

# Introduction to Statics

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## Unit 21

# Couples and Resultants with Couples

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# Unit 21

## Couples and Resultants with Couples

Frame 21-1

### **Introduction**

In the analysis of machines and machine parts the engineer often finds it necessary to represent a twisting or turning moment which acts on a body but is not associated with forces at the point of application. This unit will help you learn how to handle such situations through the use of couples.

Turn to the next frame.

Correct response to preceding frame

No response

---

Frame 21-2

**Couples**

In mechanics a "couple" is composed of forces. If this use of the term couple is similar to everyday English, how many forces make up a couple? \_\_\_\_\_

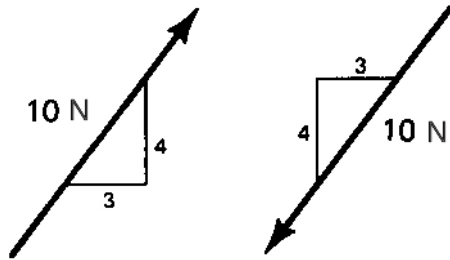
Correct response to preceding frame

two

---

Frame 21-3

**Review**



These two forces have the following characteristics:

1. (equal, unequal) magnitudes
2. (the same, different) inclinations
3. (the same, opposite) sense
4. (the same, parallel) lines of action

Two forces of these characteristics are called \_\_\_\_\_ and \_\_\_\_\_

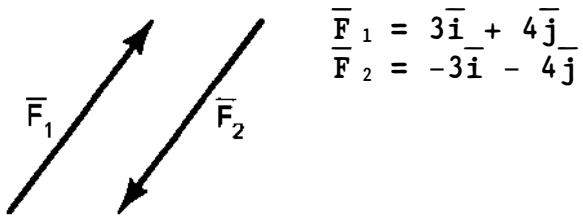
Correct response to preceding frame

1. equal
  2. the same
  3. opposite
  4. parallel
- equal and opposite
- 

Frame 21-4

### Couples

The forces shown are an example of a couple



The forces which make up a couple are \_\_\_\_\_ and \_\_\_\_\_ .

Correct response to preceding frame

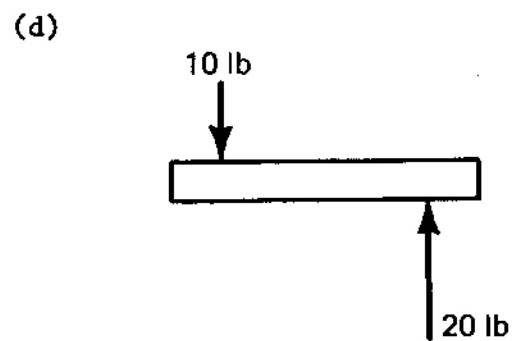
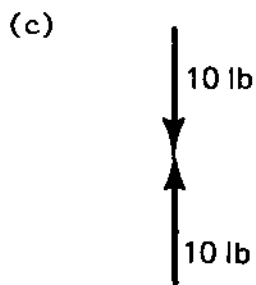
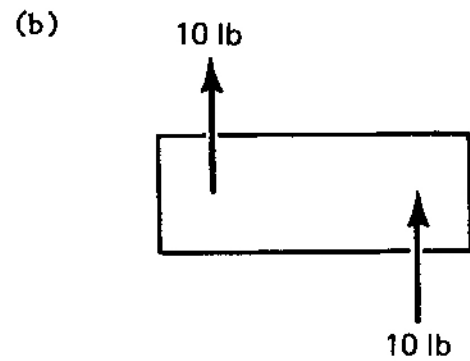
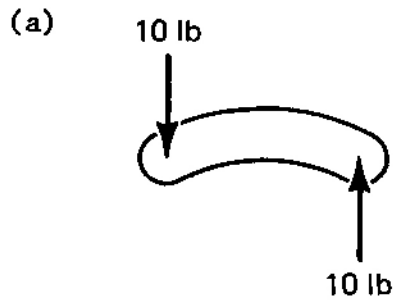
equal and opposite (Or equivalent response)

---

Frame 21-5

### Couples

A couple is composed of two equal and opposite, non-collinear forces. Which of the following show or describe couples?



(e)  $\vec{F}_1 = 5\vec{i} + 12\vec{j}$   
 $\vec{F}_2 = 5\vec{i} - 12\vec{j}$   
(not collinear)

(f)  $\vec{F}_1 = -20\vec{i} + 5\vec{j}$   
 $\vec{F}_2 = 20\vec{i} - 5\vec{j}$   
(not collinear)

Correct response to preceding frame

(a) and (f)

---

Frame 21-6

**Resultant Force Due to a Couple**

What is the resultant *force* on a body due to an applied couple? \_\_\_\_\_

Correct response to preceding frame

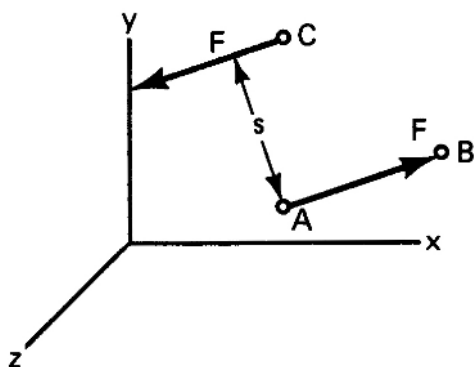
zero

---

Frame 21-7

### Moment of a Couple

The forces which make up the couple shown below lie in the x-y plane and  $s$  is the distance between the lines of action.



What is the moment of the couple about

1. Point A? \_\_\_\_\_
2. Point B? \_\_\_\_\_
3. Point C? \_\_\_\_\_
4. Any point on the line of action of one of the forces? \_\_\_\_\_



Correct response to preceding frame

$+\mathbf{s}\bar{\mathbf{F}}\mathbf{k}$  for all points on the lines of action

---

Frame 21-8

### **Moment of a Couple**

Open your notebook to page 21-1 and complete the statement at the top of the page. Then use the figure to help you answer the following questions.

Using vector products, write an expression for the moment of  $\bar{\mathbf{F}}_1$  about the general point  $(x, y, z)$

$$\bar{\mathbf{M}}_1 = \underline{\hspace{10em}}$$

Similarly, write an expression for the moment of  $\bar{\mathbf{F}}_2$  about the same point.

$$\bar{\mathbf{M}}_2 = \underline{\hspace{10em}}$$

Correct response to preceding frame

$$\bar{\mathbf{M}}_1 = \bar{\mathbf{r}}_1 \times \bar{\mathbf{F}}_1$$

$$\bar{\mathbf{M}}_2 = \bar{\mathbf{r}}_2 \times \bar{\mathbf{F}}_2$$

Record these on page 21-1 at (1) in your notebook.

---

Frame 21-9

### **Moment of a Couple**

The total moment of the forces about the point is

$$\bar{\mathbf{M}}_T = [\bar{\mathbf{r}}_1 \times \bar{\mathbf{F}}_1] + [\bar{\mathbf{r}}_2 \times \bar{\mathbf{F}}_2]$$

By definition  $\bar{\mathbf{F}}_1 = -\bar{\mathbf{F}}_2$  so that the total moment may be rewritten

$$\bar{\mathbf{M}}_T = [\bar{\mathbf{r}}_1 \times \underline{\hspace{2cm}}] + [\bar{\mathbf{r}}_2 \times \bar{\mathbf{F}}_2]$$

Correct response to preceding frame

$$\bar{\mathbf{M}}_T = [\bar{\mathbf{r}}_1 \times (-\bar{\mathbf{F}}_2)] + [\bar{\mathbf{r}}_2 \times \bar{\mathbf{F}}_2]$$

---

Frame 21-10

### **Moment of a Couple**

This equation can be shuffled to get

$$\bar{\mathbf{M}}_T = [\bar{\mathbf{r}}_2 \times \bar{\mathbf{F}}_2] - [\bar{\mathbf{r}}_1 \times \bar{\mathbf{F}}_2]$$

Make use of the distributive property of vector products,

$$[\bar{\mathbf{A}} + \bar{\mathbf{B}}] \times \bar{\mathbf{C}} = [\bar{\mathbf{A}} \times \bar{\mathbf{C}}] + [\bar{\mathbf{B}} \times \bar{\mathbf{C}}]$$

to obtain

$$\bar{\mathbf{M}}_T = ( \quad \quad \quad ) \times \bar{\mathbf{F}}_2$$

Correct response to preceding frame

$$\bar{\mathbf{M}}_T = (\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_1) \times \bar{\mathbf{F}}_2$$

---

Frame 21-11

### **Moment of a Couple**

$(\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_1) = \bar{\mathbf{r}}_{12}$  on the drawing, thus  $\bar{\mathbf{M}}_T = \bar{\mathbf{r}}_{12} \times \bar{\mathbf{F}}_2$  which is the moment of force \_\_\_\_\_ about a point located on the line of action of \_\_\_\_\_ .

Correct response to preceding frame

$$\frac{\bar{\mathbf{F}}_2}{\bar{\mathbf{F}}_1}$$

---

Frame 21-12

### **Moment of a Couple**

Write the expression

$$\bar{\mathbf{M}}_T = (\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_1) \times \bar{\mathbf{F}}_2 = \bar{\mathbf{M}}_T = \bar{\mathbf{r}}_{12} \times \bar{\mathbf{F}}_2$$

on page 21-1 of your notebook at (2) and read the theorem which we have just demonstrated.

Correct response to preceding frame

No response

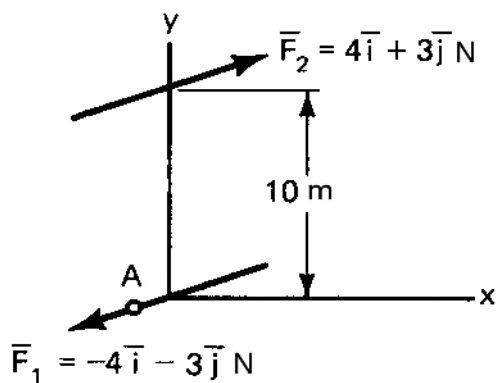
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Frame 21-13

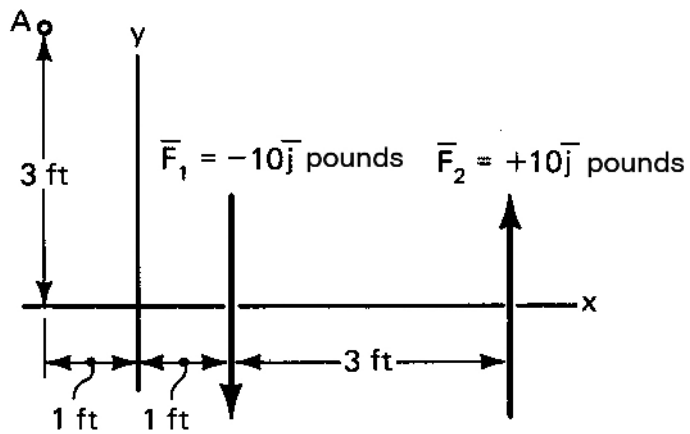
### Moment of a Couple

In each of the problems below calculate the moment of the couple about Point A.

1.



2.



Correct response to preceding frame

$$\begin{aligned} 1. \quad \mathbf{M}_A &= 10\bar{j} \times (4\bar{i} + 3\bar{j}) \\ &= -40\bar{k} \text{ N-m} \end{aligned}$$

$$\begin{aligned} 2. \quad \mathbf{M}_A &= 3\bar{i} \times 10\bar{j} \\ &= 30\bar{k} \text{ ft-lb} \end{aligned}$$

---

Frame 21-14

### **Transition**

The preceding frames have been devoted to the definition of a couple and to the computation of the moment of a couple.

The next section will be devoted to the characteristics of couples and to various ways of representing them.

Don't break yet unless it is absolutely necessary. Go to the next frame.

Correct response to preceding frame

No response

---

Frame 21-15

### **Characteristics of a Couple**

The characteristics of a force are:

- magnitude
- direction
- point of application

Place an **X** beside any characteristic of a force that you do not expect to be characteristic of a couple.



Correct response to preceding frame

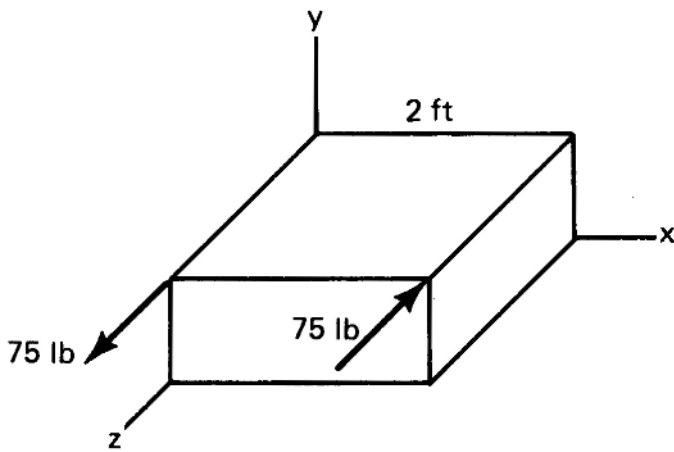
point of application

---

Frame 21-16

### Characteristics of a Couple

The characteristics of a couple are (1.) the magnitude of the moment (2.) the direction of the moment vector -- this includes both its line of action and its sense.



Give the characteristics of the couple shown.

Correct response to preceding frame

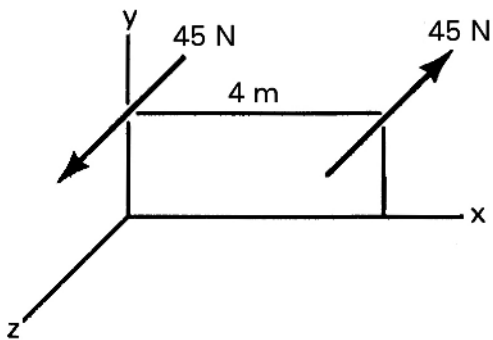
magnitude\_ 150 foot-pounds  
direction +j

---

Frame 21-17

### Characteristics of a Couple

Another way of describing the direction of a couple is to give the slope of the plane in which the forces are applied and the sense of the rotation.



The sketch shows a couple whose magnitude is

\_\_\_\_\_ applied in a

\_\_\_\_\_ plane with a

\_\_\_\_\_ rotation.

Correct response to preceding frame

180 N-m

horizontal

positive, or counterclockwise (Or equivalent response)

---

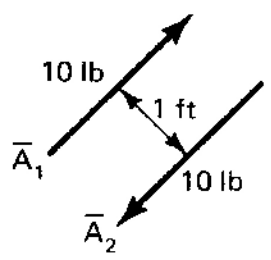
Frame 21-18

### **Representation of Couples**

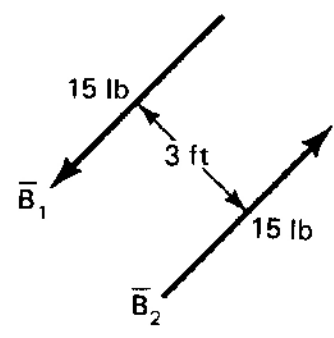
Since the forces which make up a couple have no resultant effect on the body except their moment, any other pair of forces which gives the same moment is equivalent.

Which of the following pairs of forces form equivalent couples?

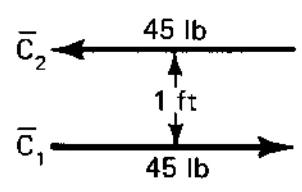
(a)



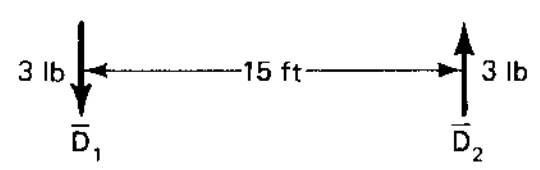
(b)



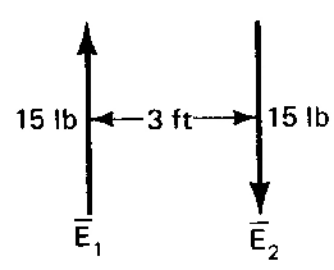
(c)



(d)



(e)



Correct response to preceding frame

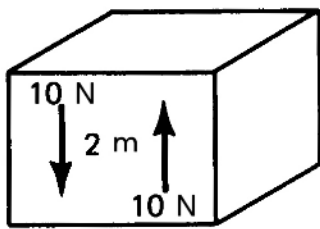
(b), (c), and (d) are equivalent

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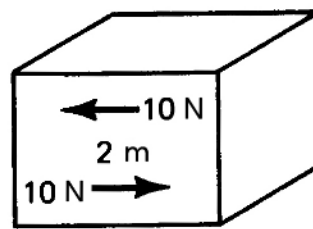
Frame 21-19

### Transformations of a Couple

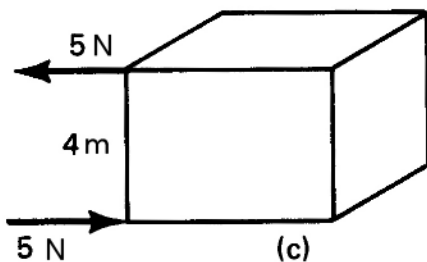
Transformations of a couple are operations which may be performed on the couple without changing its characteristics. Which of the following are not valid transformations of the couple shown in (a) ?



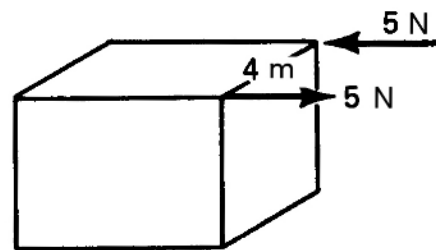
(a)



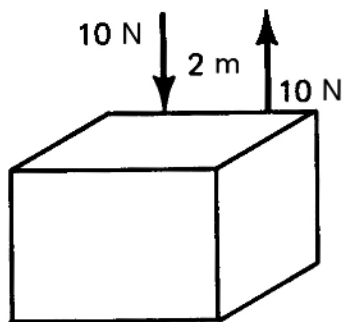
(b)



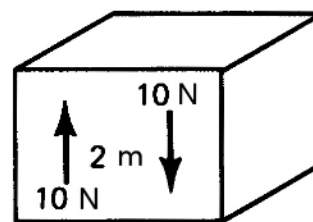
(c)



(d)



(e)



(f)

Correct response to preceding frame

Transformations (d) and (f) are not valid

(a), (b), (c) and (e) all show a couple of  $+20\bar{k}$  N-m

(d) shows a couple of  $20\bar{j}$  N-m

(f) shows a couple of  $-20\bar{k}$  N-m

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Frame 21-20

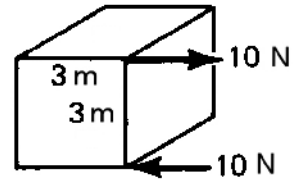
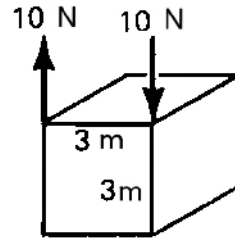
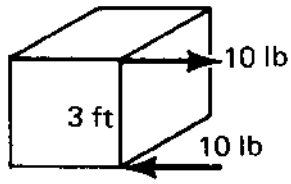
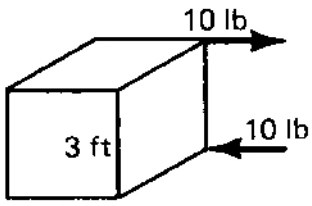
### **Transformations of a Couple**

The following operations may be performed without changing the characteristics of a couple.

The couple may be

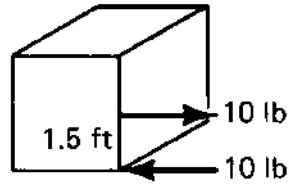
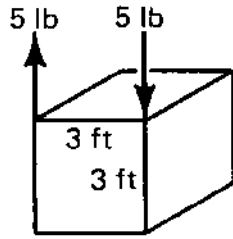
1. rotated in its plane
2. moved to a parallel position in its plane
3. moved to a parallel plane
4. recomposed into a different combination of force and distance provided the moment remains the same.

Each sketch shows a transformed couple. Identify which of the above steps was used and write the number, or numbers, in the appropriate blank.



(a) \_\_\_\_\_

(b) \_\_\_\_\_



(c) \_\_\_\_\_

Correct response to preceding frame

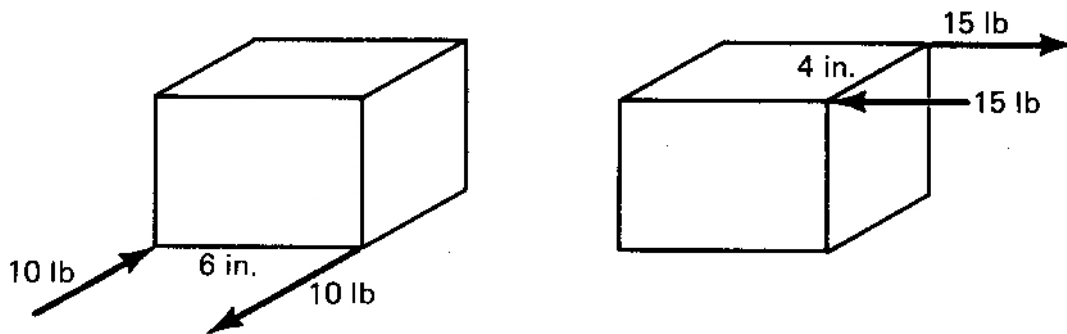
- (a) 3
  - (b) 1
  - (c) 1 and 4
- 

Frame 21-21

### Transformations of Couples

Couples are equivalent if they have the same moment or, in other words, if one may be transformed into the other.

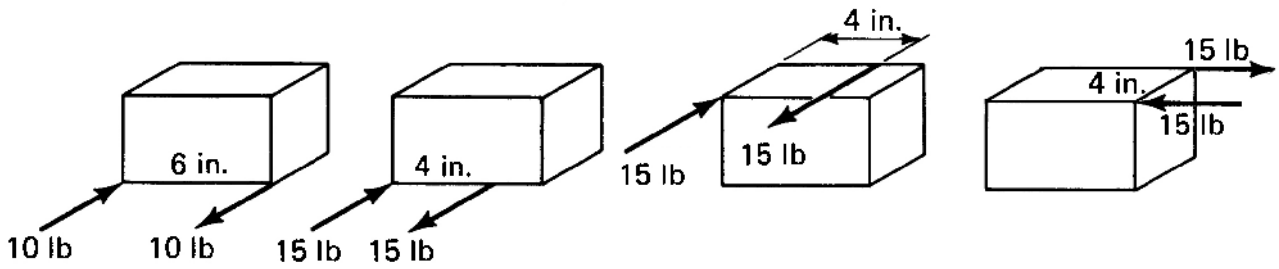
Are the two couples below equivalent? If so, prove it by making use of the operations listed in the preceding frame. If not, change the second couple so that it is equivalent to the first.





Correct response to preceding frame

The couples are equivalent



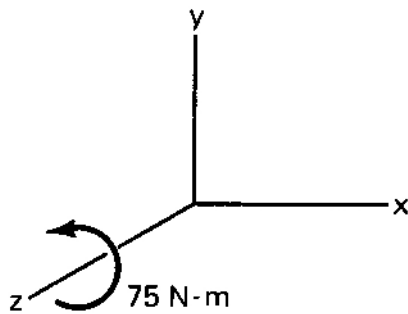
The steps may be taken in any order.

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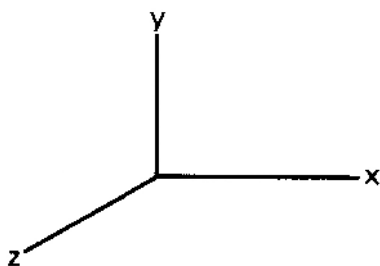
Frame 21-22

### Representation of Couples

We often picture a couple as a curved arrow like this:

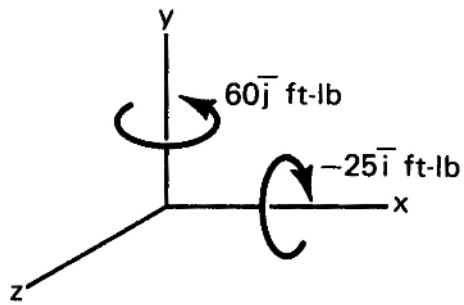


Show and label on the sketch below



1. a clockwise moment of 25 ft-lb about the x axis
2. a counterclockwise moment of 60 ft-lb about the y axis

Correct response to preceding frame

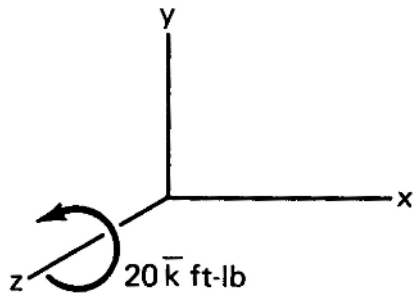


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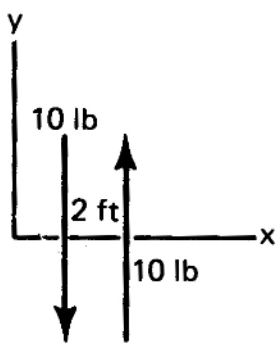
Frame 21-23

### Representation of Couples

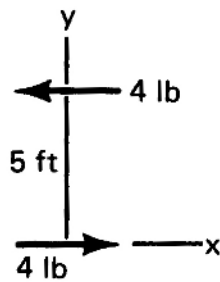
Suppose you were given a couple in this form:



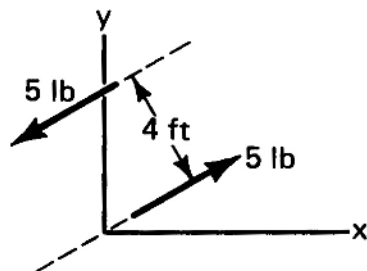
Which of these force pairs are equivalent to the couple above?



(a)



(b)



(c)

Can a couple be resolved into a unique pair of forces?  Yes  No

Correct response to preceding frame

all are equivalent

No

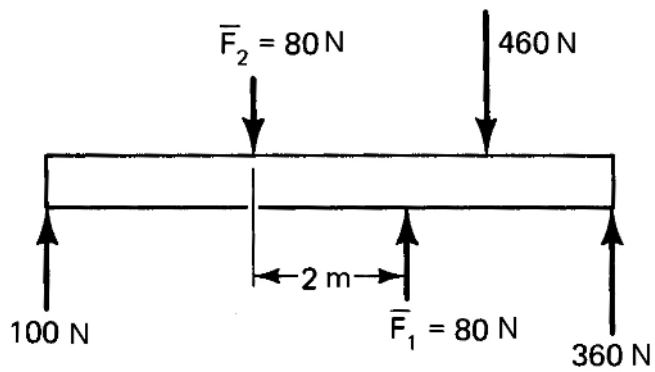
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Frame 21-24

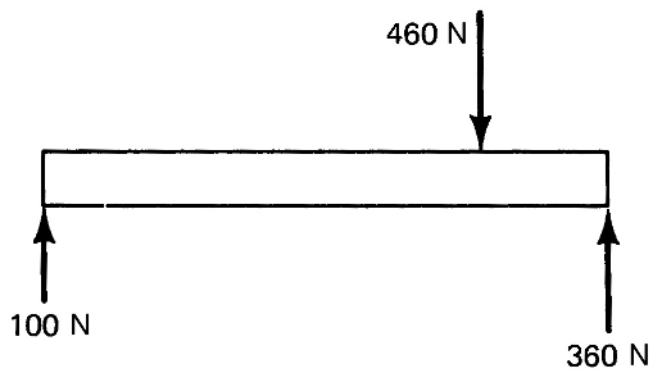
### Representation of Couples

Use the new notation to show the couple caused by  $\bar{F}_1$  and  $\bar{F}_2$  on sketch (b).

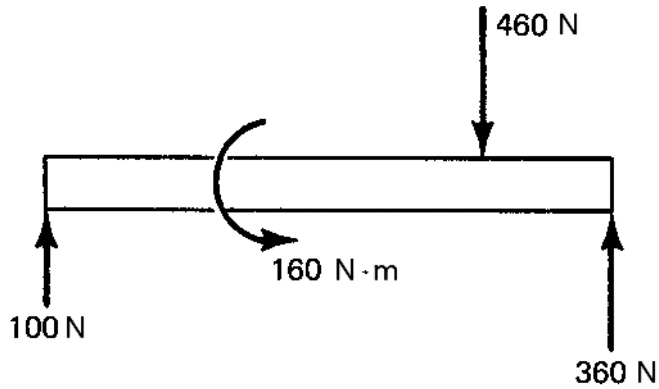
(a)



(b)



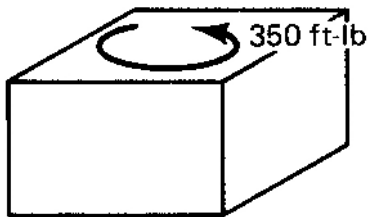
Correct response to preceding frame



Frame 21-25

### Characteristics of a Couple

The direction of a couple is often given by describing the axis about which it would cause rotation and the sense of the rotation.



The couple shown has a magnitude of \_\_\_\_\_ and causes rotation about a \_\_\_\_\_ axis in a \_\_\_\_\_ direction.

Correct response to preceding frame

350 ft-lb  
vertical axis  
positive, or counterclockwise

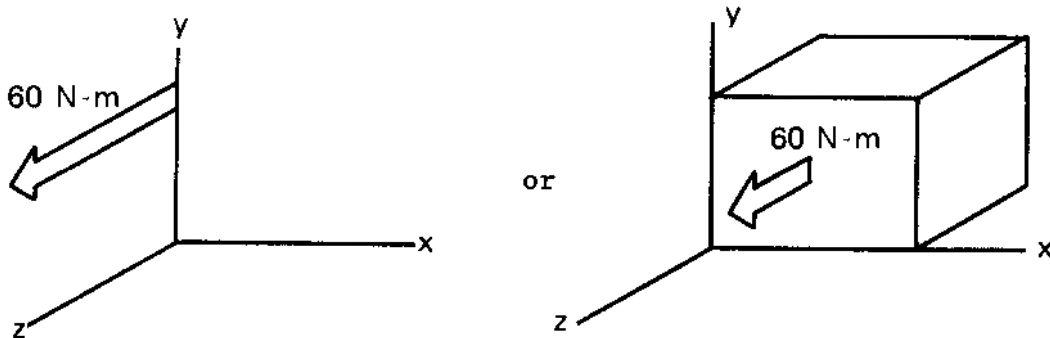
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Frame 21-26

### Representation of a Couple

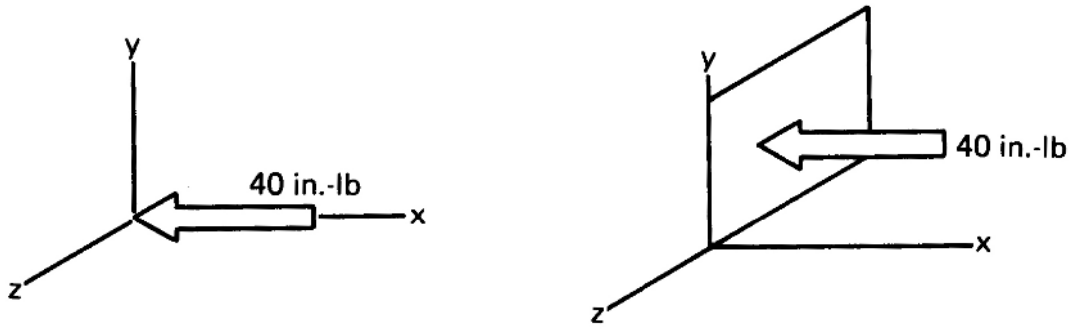
Another common representation of a couple is by a vector arrow in the direction of the moment of a couple. Since a couple has no particular location such a vector can be shown anywhere so long as it is properly directed.

In order to avoid the dire consequences of confusing forces and couples, the vector representing the couple is often drawn as an outlined arrow. Thus, a moment of  $60\mathbf{k}$  N-m might be shown as



Draw a vector representation of a couple of  $-40\bar{\mathbf{i}}$  inch-pounds.

Correct response to preceding frame



(Or equivalent response)

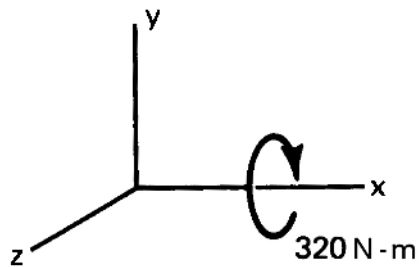
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Frame 21-27

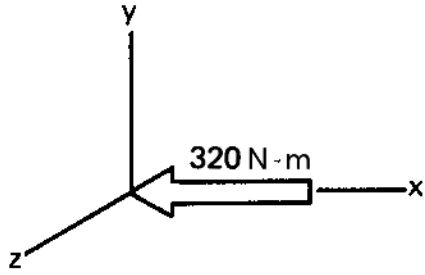
### Representation of a Couple

When a couple is represented as a vector the vector is drawn along or parallel to the axis about which the couple would cause rotation. The sense is determined by the right hand rule.

Draw a vector representing the couple shown below.



Correct response to preceding frame



(Other locations are just as good.)

---

Frame 21-28

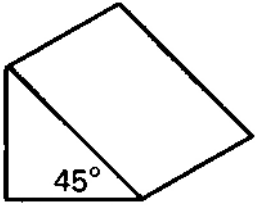
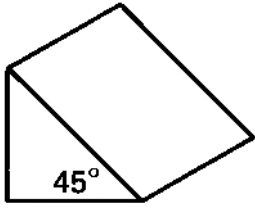
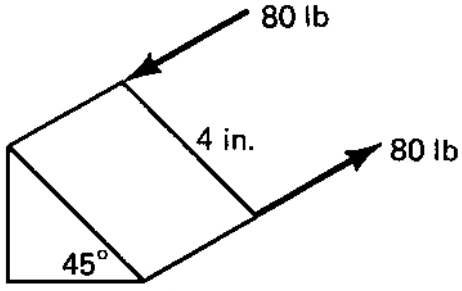
### **Representation of a Couple**

Draw two other representations of the couple shown below.

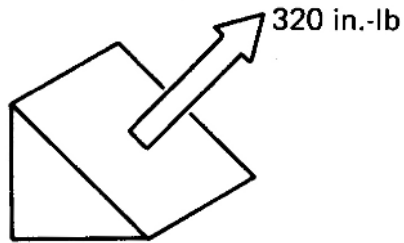
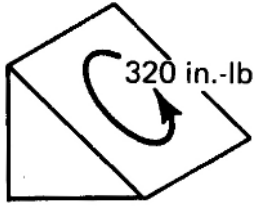








Correct response to preceding frame



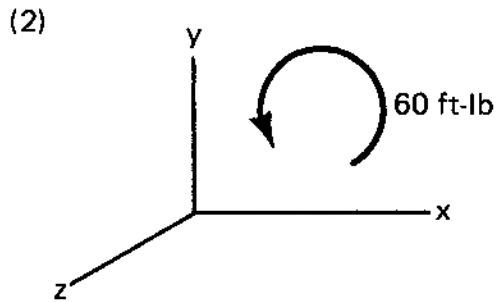
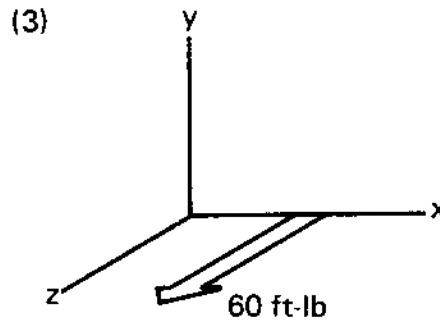
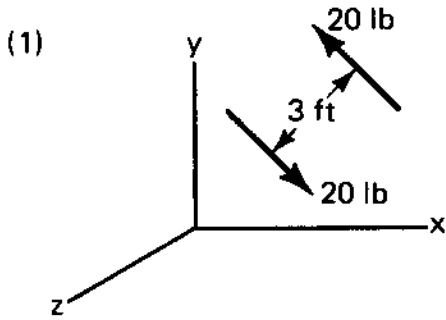
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Frame 21-29

**Recapitulation**

Sum up what you have learned so far on page 21-2 of your notebook.

Correct response to preceding frame



---

Frame 21-30

### Transition

The next sort of problem we will look at involves replacing a force in one location with an equal parallel force in another location plus a couple, without changing the resultant of the system.

This is a strategy that is often used in the solution of problems in strength of materials since it enables one to do a better job of visualizing the effects of a loading.

This will be the last section of this unit and will only take about 10 minutes.

Go the next frame.

Correct response to preceding frame

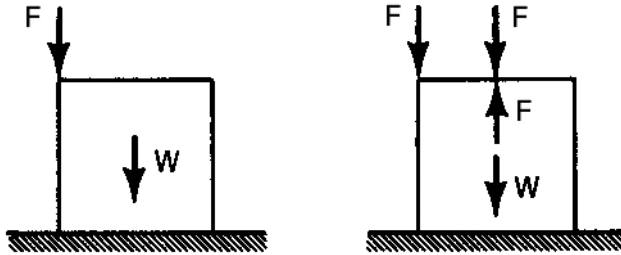
No response

---

Frame 21-31

**Resolution of a Force into a Force and a Couple**

Two loadings are shown below.



Are the force systems equivalent?

Yes

No

Correct response to preceding frame

Yes. The systems are equivalent since the addition of two equal, opposite collinear forces does not affect the resultant.

---

Frame 21-32

### Resolution of a Force into a Force and a Couple

Figures A and B show (equivalent, non-equivalent) force systems.

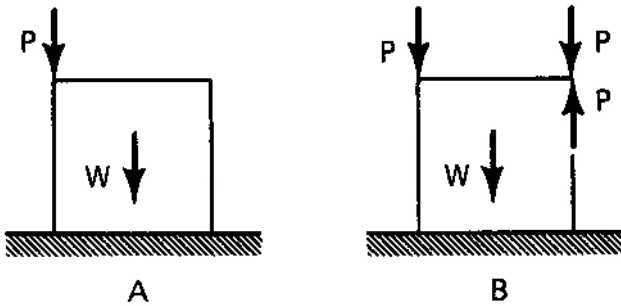
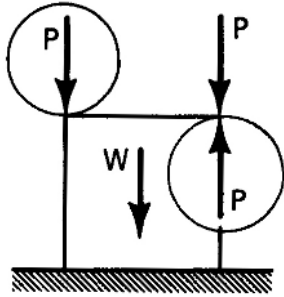


Figure B contains a couple. Circle the two forces which comprise it.

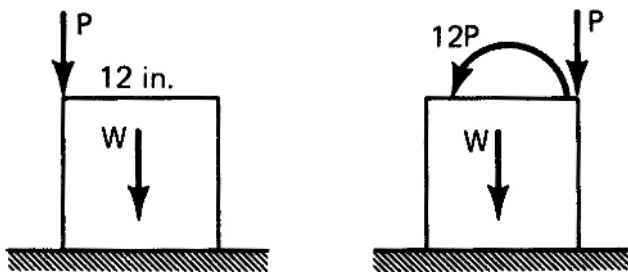
Correct response to preceding frame equivalent



---

Frame 21-33

**Resolution of a Force into a Force and a Couple**



Are the two force systems shown equivalent?

Yes

No

Correct response to preceding frame

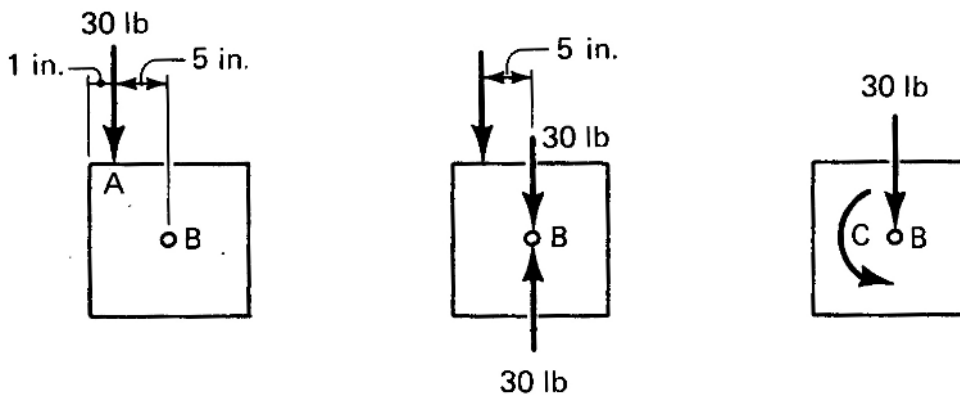
Yes

---

Frame 21-34

### Resolution of a Force into a Force and a Couple

The sketches below show the steps in replacing a force through A with a force through B and a couple.



What is the force through A? \_\_\_\_\_

What is the force through B? \_\_\_\_\_

What is the value of couple C? \_\_\_\_\_



Correct response to preceding frame

$$\bar{F}_A = -30\bar{j} \text{ lb}$$

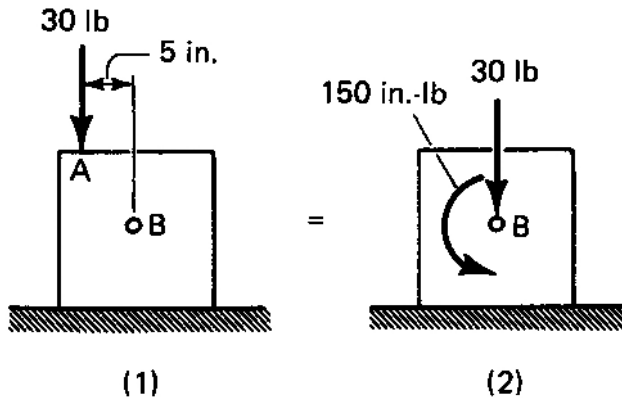
$$\bar{F}_B = -30\bar{j} \text{ lb}$$

$$\bar{C} = +150\bar{k} \text{ in.-lb}$$

---

Frame 21-35

### Resolution of a Force into a Force and a Couple



The two systems shown above are equivalent. Point B is the mass center of the block. In loading (1) the 30 pound force will cause the block to tend to tip about an axis through the mass center.

Does the 30 pound force in loading (2) tend to tip the body about B?

Yes     No

What does tend to tip the body? \_\_\_\_\_

Correct response to preceding frame

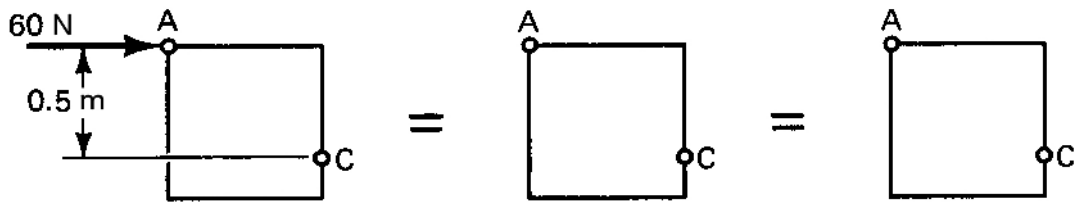
No  
The couple

---

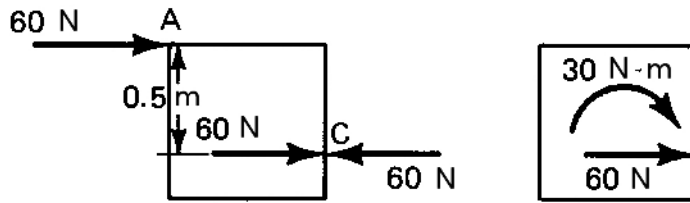
Frame 21-36

**Resolution of a Force into a Force and a Couple**

Replace the force at A with a force through C plus a couple by completing the sketches.



Correct response to preceding frame



Frame 21-37

### Resolution of a Force into a Force and a Couple

Do Problems 21-1 and 21-2 in your notebook.

Correct response to preceding frame

Answers to 21-1

$$\bar{F}_B = -20 (.707\bar{i} + .707\bar{j}) \text{ lb}$$

$$\bar{C} = 141.4\bar{k} \text{ N-m}$$

Answers to 21-2

$$\bar{F}_G = -20\bar{j} \text{ kips}$$

$$\bar{C} = + 200\bar{k} \text{ in-kips}$$

---

Frame 21-38

### Summary

That completes this unit. You now know how to define a couple, how to compute its moment, how to work simple problems which involve couples, and how to resolve a force into a force and a couple.

In the next unit you will use these ideas to work more involved problems of the equilibrium of machine elements.