Introduction to Statics

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Unit 20
Equilibrium of Non-Coplanar Force Systems

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Unit 20
Equilibrium of Non-Coplanar Force Systems

Frame 20-1

Introduction

Except for Unit 6, where we only looked at particles, all the problems you have solved so far have dealt with bodies loaded by coplanar force systems. However, we do inhabit a three-dimensional universe so you must learn to cope with problems involving the equilibrium of bodies subject to loads in three dimensions.

Doing so involves no new theory but the practice can be a mite messy. This unit will therefore consist of several somewhat messy problems.

After taking a look at supports in 3-d, you will review problems involving concurrent non-coplanar force systems. Next you will consider systems of parallel non-coplanar forces. Last you will work problems with non-concurrent, non-parallel, non- coplanar forces. (That's when the party gets rough.)

However you can worry about that when you get there.

Go to the next frame to begin.

Correct response to preceding frame

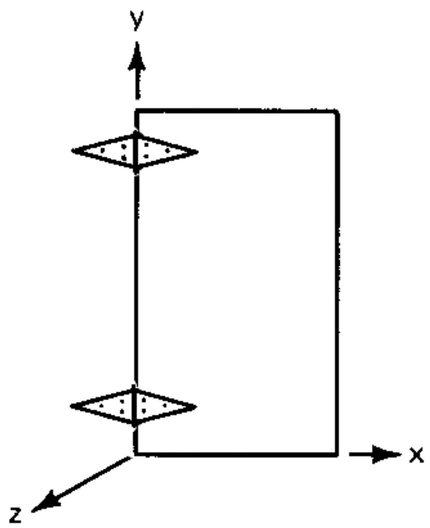
No response

Frame 20-2

Supports and Connections

There are so many sorts of supports and connections that can be used in three-dimensional problems that it is impossible to even attempt to discuss them all. We will examine a few common cases, but beyond that, the only advice that can be given is, "Figure out how each device works in terms of what resisting forces and couples it can provide and draw your free body diagrams accordingly."

Which of the following loads can the hinges resist?



- ☐ Force parallel to x axis
- ☐ Force parallel to y axis
- ☐ Force parallel to z axis
- ☐ Moment about x axis
- ☐ Moment about y axis
- ☐ Moment about z axis

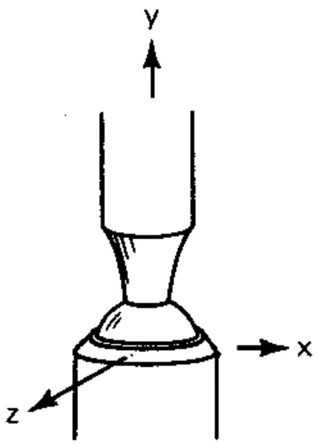
Correct response to preceding frame

The hinges shown can resist all loads except a moment about the y axis.

Frame 20-3

Supports and Connections

Check the reactions which the ball-and-socket joint can provide.



- ☐ Force parallel to x axis
- ☐ Force parallel to y axis
- ☐ Force parallel to z axis
- ☐ Moment about x axis
- ☐ Moment about y axis
- ☐ Moment about z axis

Correct response to preceding frame

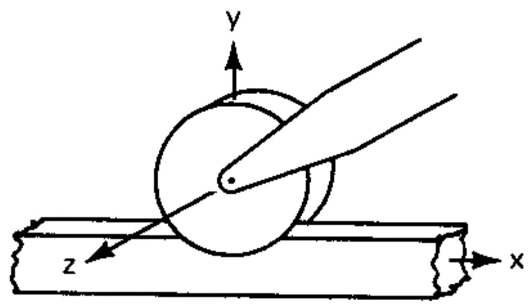
The ball-and-socket shown can resist forces in all three directions, but no moments.

Frame 20-4

Supports and Connections

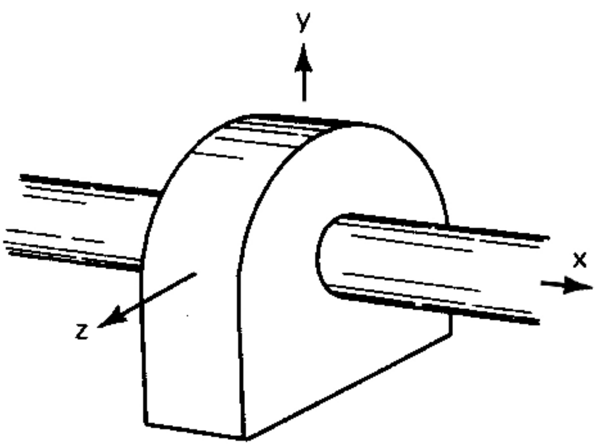
Check the reactions each of the following can provide.

Flanged wheel on rail



- [ ] Force parallel to x axis
- [ ] Force parallel to y axis
- [ ] Force parallel to z axis
- [ ] Moment about x axis
- [ ] Moment about y axis
- [ ] Moment about z axis

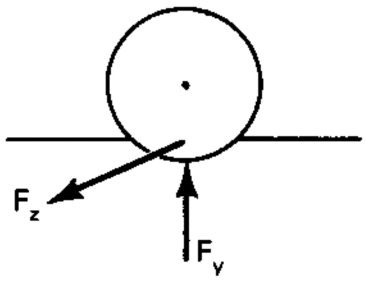
Shaft in a bearing



- [ ] Force parallel to x axis
- [ ] Force parallel to y axis
- [ ] Force parallel to z axis
- [ ] Moment about x axis
- [ ] Moment about y axis
- [ ] Moment about z axis

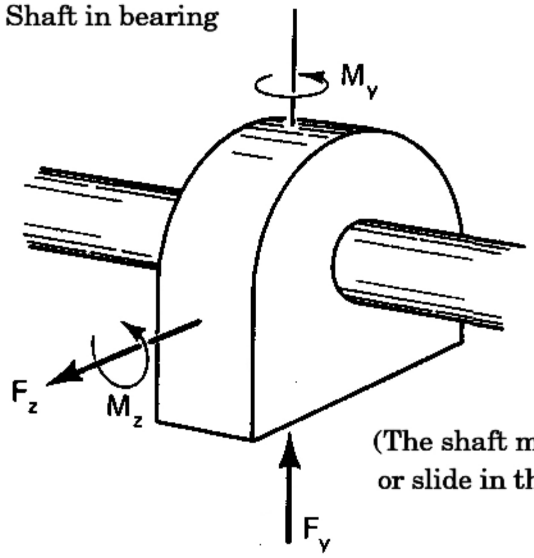
Correct response to preceding frame

Flanged wheel



- ☐ F_x
- ☑ F_y
- ☑ F_z
- ☐ M_x
- ☐ M_y
- ☐ M_z

Shaft in bearing

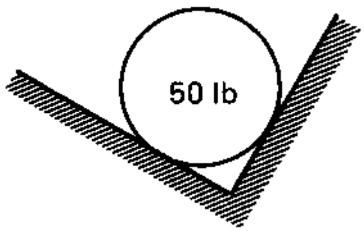


- ☐ F_x
- ☑ F_y
- ☑ F_z
- ☐ M_x
- ☑ M_y
- ☑ M_z

(The shaft may rotate about the x axis or slide in the x direction.)

Frame 20-5

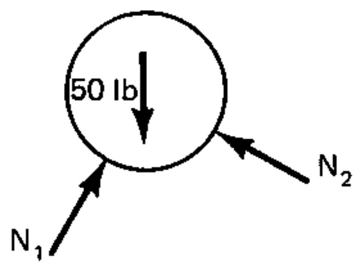
Review



Here is a two-dimensional problem in concurrent forces – a cylinder resting in a groove.

To solve it which equation would you write?

- ☐ $\sum \overline{F} = 0$
- ☐ $\sum \overline{M}_G = 0$



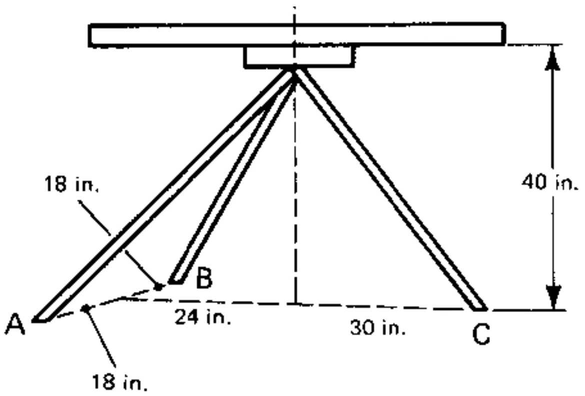
Correct response to preceding frame

$\sum \overline{F} = 0$
($\sum \overline{M}_G = 0$ is true, of course, but would give us no information, since all three forces have a moment arm of zero about the center of the cylinder.)

Frame 20-5

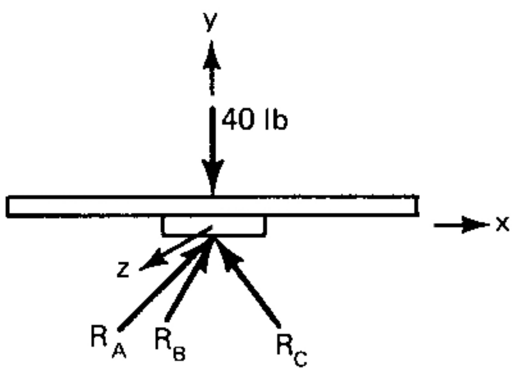
Concurrent Force System in Space

The same method used to solve coplanar concurrent force systems is used to solve non-coplanar concurrent systems.



The plane-table (an early surveying instrument) weighs 40 pounds and is supported by a tripod, the legs of which are pushed into the ground. The force in each leg may be considered to act along the leg.

Using the free body and the equations given, solve for the magnitude of the force in each leg.



$$\overline{R}_C = R_C \left(\frac{-3\overline{i} + 4\overline{j}}{5}\right)$$

$$\overline{R}_B = R_B \left(\frac{+24\overline{i} + 40\overline{j} + 18\overline{k}}{\sqrt{24^2 + 40^2 + 18^2}}\right)$$

$$\overline{R}_A = R_A \left(\frac{24\overline{i} + 40\overline{j} - 18\overline{k}}{50}\right)$$

$$\Sigma\overline{F} = 0$$

Correct response to preceding frame

$R_C = 22.2$ lb

$R_B = 13.9$ lb

$R_A = 13.9$ lb

Solution:

$$\Sigma\overline{F} = 0 = \overline{R}_A + \overline{R}_B + \overline{R}_C - 40\overline{j}$$

$$\left[\frac{24}{50} R_A\overline{i} + \frac{40}{50} R_A\overline{j} - \frac{18}{50} R_A\overline{k}\right]$$

$$+ \left[\frac{24}{50} R_B\overline{i} + \frac{40}{50} R_B\overline{j} + \frac{18}{50} R_B\overline{k}\right]$$

$$+ \left[\frac{-30}{50} R_C\overline{i} + \frac{40}{50} R_C\overline{j}\right] - 40\overline{j} = 0$$

coefficient equations

$$\frac{-30R_C + 24 R_B + 24R_A}{50} = 0$$

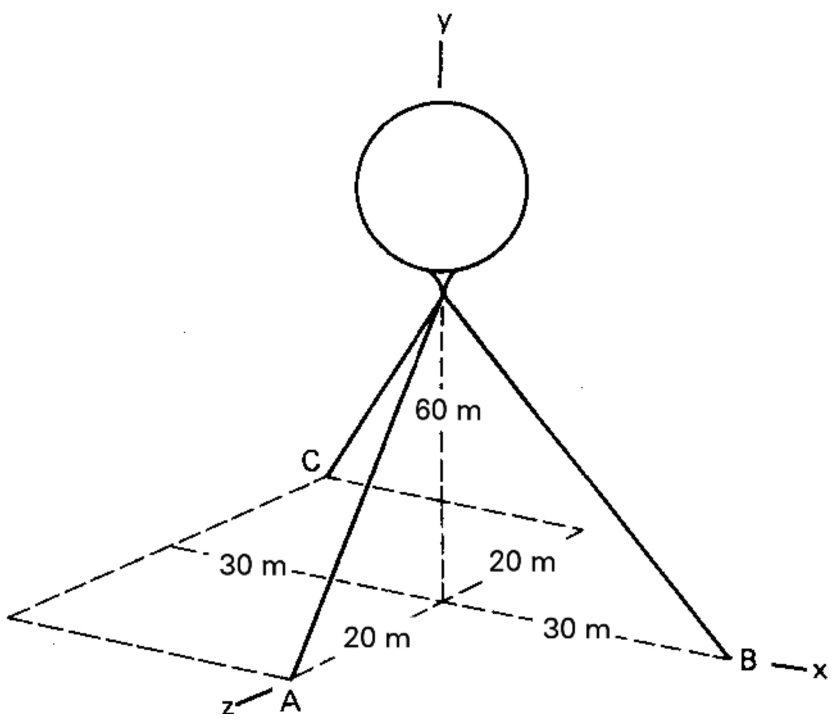
$$\frac{40}{50}(R_A + R_B + R_C) - 40 = 0$$

$$\frac{18}{50}(R_B - R_A) = 0$$

solve simultaneously

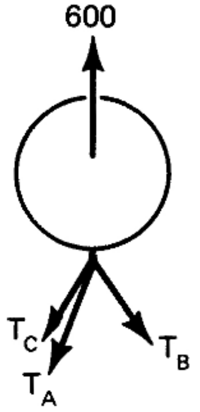
Frame 20-7

Concurrent Forces in Space



A balloon is moored by three cables as shown. The lift on the balloon is 600 Newtons. Draw the FBD of the balloon and write a vector expression for the tension in each cord.

Correct response to preceding frame



$$\bar{T}_A = T_A \left(\frac{-60\bar{j} + 20\bar{k}}{\sqrt{4000}}\right)$$

$$\bar{T}_B = T_B \left(\frac{+30\bar{i} - 60\bar{j}}{\sqrt{4500}}\right)$$

$$\bar{T}_C = T_C \left(\frac{-30\bar{i} - 60\bar{j} - 20\bar{k}}{\sqrt{4900}}\right)$$

Frame 20-8

Concurrent Forces in Space

Using your answers from the preceding frame, write $\sum \bar{F} = 0$ and form the coefficient equations.

Correct response to preceding frame

$$\Sigma\bar{F} = 0$$

$$600\bar{j} - \frac{6T_A}{\sqrt{40}}\bar{j} + \frac{2T_A}{\sqrt{40}}\bar{k} + \frac{3T_B}{\sqrt{45}}\bar{i} - \frac{6T_B}{\sqrt{45}}\bar{j}$$

$$- \frac{3}{7}T_C\bar{i} - \frac{6}{7}T_C\bar{j} - \frac{2}{7}T_C\bar{k} = 0$$

$\bar{i}$ coefficients give

$$\frac{3T_B}{\sqrt{45}} - \frac{3}{7}T_C = 0$$

$\bar{j}$ coefficients give

$$600 - \frac{6T_A}{\sqrt{40}} - \frac{6T_B}{\sqrt{45}} - \frac{6}{7}T_C = 0$$

$\bar{k}$ coefficients give

$$\frac{2}{\sqrt{40}}T_A - \frac{2}{7}T_C = 0$$

Frame 20-9

Concurrent Forces in Space

Using your coefficient equations from the preceding frame, solve for the magnitude of the tension in each cable.

Correct response to preceding frame

$T_A = 211$ N

$T_B = 224$ N

$T_C = 233$ N

<u>Solution</u>:

from $\overline{k}$ coefficients

$$T_A = \frac{\sqrt{40}}{7} T_C$$

From $\overline{i}$ coefficients

$$T_B = \frac{\sqrt{45}}{7} T_C$$

substituting into $\overline{j}$ coefficients

$$600 - \frac{6}{\sqrt{40}} \frac{\sqrt{40}}{7} T_C - \frac{6}{\sqrt{45}} \frac{\sqrt{45}}{7} T_C - \frac{6}{7} T_C = 0$$

$$600 - \frac{18}{7} T_C = 0$$

$$T_C = \frac{700}{3} = 233$$

$$T_A = \frac{700}{3} \frac{\sqrt{40}}{7} = 211$$

$$T_B = \frac{700}{3} \frac{\sqrt{45}}{7} = 224$$

Frame 20-10

Concurrent Forces in Space

List (from memory if possible) the steps taken in solving the preceding problem.

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

Correct response to preceding frame

1. Draw FBD
2. Write forces as vectors
3. Write $\sum \overline{F} = 0$
4. Write coefficient equations
5. Solve simultaneously

Frame 20-11

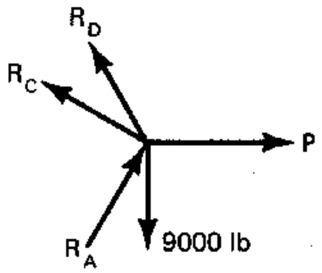
Concurrent Forces in Space

Do Problem 20-1 in your notebook.

Correct response to preceding frame

P = 9000 lb **Solution:**

AB = 0



$$\bar{R}_A = R_A\left(\frac{6\bar{i} + 3\bar{j} - 2\bar{k}}{7}\right)$$

$$\bar{R}_C = R_C\left(\frac{-6\bar{i} + 6\bar{j} + 3\bar{k}}{9}\right)$$

$$\bar{R}_D = R_C\left(\frac{-6\bar{i} + 6\bar{j} - 3\bar{k}}{9}\right)$$

$$\bar{P} = P\bar{i}$$

Since tensions are equal

$$R_C = R_D$$

$$\Sigma\bar{F} = 0$$

$$\left(P + \frac{6}{7}R_A - \frac{6}{9}R_C(2)\right)\bar{i} + \left(\frac{3}{7}R_A + \frac{6}{9}R_C(2) - 9000\right)\bar{j} + \left(-\frac{2}{7}R_A + \frac{3}{9}R_C - \frac{3}{9}R_C\right)\bar{k} = 0$$

$\bar{k}$ coefficients

$$\frac{2}{7}R_A - \frac{1}{3}R_C + \frac{1}{3}R_C = 0$$

$$R_A = 0$$

$\bar{i}$ coefficients

$$P + \frac{6}{7}(0) - \frac{4}{3}\left(\frac{27000}{4}\right) = 0$$

$$P = 9000$$

$\bar{j}$ coefficients

$$\frac{3}{7}(0) + \frac{4}{3}R_C - 9000 = 0$$

$$R_C = \frac{27000}{4}$$

Frame 20-12

Transition

You should now be able to work any problem dealing with forces in three dimensions -- as long as they meet at a point.

The next section will deal with systems of parallel forces in space. Once more you will find that your problems are very similar to ones you worked in two dimensions -- but more tedious.

With that cheering thought in mind, lay in whatever supplies you need for a 15-minute seige and turn to the next frame.

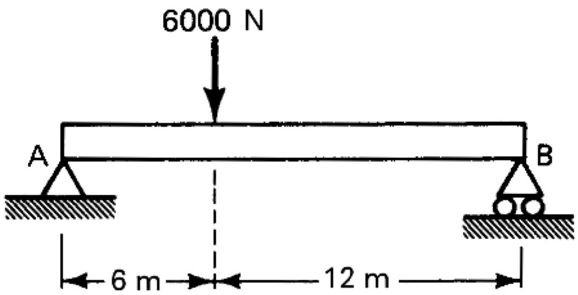
Correct response to preceding frame

No response

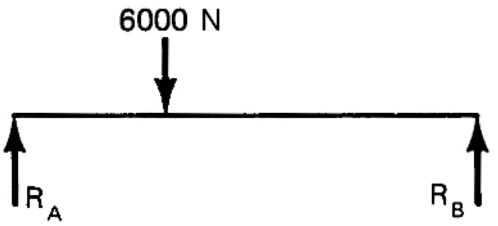
Frame 20-13

Review--Parallel Forces in a Plane

Draw the FBD and write the equations necessary to solve for the reactions at A and B .



Correct response to preceding frame



$\Sigma\overline{F} = 0$

$R_A\overline{j} + R_B\overline{j} - 6000\overline{j} = 0$

$\Sigma\overline{M}_A = 0$

$18\overline{i} \times R_B\overline{j} + 6\overline{i} \times (-6000\overline{j}) = 0$

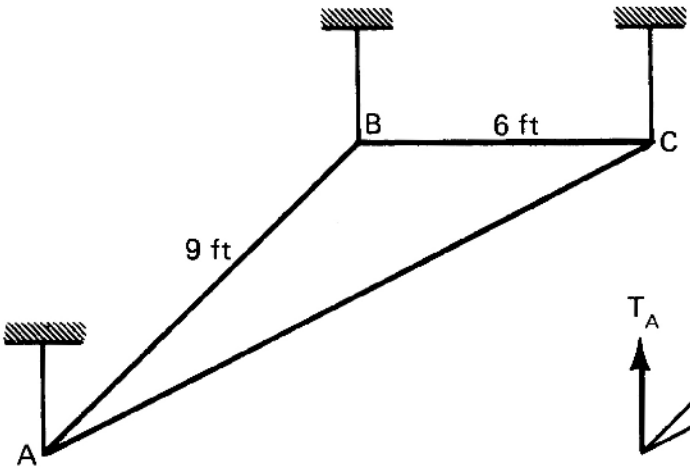
$18R_B\overline{k} - 36000\overline{k} = 0$

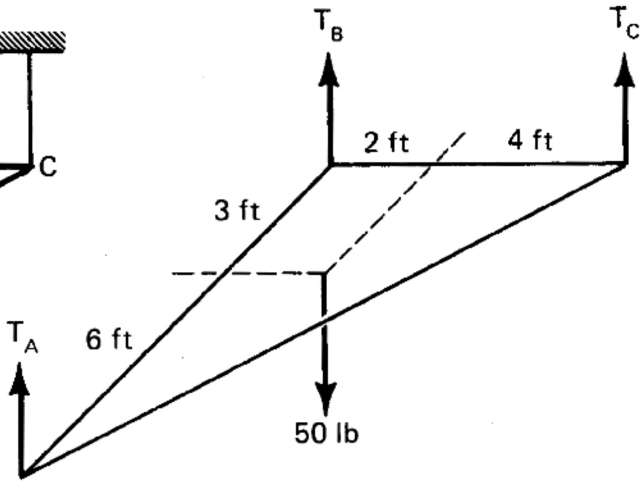
(Other moment equations are also possible.)

Frame 20-14

Parallel Forces in Space

A homogeneous triangular plate weighing 50 lb is suspended in a horizontal position by three vertical strings.





Using the FBD given, write $\Sigma\overline{F} = 0$ and $\Sigma\overline{M}_B = 0$.

Correct response to preceding frame

$$\Sigma \overline{F} = 0$$

$$T_A \overline{j} + T_B \overline{j} + T_C \overline{j} - 50\overline{j} = 0$$

$$\Sigma \overline{M}_B = 0$$

$$9\overline{k} \times T_A \overline{j} + 6\overline{i} \times T_C \overline{j} + (2\overline{i} + 3\overline{k}) \times (-50\overline{j}) = 0$$

$$-9T_A \overline{i} + 6T_C \overline{k} - 100\overline{k} + 150\overline{i} = 0$$

Frame 20-15

Parallel Forces in Space

Reduce the equations from the preceding frame to coefficient equations and solve for the tension in each cord.

Correct response to preceding frame

$T_A = T_B = T_C = \frac{50}{3}$

Solution:

$\bar{i}$ coefficients

$$-9T_A + 150 = 0$$

$\bar{j}$ coefficients

$$T_A + T_B + T_C - 50 = 0$$

$\bar{k}$ coefficients

$$6T_C - 100 = 0$$

Solving

$$T_A = \frac{50}{3}$$

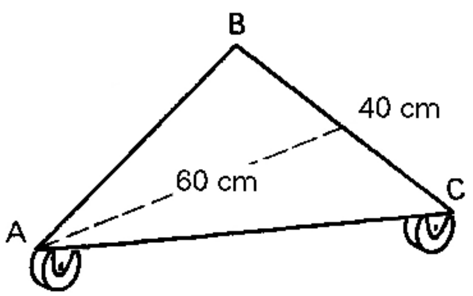
$$T_C = \frac{50}{3}$$

$$T_B = \frac{50}{3}$$

(This result might have been predicted from the symmetry of the figure but such predictions should be made with extreme caution.)

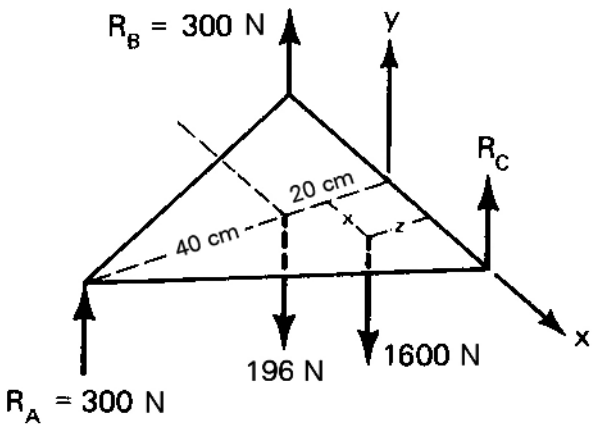
Frame 20-16

Parallel Forces in Space



A three wheeled dolly weighs 20 kilograms. When carrying a 1600 Newton load, the wheel pressure at A is 300 Newtons, as is the wheel pressure at B. The problem is-- where is the center of gravity of the load located? Draw a FBD of the dolly.

Correct response to preceding frame

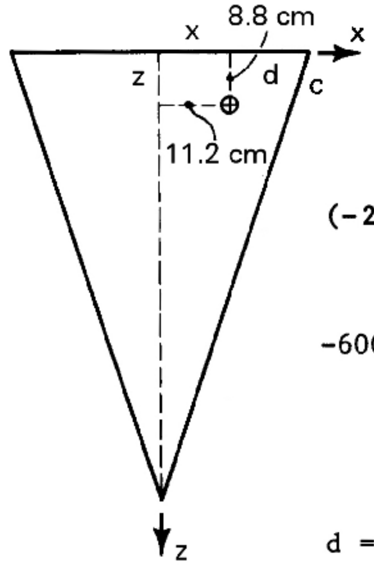


Frame 20-17

Parallel Forces in Space

Using your FBD from the preceding frame write the necessary equations, reduce to coefficient equations, and solve. Locate the load on a top view of the platform.

Correct response to preceding frame



Solution:

$$\Sigma \overline{M}_C = 0$$

Let d = 20 - x

$$(-20\overline{i} + 60\overline{k}) \times 300\overline{j} + (-40\overline{i}) \times 300\overline{j} + (-20\overline{i} + 20\overline{k}) \times (-196\overline{j})$$

$$+ (-d\overline{i} + z\overline{k}) \times (-1600\overline{j}) = 0$$

$$-6000\overline{k} - 18000\overline{i} - 12000\overline{k} + 3920\overline{k} + 3920\overline{i} + 1600d\overline{k} + 1600z\overline{i} = 0$$

$$-6000 - 12000 + 3920 + 1600d = 0$$

$$-18000 + 3920 + 1600z = 0$$

$$d = \frac{14080}{1600} = 8.8 \text{ cm} \qquad x = 20 - d = 11.2 \text{ cm}$$

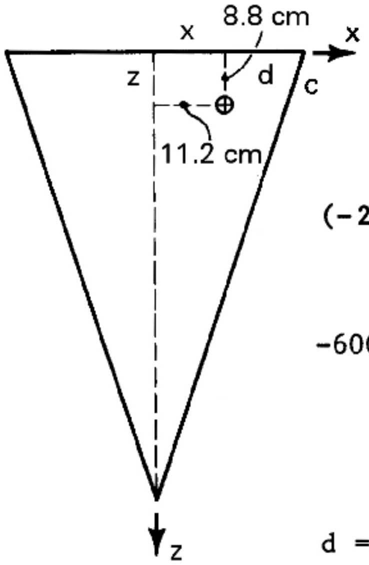
$$z = \frac{14080}{1600} = 8.8 \text{ cm}$$

Frame 20-18

Parallel Forces in Space

Do Problem 20-2 in your notebook.

Correct response to preceding frame



Solution:

$$\Sigma \overline{M}_C = 0$$

Let $d = 20 - x$

$$(-20\bar{i} + 60\bar{k}) \times 300\bar{j} + (-40\bar{i}) \times 300\bar{j} + (-20\bar{i} + 20\bar{k}) \times (-196\bar{j}) + (-d\bar{i} + z\bar{k}) \times (-1600\bar{j}) = 0$$

$$-6000\bar{k} - 18000\bar{i} - 12000\bar{k} + 3920\bar{k} + 3920\bar{i} + 1600d\bar{k} + 1600z\bar{i} = 0$$

$$-6000 - 12000 + 3920 + 1600d = 0$$

$$-18000 + 3920 + 1600z = 0$$

$$d = \frac{14080}{1600} = 8.8 \text{ cm} \qquad x = 20 - d = 11.2 \text{ cm}$$

$$z = \frac{14080}{1600} = 8.8 \text{ cm}$$

Frame 20-19

Transition

Now you can work the easy ones in three dimensions. The rest of the unit will show you how to do the hard ones.

The problems coming up will deal with non-concurrent, non-parallel, non- coplanar systems. (Unfortunately, they are not non-existent.)

The method is the same: draw FBD, write $\Sigma \overline{F} = 0$, write $\Sigma \overline{M}_P = 0$, form coefficient equations and solve. The complications arise from the fact that most single body problems have six unknowns! Patience and attention to detail will help but mostly it will take time -- at least an hour to finish the unit.

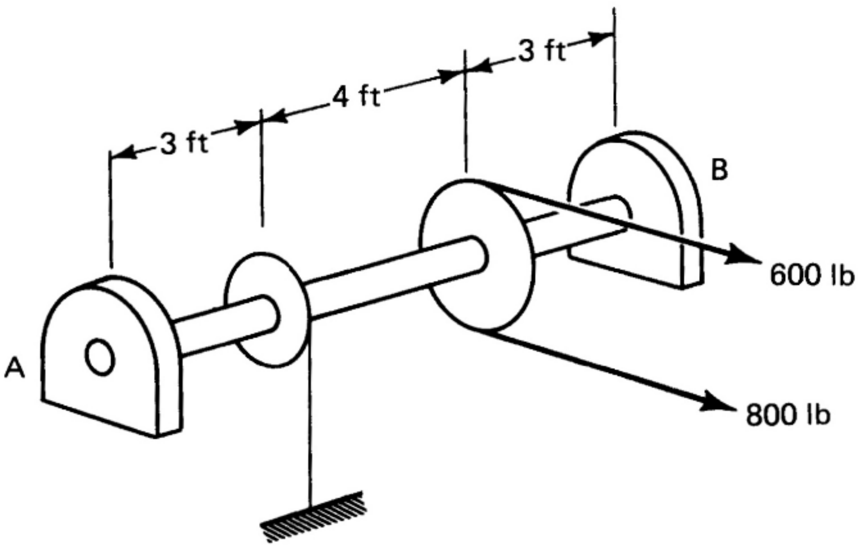
The first problem is a relatively easy one. Sharpen your pencil, find your calculator and begin.

Correct response to preceding frame

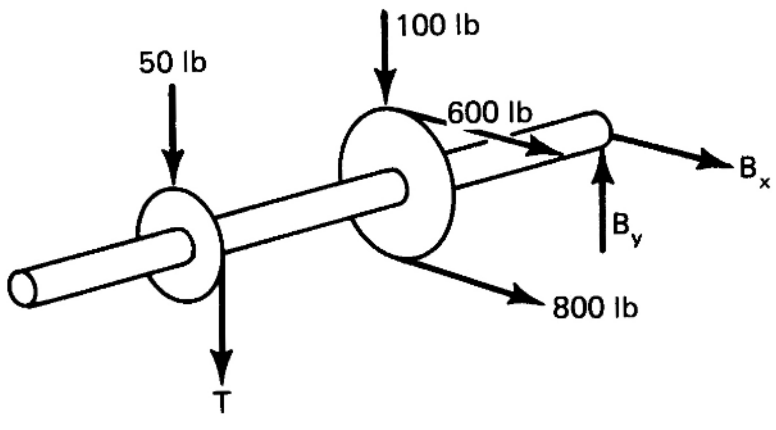
No response

Frame 20-20

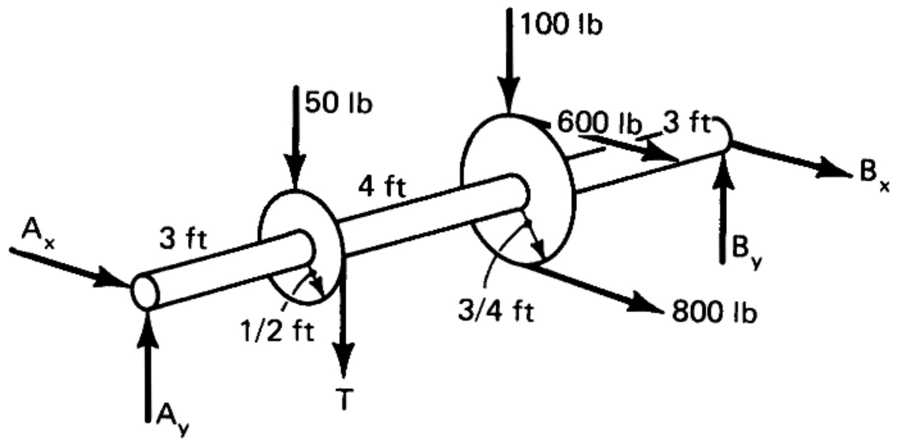
Forces in Parallel Planes



The larger pulley weighs 100 pounds and is 18 inches in diameter. The smaller pulley weighs 50 pounds and is 12 inches in diameter. Complete the free body of the pulleys and shaft.



Correct response to preceding frame



Frame 20-21

Forces in Parallel Planes

Using the free body from the preceding frame, write the equation $\sum \overline{F} = 0$ and $\sum \overline{M}_B = 0$.

(Note: Since these problems are quite laborious you will not likely want to work very many. Consequently it might be a good idea to solve them in a form suitable for inclusion in your notebook.)

Correct response to preceding frame

$$\Sigma\bar{F} = 0$$

$$A_y\bar{j} + A_x\bar{i} + B_y\bar{j} + B_x\bar{i} - T\bar{j} - 100\bar{j} - 50\bar{j} + 600\bar{i} + 800\bar{i} = 0$$

$$\Sigma\bar{M}_B = 0$$

$$\left[3k \times (-100\bar{j})\right] + \left[(3\bar{k} + \frac{3}{4}\bar{j}) \times 600\bar{i}\right] + \left[(3\bar{k} - \frac{3}{4}\bar{j}) \times 800\bar{i}\right] + \left[7\bar{k} \times (-50\bar{j})\right]$$
$$+ \left[(7\bar{k} + \frac{1}{2}\bar{i}) \times (-T\bar{j})\right] + \left[10\bar{k} \times (A_x\bar{i} + A_y\bar{j})\right] = 0$$

Frame 20-22

Forces in Parallel Planes

1. Work out the cross products in the moment equation from the preceding frame.

2. How many coefficient equations can you get from the moment equation? _____

3. How many coefficient equations can you get from the force equation? _____

4. How many unknowns have you? _____

5. Can you solve the problem?

6. Do you want to?

Correct response to preceding frame

1.

$$+300\bar{i} + 1800\bar{j} - 450\bar{k} + 2400\bar{j} + 600\bar{k} + 350\bar{i} + 7T\bar{i} - \frac{T}{2}\bar{k} + 10A_x\bar{j} - 10A_y\bar{i} = 0$$

$$650\bar{i} + 4200\bar{j} + 150\bar{k} + 7T\bar{i} - \frac{T}{2}\bar{k} + 10A_x\bar{j} - 10A_y\bar{i} = 0$$

2. 3
3. 2
4. 5
5. Yes
6. I certainly don't want to.

Frame 20-23

Rigid Bodies--Noncoplanar Forces

Write (preferably from memory) the steps you took in working the preceding problem:

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

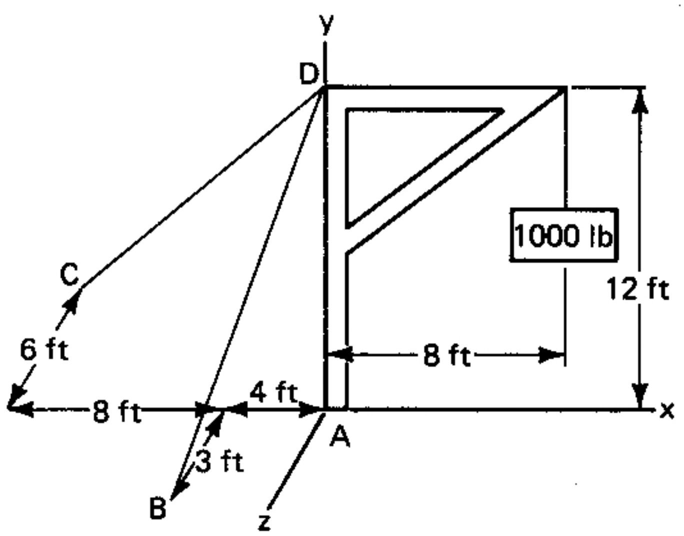
5. ______________________________

Correct response to preceding frame

1. Draw free body diagram
2. Write all forces as vectors
3. Write $\sum \overline{F} = 0$ and $\sum \overline{M}_0 = 0$
4. Break into coefficient equations
5. Solve

Frame 20-24

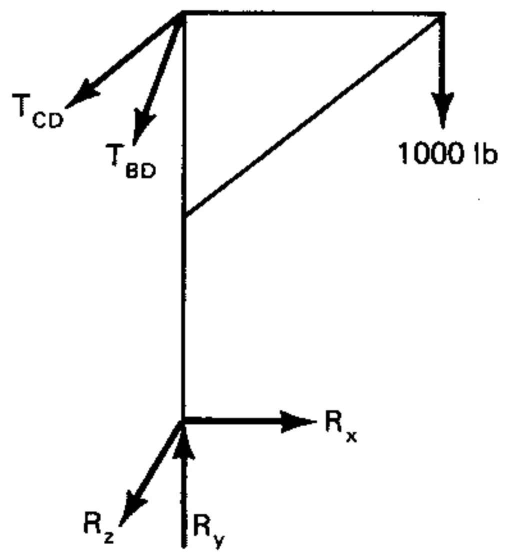
Rigid Bodies--Noncoplanar, NonConcurrent, NonParallel Forces



The derrick shown supports a weight of 1000 pounds. There is a ball and socket at A and Cables run from D to C and from D to B .

The free body is shown.

Write the tension in each cable in vector form.



Correct response to preceding frame

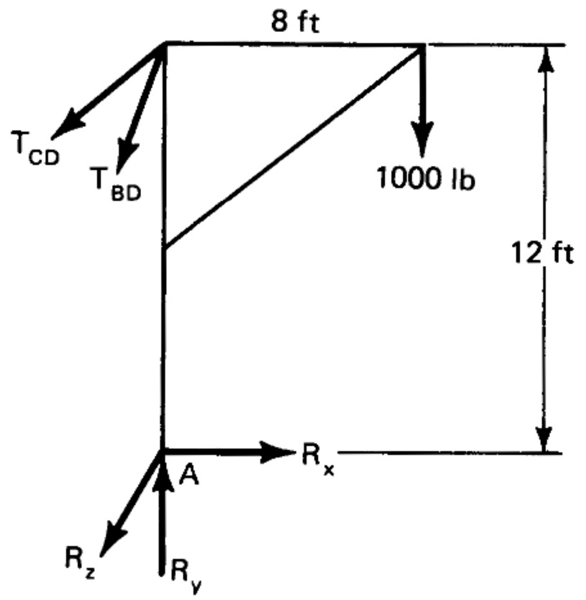
$$\overline{T}_{CD} = T_{CD}\left(\frac{-12\overline{i} - 12\overline{j} - 6\overline{k}}{18}\right)$$

$$= T_{CD}\left(-\frac{2}{3}\overline{i} - \frac{2}{3}\overline{j} - \frac{1}{3}\overline{k}\right)$$

$$\overline{T}_{BD} = T_{BD}\left(\frac{-4\overline{i} - 12\overline{j} + 3\overline{k}}{13}\right)$$

Frame 20-25

Rigid Bodies--NonCoplanar, NonConcurrent, NonParallel Forces



The free body of the derrick from the preceding frame is redrawn for your convenience. Write $\sum\overline{F} = 0$ and $\sum\overline{M}_A = 0$.

Correct response to preceding frame

$\Sigma\overline{F} = 0$

$$R_x\overline{i} + R_y\overline{j} + R_z\overline{k} - 1000\overline{j} - \frac{2}{3}T_{CD}\overline{i} - \frac{2}{3}T_{CD}\overline{j} - \frac{1}{3}T_{CD}\overline{k} - \frac{4}{13}T_{BD}\overline{i} - \frac{12}{13}T_{BD}\overline{j} + \frac{3}{13}T_{BD}\overline{k} = 0$$

$\Sigma\overline{M}_A = 0$

$$\left[8\overline{i} \times (-1000\overline{j})\right] + \left[12\overline{j} \times T_{CD}\left(-\frac{2}{3}\overline{i} - \frac{2}{3}\overline{j} - \frac{1}{3}\overline{k}\right)\right] + \left[12\overline{j} \times T_{BD}\left(\frac{-4\overline{i} - 12\overline{j} + 3\overline{k}}{13}\right)\right] = 0$$

$$-8000\overline{k} + 12T_{CD}\frac{2}{3}\overline{k} - \frac{12T_{CD}}{3}\overline{i} + 12T_{BD}\frac{4}{13}\overline{k} + 12T_{BD}\frac{3}{13}\overline{i} = 0$$

Frame 20-26

Rigid Bodies--NonCoplanar Forces

Break the Force Equation and the Moment Equation you just wrote into component equations.

How many equations have you? _____

How many unknowns? _____

Correct response to preceding frame

You should have 5 equations - 5 unknowns.

$$R_x - \frac{2}{3} T_{CD} - \frac{4}{13} T_{BD} = 0$$

$$R_y - 1000 - \frac{2}{3} T_{CD} - \frac{12}{13} T_{BD} = 0$$

$$R_z - \frac{1}{3} T_{CD} + \frac{3}{13} T_{BD} = 0$$

$$-8000 + 8T_{CD} + \frac{48}{13} T_{BD} = 0$$

$$-4 T_{CD} + \frac{36}{13} T_{BD} = 0$$

Frame 20-27

Rigid Bodies--NonCoplanar Forces

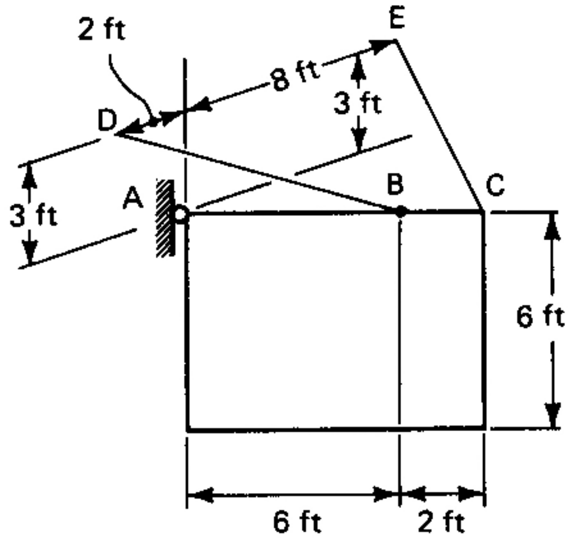
If you need practice in solving simultaneous equations solve for the reactions at A and the tensions in the cables. Otherwise go to the next frame.

Correct response to preceding frame

T_{BD} = 867 lb
T_{CD} = 600 lb
R_x = 667 lb
R_y = 2200 lb
R_z = 0 lb

Frame 20-28

General Equilibrium



A large sign (supply your own message!) of uniform density weighs 360 pounds and is supported by two cables and a ball and socket at A. Find the tension in each cable and the components of the reaction at A.

Correct response to preceding frame

$T_D = 448$ lb

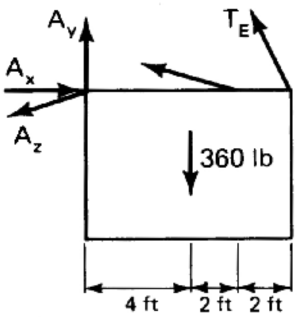
$T_E = 140$ lb

$A_x = 480$ lb

$A_y = 132$ lb

$A_z = -32$ lb

Solution:



$$\overline{T}_D = T_D\left(\frac{3\overline{j} + 2\overline{k} - 6\overline{i}}{\sqrt{9 + 36 + 4}}\right) = T_D\left(\frac{3}{7}\,\overline{j} + \frac{2}{7}\,\overline{k} - \frac{6}{7}\,\overline{i}\right)$$

$$\overline{T}_E = T_E\left(\frac{3\overline{j} - 8\overline{i} - 8\overline{k}}{\sqrt{9 + 64 + 64}}\right) = T_E\left(\frac{3\overline{j} - 8\overline{i} - 8\overline{k}}{\sqrt{137}}\right)$$

$$\Sigma\overline{M}_A = \left[4\overline{i} \times (-360\overline{j})\right] + \left[6\overline{i} \times T_D\left(\frac{3\overline{j} + 2\overline{k} - 6\overline{i}}{7}\right)\right] + \left[8\overline{i} \times T_E\left(\frac{3\overline{j} - 8\overline{k} - 8\overline{i}}{\sqrt{137}}\right)\right] = 0$$

$$-1440\overline{k} + \frac{18}{7}T_D\overline{k} - \frac{12}{7}T_D\overline{j} + \frac{24}{\sqrt{137}}T_E\overline{k} + \frac{64}{\sqrt{137}}T_E\overline{j} = 0$$

$\overline{j}$ coefficients

$$-\frac{12}{7}T_D + \frac{64}{\sqrt{137}}T_E = 0 \quad \text{Thus, } T_E = \frac{\sqrt{137}}{16}\,\frac{3}{7}\,T_D$$

$\overline{k}$ coefficients

$$-1440 + \frac{18}{7}T_D + \frac{24}{\sqrt{137}}T_E = 0, \text{ substituting } T_E$$

$$-1440 + \frac{18}{7}T_D + \frac{24}{\sqrt{137}}\,\frac{\sqrt{137}}{16}\,\frac{3}{7}T_D = 0$$

$$T_D = \frac{1440 \times 14}{45} = 448 \text{ and } T_E = \frac{448 \times 3\sqrt{137}}{7 \times 16}$$

$$= 140 \text{ lb}$$

$$\Sigma\overline{F} = A_x\overline{i} + A_y\overline{j} + A_z\overline{k} + 140\left(\frac{3\overline{j} - 8\overline{i} - 8\overline{k}}{\sqrt{137}}\right) + 448\left(\frac{3\overline{j} + 2\overline{k} - 6\overline{i}}{7}\right) - 360\overline{j} = 0$$

Frame 20-29

General Equilibrium

To make sure you have at least one of these little joys in your notes, do Problem 20-3 in your notebook. (Cheer up, it's not as bad as the last one-- quite.)

Correct response to preceding frame

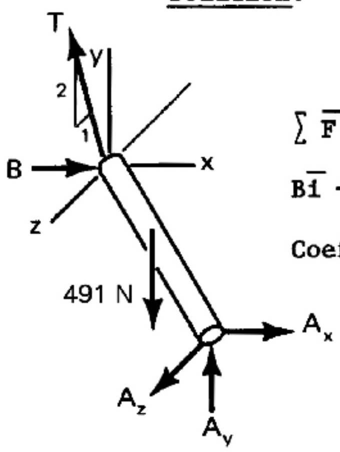
$A_x = -34.1$ N

$A_y = 300$ N

$A_z = -95.3$ N

$B = 34.1$ N

$T = 213$ N



Solution:

$$\sum \bar{F} = 0$$

$$B\bar{i} + A_x\bar{i} + A_y\bar{j} + A_z\bar{k} - 491\bar{j} + T\left(\frac{2}{\sqrt{5}}\bar{j} + \frac{1}{\sqrt{5}}\bar{k}\right) = 0$$

Coefficient equations give

$$A_x + B = 0$$

$$A_y - 491 + \frac{2T}{\sqrt{5}} = 0$$

$$A_z + \frac{T}{\sqrt{5}} = 0$$

$$\sum \bar{M}_A = 0$$

$$\left(-\frac{5}{2}\bar{i} + 4\bar{j} - 7\bar{k}\right) \times \left(-491\bar{j}\right) +$$

$$\left(-5\bar{i} + 8\bar{j} - 14\bar{k}\right) \times \left[B\bar{i} + T\left(\frac{2}{\sqrt{5}}\bar{j} + \frac{1}{\sqrt{5}}\bar{k}\right)\right] = 0$$

$$1228\bar{k} - 3437\bar{i} + \left(\frac{8T}{\sqrt{5}} + \frac{28T}{\sqrt{5}}\right)\bar{i} + \left(-14B + \frac{5T}{\sqrt{5}}\right)\bar{j} + \left(\frac{-10T}{\sqrt{5}} - 8B\right)\bar{k} = 0$$

Coefficient equations give

$$1228 - \frac{10T}{\sqrt{5}} - 8B = 0$$

$$-3437 + \frac{36T}{\sqrt{5}} = 0$$

$$-14B + \frac{5T}{\sqrt{5}} = 0$$

Frame 20-30

Closure

That does it! Hallelujah! By this time you undoubtedly agree with the_ thesis that these problems are tedious. It is to be hoped that you also agree that given enough time and patience you can solve three-dimensional equilibrium problems exactly as you did those in two dimensions.

In fact all statically determinate equilibrium problems will yield to the two equations: Ei = 0 and M0 = 0.

Possibly that is why they are known as the equilibrium equations.