for the *couples* on the ends of beams. The convention in beam analysis is to use M for *internal moment*.

It is also the convention to designate the force on the section as V . In Mechanics of Materials, such a force is known as a *shear force*, or *shearing force*, because it is what would be generated if you were to cut the beam with shears or scissors. The reasons for using a V are lost in time, but it may be because it looks a bit like a pair of shears.

Complete the FBDs below using the shear and moment conventions.



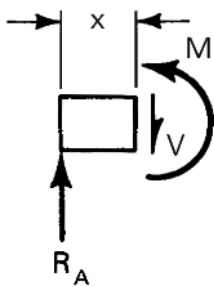
Correct response to preceding frame



Frame 23-39

Shear and Moment on a Section

In statics we have worked most problems using numeric values. In Mechanics of Materials you will do a lot more symbolic work, so we will do so here.



Find the shear and moment on the section of the beam shown.

$$V = \underline{\hspace{2cm}}$$

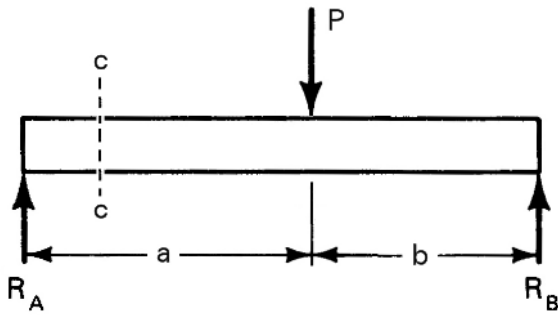
$$M = \underline{\hspace{2cm}}$$

Correct response to preceding frame

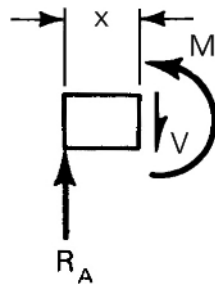
$$V = R_A$$
$$M = R_A x$$

Frame 23-40

Shear and Moment on a Section



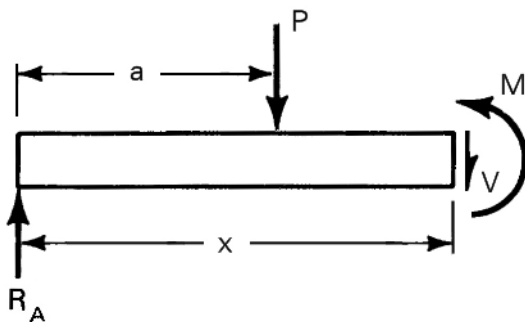
For our beam we get the result



$$V = R_A$$
$$M = R_A x$$

Only as long as our section c - c falls between R_A and P .

If the cut is between P and R_B we must draw the following FBD.



Find the shear and moment on this section of the beam shown.

$$V = \underline{\hspace{4cm}}$$

$$M = \underline{\hspace{4cm}}$$

Correct response to preceding frame

$$V = R_A - P$$

$$M = R_A x - P (x-a)$$

Frame 23-41

Shear and Moment on a Section

OK, that is the basic way of finding shear and moment on a given section if you have concentrated loads.

The steps are:

- Draw a FBD of the beam to the left of the section
- Write $\sum F = 0$ and solve for V .
- Write $\sum M = 0$ and solve for M .

Later you will learn to create a set of equations and limits. You will also learn to sketch graphs of the shear and moment known as ***shear and moment diagrams***.

We will conclude this unit with a quick look at shear and moment when the section passes through a distributed load.

Go to the next frame.

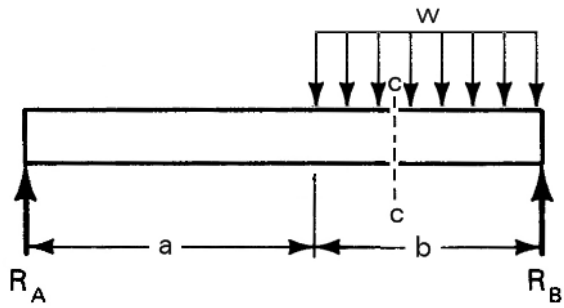
Correct response to preceding frame

No response

Frame 23-42

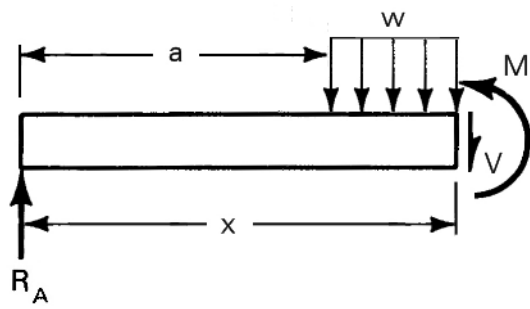
Shear and Moment on a Section

Let's consider a beam which has a distributed load on a portion at its right hand end.



Draw a FBD for section c-c.

Correct response to preceding frame



Frame 23-43

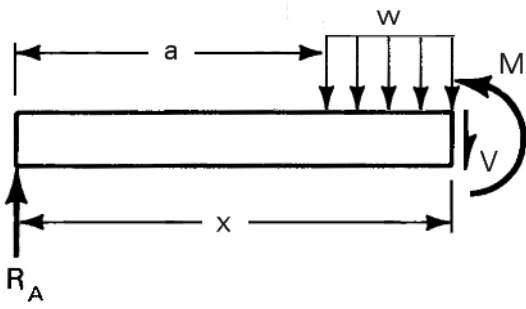
Shear and Moment on a Section

Now find the shear and moment on the section. You'll need to be careful in setting out some of the dimensions.

$$V = \underline{\hspace{2cm}}$$

$$M = \underline{\hspace{2cm}}$$

Correct response to preceding frame



$$V = R_A - w [x - a]$$

$$M = R_A x - w [x - a] \left[\frac{x - a}{2} \right]$$

$$= R_A x - \frac{w}{2} [x - a]^2$$

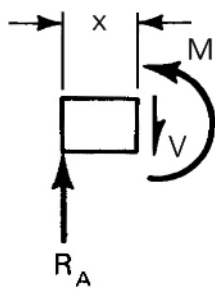
Frame 23-44

Notebook

Wrap this unit up by completing page 23-6 in your notebook.

Correct response to preceding frame

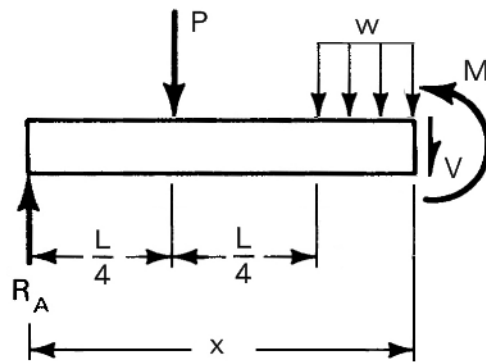
For section a-a



$$V = R_A$$

$$M = R_A x$$

For section b-b



$$V = R_A - P - w \left[x - \frac{L}{2} \right]$$

$$M = R_A x - P \left[x - \frac{L}{4} \right] - \frac{w}{2} \left[x - \frac{L}{2} \right]^2$$

Frame 23-45

Closure

Another unit down!

You should now have little trouble working many of the problems in which a couple is either applied to a member as a specified load or is a resultant at one of its supports.

You have also gotten a bit of a start on shear and moment in beams. This should help in your materials course, and may eliminate a "surprise" that a few instructors put on statics tests. "You knew how to do this," you just hadn't seen it.

In the process of adding these new ideas you should also have gotten some helpful exercise in setting up and solving equilibrium problems to improve your working speed. You should be more concerned with being correct than with being fast, nevertheless you will find it a big advantage to be able to do work of this type efficiently when you move on into dynamics and mechanics of materials.