Introduction to Statics

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Unit 17
Equilibrium of Frames

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Unit 17
Equilibrium of Frames

Frame 17-1

Introduction

In an earlier unit you learned how to determe reactions on single bodies. In this unit you will learn to find reactions on systems of interconnected bodies, as well as the forces exerted on one of the bodies by another. The method you will use is unchanged but you will use that method to solve increasingly complex problems. You will do so by considering free body diagrams of the system as a whole and of the individual components.

Some students consider the ability to solve this type of problem to be the most useful skill they develop in a statics course. It is also a skill which is essential to the solution of many mechanics of materials problems.

Go to the next frame.

Correct response to preceding frame

No response

Frame 17-2

Review

What were the necessary and sufficient equations for determining reactions on a rigid body?

1. ______________________________

2. ______________________________

Correct response to preceding frame

1. $\sum \overline{F} = 0$
2. $\sum \overline{M}_A = 0$

Frame 17-3

Review

In a preceding unit you learned a five step system for solving problems dealing with the equilibrium of rigid bodies. List the steps:

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

Correct response to preceding frame

1. Draw a free body diagram.
2. Write vector expressions for all forces in the problem.
3. Write $\sum \overline{F} = 0$
4. Write $\sum \overline{M}_A = 0$
5. Break into component equations and solve.

Frame 17-4

Equilibrium of Systems

In solving problems involving the equilibrium of systems you will use exactly the same method you used to solve single body problems but you will use it several times in each problem.

A system will consist of several interconnected bodies -- each of which is in static equilibrium.

You will find that by drawing a free body of each element and writing $\sum \overline{F} = 0$ and $\sum \overline{M}_A = 0$ for each you will have enough equations to obtain your unknowns.

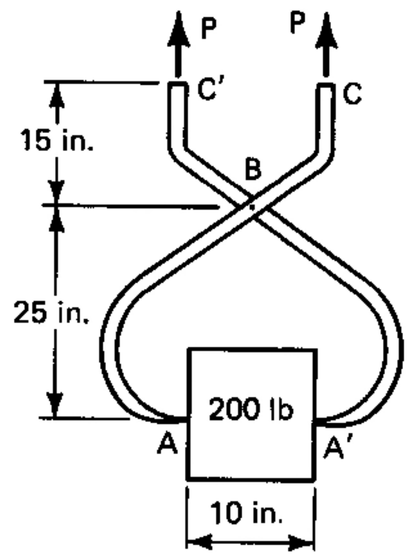
Go to the next frame.

Correct response to preceding frame

No response

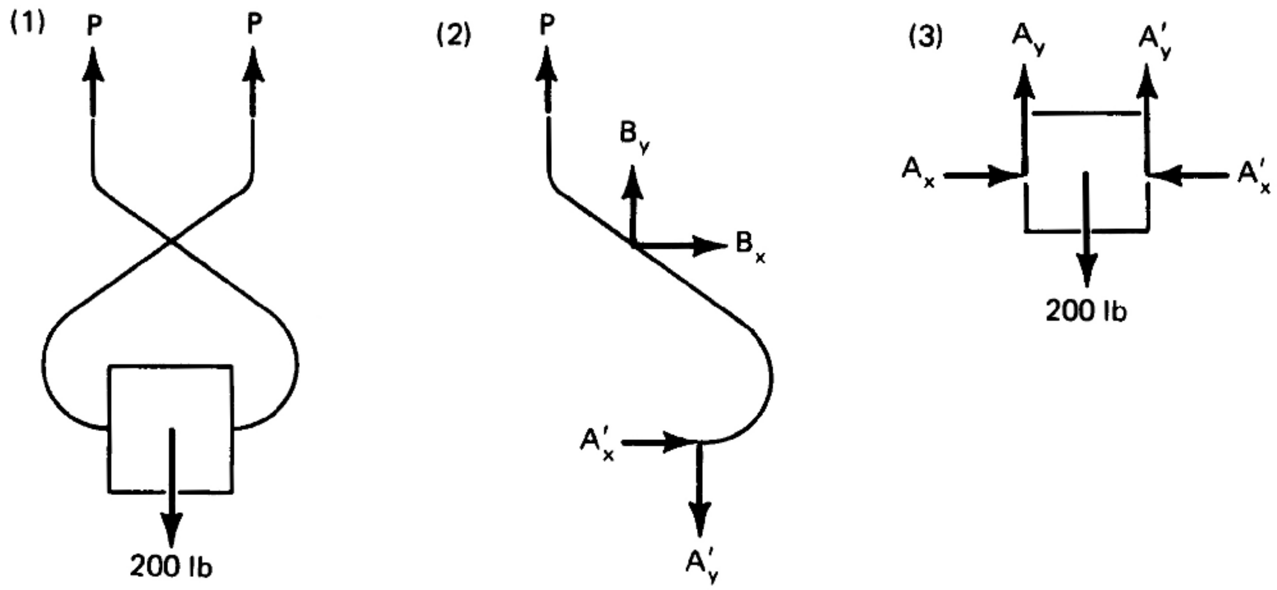
Frame 17-5

Equilibrium of Systems



The problem is to determine the force exerted on the 200 lb block at A and A′. (The lift consists of two arms pinned at B.)

Three free body diagrams are shown below. (1)



Which free body would you use to find the following:

Force P	Free body	___________
Force A'_y	Free body	___________
Force A'_x	Free body	___________

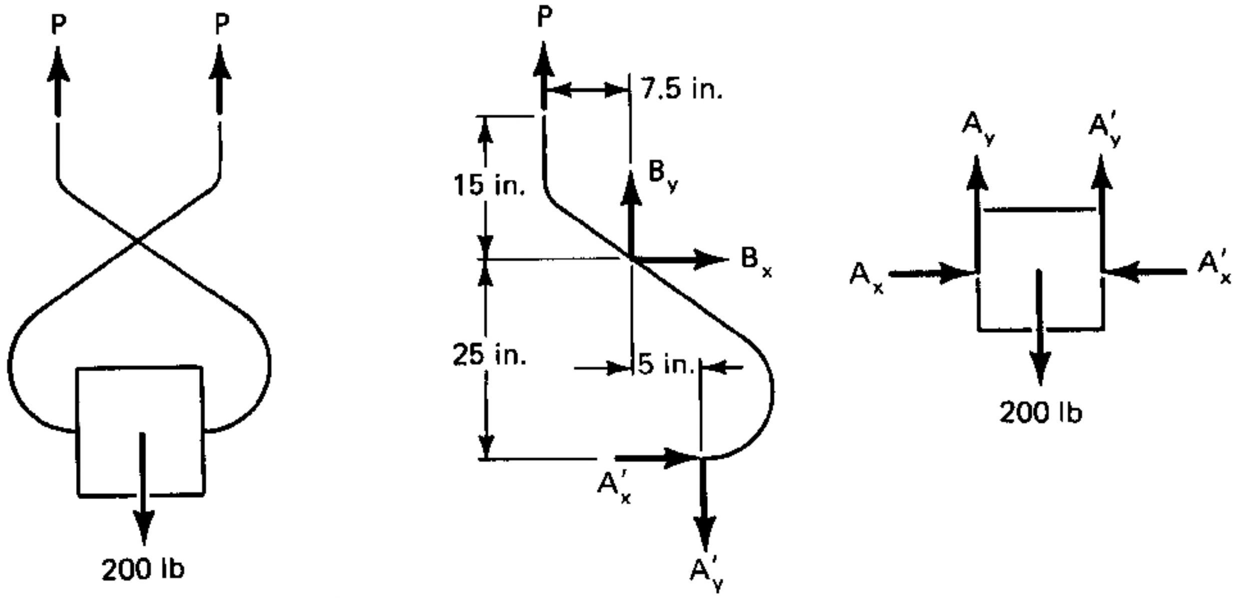
Correct response to preceding frame

Force P Free body (1)
Force A'_y Free body (2)
Force A'_x Free body (3)

(A'_x could not be evaluated, however, until both P and A'_y were found.)

Frame 17-6

Equilibrium of a System



Using the appropriate free body solve for the following. As you evaluate a quantity write its value on the other free bodies.

P = ____________________

A'_y = ____________________

A'_x = ____________________

Correct response to preceding frame

$P = 100$ lb

$A'_y = 100$ lb

$A'_x = 50$ lb

<u>Solution</u>:

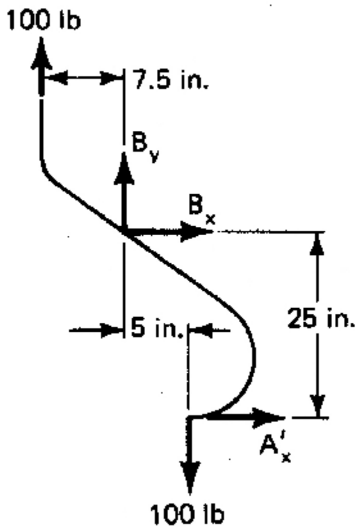
From first FBD $\Sigma\overline{F} = 0$

$P = 100$ lb

From third FBD $\Sigma\overline{F} = 0$

$A'_y = 100$ lb

From second FBD



$\Sigma\overline{M}_B = 0$

$(-7.5\overline{i} \times 100\overline{j}) + (-25\overline{j} \times A'_x\overline{i})$

$+ (5\overline{i} \times -100\overline{j}) = 0$

$(-750 + 25A'_x - 500)\overline{k} = 0$

$A'_x = 50$ lb

Frame 17-7

Equilibrium of a System

Complete the problem by finding the magnitudes of the pin reactions B_x and B_y..

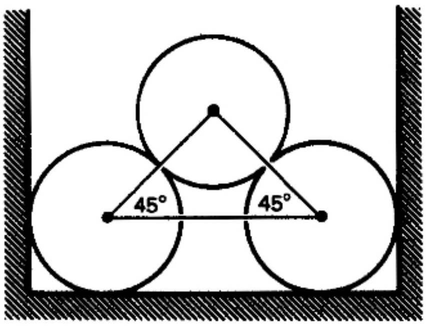
Correct response to preceding frame

$B_y = 0$

$B_x = -50$ lb
(It turns out that the direction is opposite to that assumed in free body (2), on Frame 17-6.)

Frame 17-8

Systems--Free Bodies

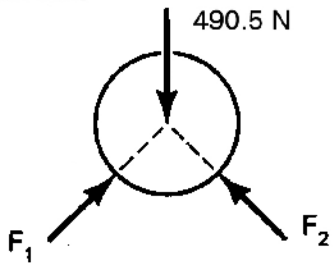


The three cylinders shown are identical. Each weighs 50 kilograms. The problem is to find the magnitude of the force exerted by each wall. (All contact surfaces are frictionless.)

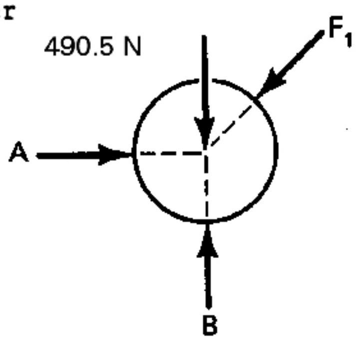
Draw the free bodies necessary to solve.

Correct response to preceding frame

top cylinder



one lower cylinder



You could draw others but these are sufficient.

Frame 17-9

Systems Solutions

1. Each of the free bodies you drew for the preceding frame was acted upon by a

    ________________________ force system.

2. Do the vertical walls exert forces of equal magnitude? ☐ Yes ☐ No

3. Using the free bodies from the preceding frame solve for the magnitude of the force exerted by the vertical wall.

Correct response to preceding frame

1. concurrent
2. Yes, since the system is symmetrical
3. A = 245 N

Solution:

From top cylinder

$$F_1 \cos 45° \bar{i} - F_2 \cos 45° \bar{i} + F_1 \sin 45° \bar{j} + F_2 \sin 45° \bar{j} - 490.5 \bar{j} = 0$$

$$F_1(.707) - F_2(.707) = 0$$

$$F_1 = F_2$$

$$F_1(.707) + F_2(.707) - 490.5 = 0$$

$$2F_1(.707) = 490.5$$

$$F_1 = \frac{490.5}{\sqrt{2}} = 347 \text{ N}$$

From lower cylinder

$$A\bar{i} - F_1(.707)\bar{i} - F_1(.707)\bar{j} - 490.5 \bar{j} + B\bar{j} = 0$$

$$A - F_1(.707) = 0$$

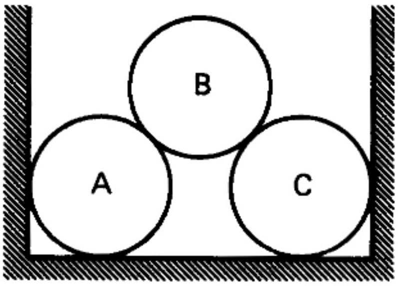
$$A = 245 \text{ N}$$

Frame 17-10

Sign of Reactions

You may have noticed that we have been avoiding any discussion of the issue of sign in finding reactions. In order to write a reaction as a vector, complete with direction, it is necessary to be very specific about which body it acts upon.

In some cases the sign is determined by reality. For instance a normal force is "always a push."



In the previous problem we found that the magnitude of the force exerted by the wall on body A was 245 Newtons.

1. Write the force the wall exerts on A as a vector.
2. Write the force A exerts on the wall as a vector.
3. What force does C exert on the wall?
4. What force does the wall exert on C?

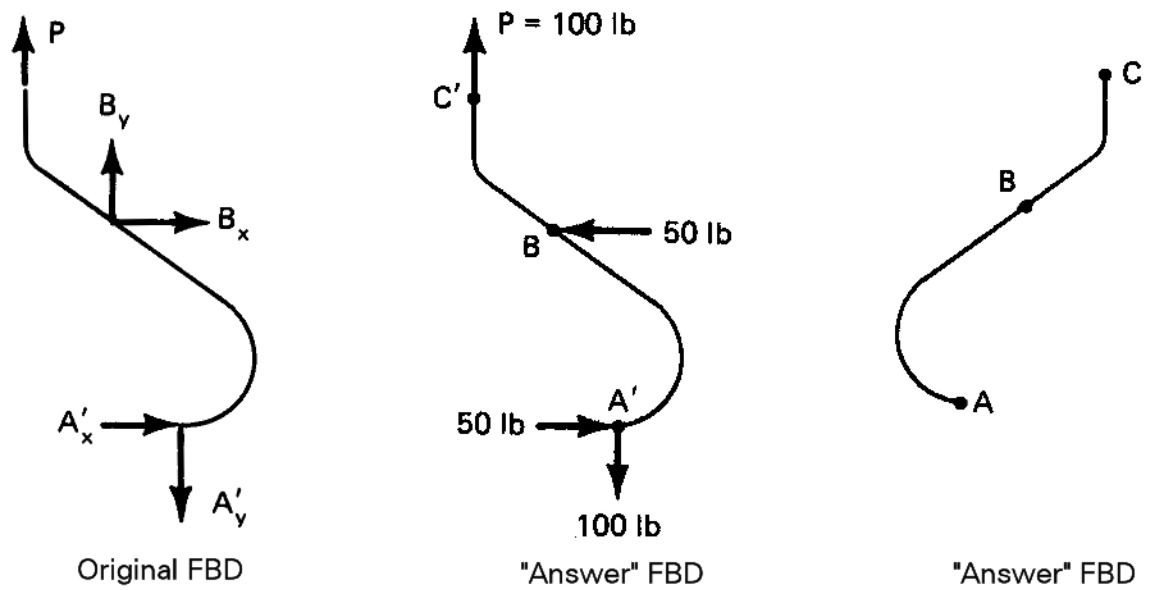
Correct response to preceding frame

1. Wall on A $245\bar{i}$
2. A on wall $-245\bar{i}$
3. C on wall $245\bar{i}$
4. Wall on C $-245\bar{i}$

Frame 17-11

Sign of Reactions

Let's take another look at the lifting device you analyzed a couple of problems back.



On the left is the free body diagram of one half which we made with no knowledge of the actual directions of the force at C. I chose the positive coordinates just for convenience. The middle diagram might be called an "Answer FBD" which shows the numerical results.

Remembering symmetry, complete the answer FBD of the other half.

1. Write a vector expression for the force exerted by the pin at B on ABC. Using the numerical values.

$\bar{B}$ = ________________________________

2. What is the force exerted by the pin at B on A'B'C' ?

$\bar{B}'$ = ________________________________

Correct response to preceding frame

1. $50\bar{i}$ lb
2. $-50\bar{i}$ lb

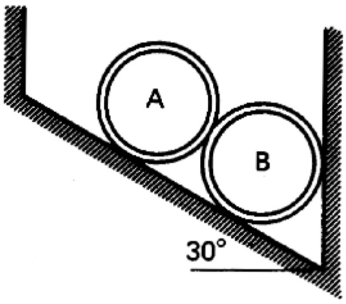
Frame 17-12

Equilibrium of Simple Systems

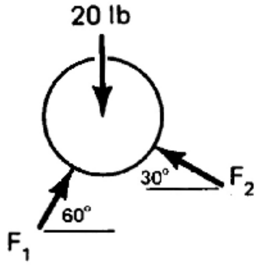
Now do problem 17-1 in your notebook.

Correct response to preceding frame

Force exerted by vertical wall on B is $-23.06\bar{i}$ lb



Solution:



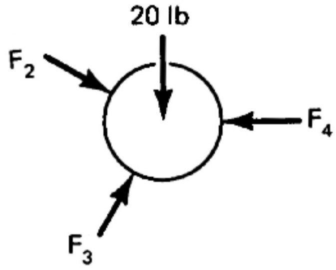
Body A

$\Sigma\bar{F} = 0$

$-.866F_2\bar{i} + .5F_1\bar{i} = 0$

$.866F_1\bar{j} + .5F_2\bar{j} - 20\bar{j} = 0$

$F_1 = 17.32$ lb $\quad F_2 = 10$ lb



Body B

$\Sigma\bar{F} = 0$

$-.5F_2\bar{j} + .866F_3\bar{j} - 20\bar{j} = 0$

$.866F_3 = 25 \qquad F_3 = 28.8$ lb

$.866F_2\bar{i} + .5F_3\bar{i} - F_4\bar{i} = 0$

$8.66 + 14.4 = F_4$

$F_4 = 23.06$ lb

Frame 17-13

Review

You have worked three problems dealing with systems of bodies. The first five steps in your solution were as follows:

1. Draw a FBD of the system or of a part of the system.
2. Write all forces as vectors.
3. Write $\Sigma\bar{F} = 0$.
4. Write $\Sigma\bar{M}_P = 0$ about some point.
5. Break your equations into coefficient equations.

What else must you do?

6. ____

7. ____

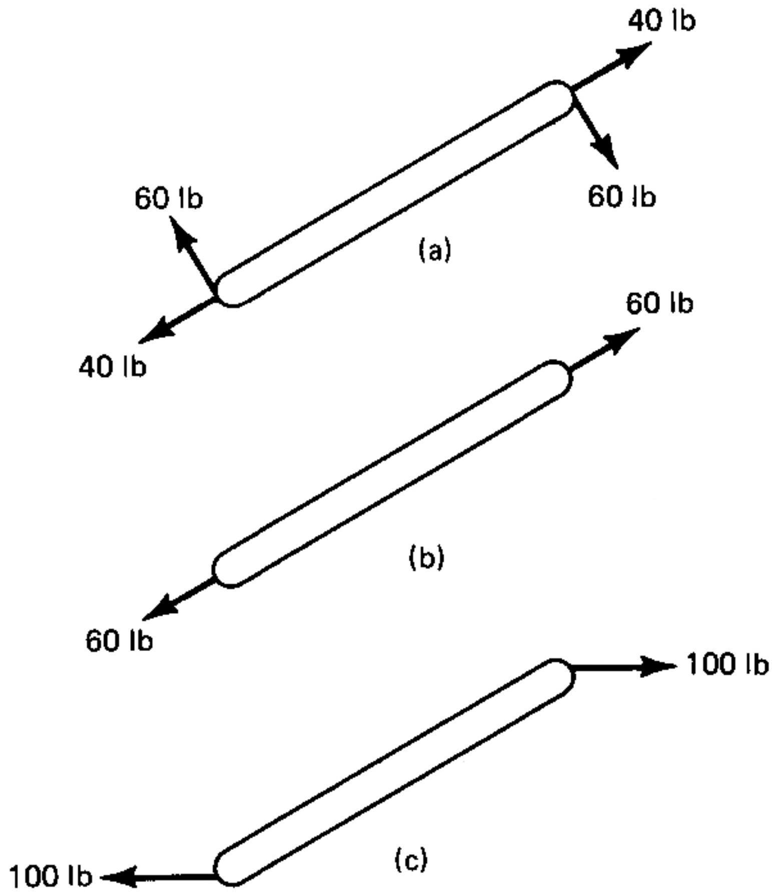
Correct response to preceding frame

6. Repeat steps 1 to 5 until you have enough equations to solve.
7. Solve the simultaneous equations

Frame 17-14

Two-Force Member

Before working more complex problems, let's consider the simple case of a two force member.



Each of the members shown can be described as a two force member with a force acting at each end.

Which are in equilibrium?

____

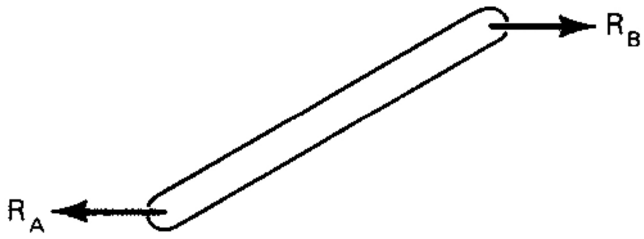
Correct response to preceding frame

Only (b) is in equilibrium.

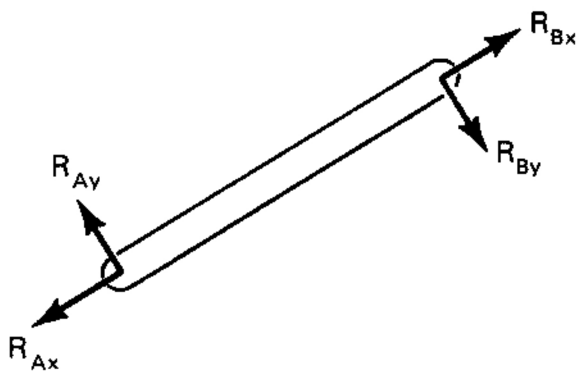
Frame 17-15

Two-Force Members

The free body below shows a member pinned at each end. Each pin can exert a force in any direction.



Splitting R_A and R_B into components, along and perpendicular to, the member results in the following free body.

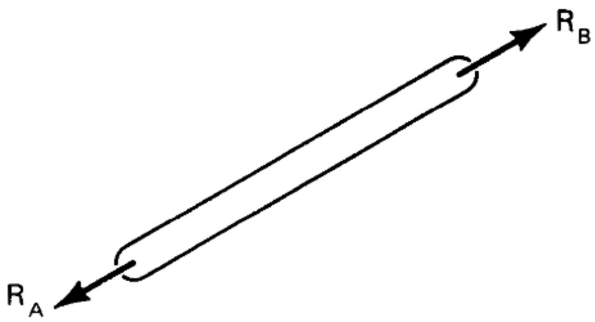


By taking moments, evaluate R_{Ay} and R_{By}.

Draw a free body of the member that shows all forces acting on it in their true directions.

Correct response to preceding frame

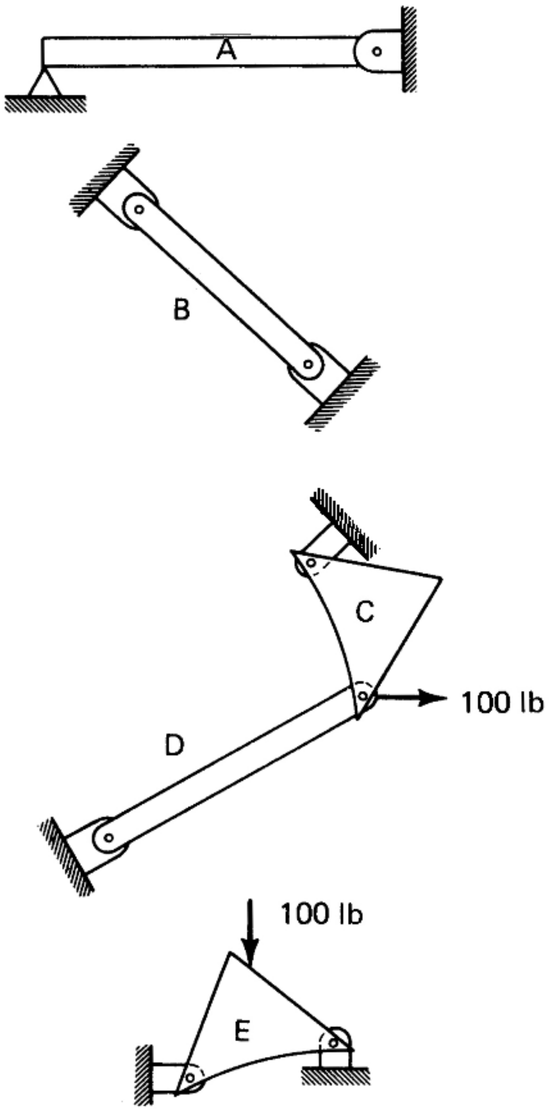
$R_{Ay} = 0$ and $R_{By} = 0$



Frame 17-16

Two-Force Members

Any member which is pinned at two points is a two-force member if the only forces acting on the body are applied at the pins. Which of the following are two-force members? Consider the members to be of negligible weight.



Two-Force Members are

☐ A

☐ B

☐ C

☐ D

☐ E

Correct response to preceding frame

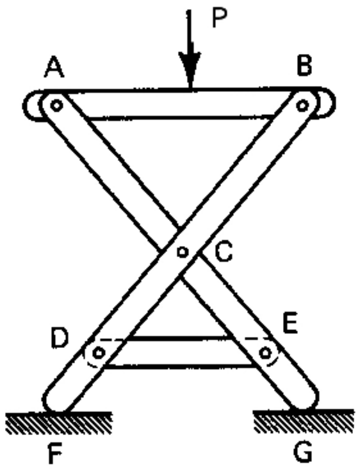
B, C, and D are two-force members.

Frame 17-17

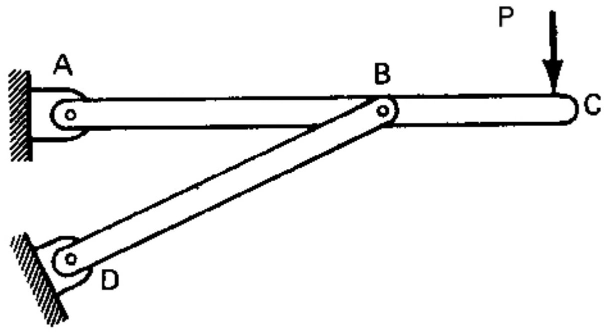
Two-Force Members

In a pin-connected two-force member, the lines of action of the forces pass through both pins. Identify the two-force members in the frames below and draw a free body of each, showing each force with its true line of action.

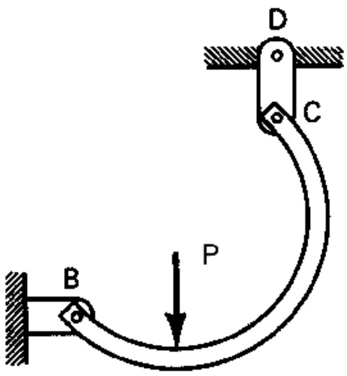
1.



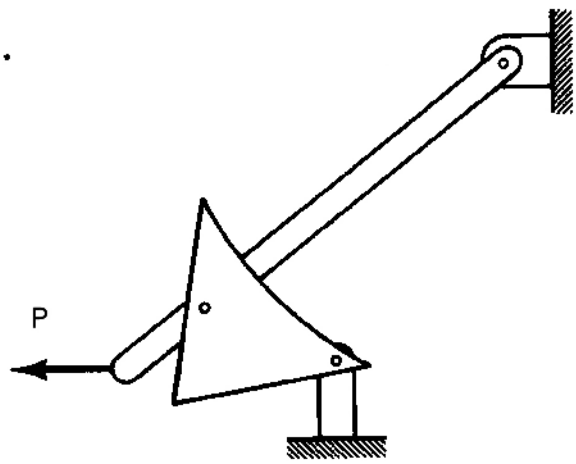
2.



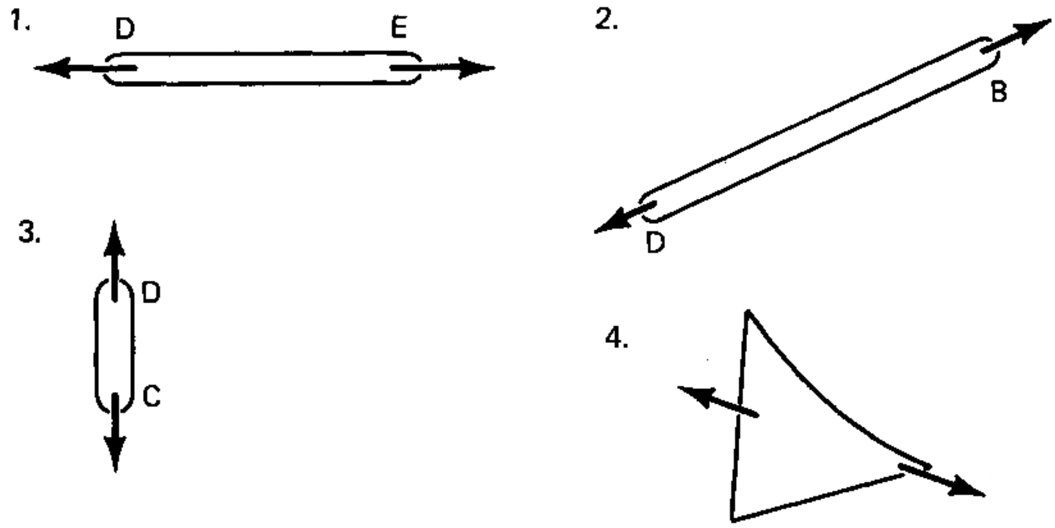
3.



4.



Correct response to preceding frame



Frame 17-18

Transition

The problems you just finished were relatively simple, but problems of greater complexity can be worked just the same way. The trick is to draw enough FBDs and write enough equations to get rid of all unknowns.

The remainder of this unit will introduce you to some more complex problems, and afford you some practice in working them.

The next section will take about 30 minutes. Whenever you feel up to it -- Go to the next frame.

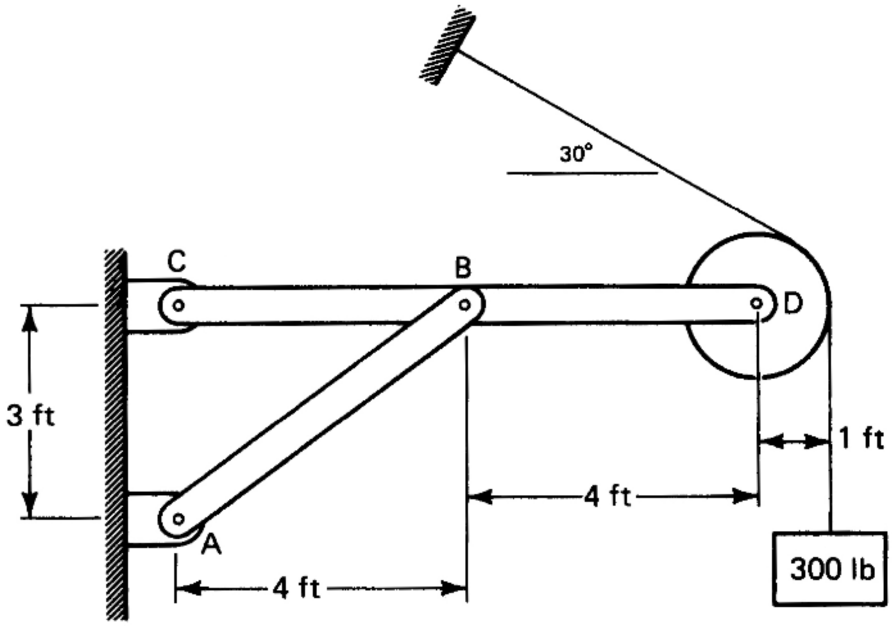
Correct response to preceding frame

No response

Frame 17-19

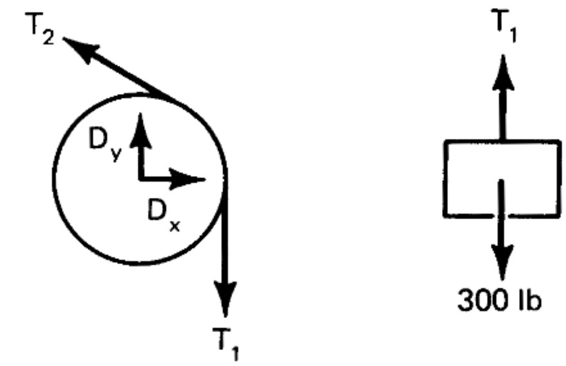
Frames

The weights of the frame and pulley are negligible.



Draw free body diagrams of the pulley and weight and from them compute the magnitude of the tension in the cord and the reactions of pin D on the pulley. (Remember, it is a good idea to assume all unknown senses positive until proven otherwise.)

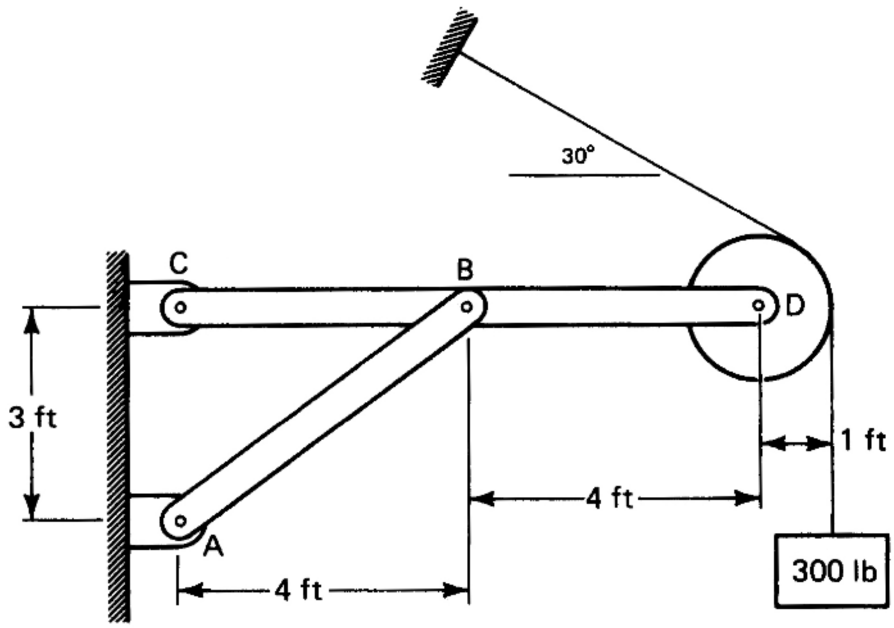
Correct response to preceding frame



$T_1 = 300$ lb
$T_2 = T_1 = 300$ lb
$D_y = 150$ lb
$D_x = 260$ lb

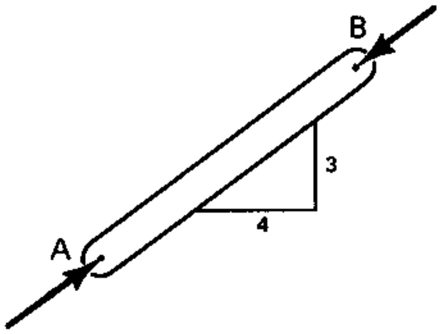
Frame 17-20

Frames



Now make an FBD of member AB. Notice that AB is loaded only at the two pins, so that it must be subject to a collinear force system.

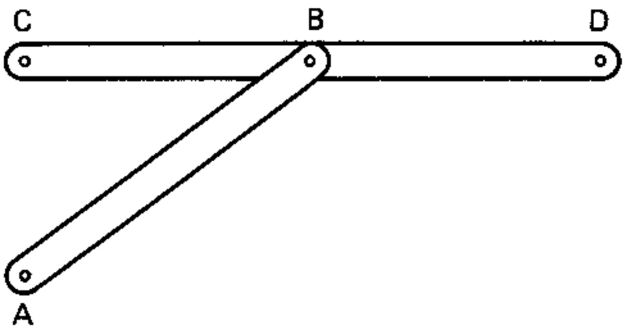
Correct response to preceding frame



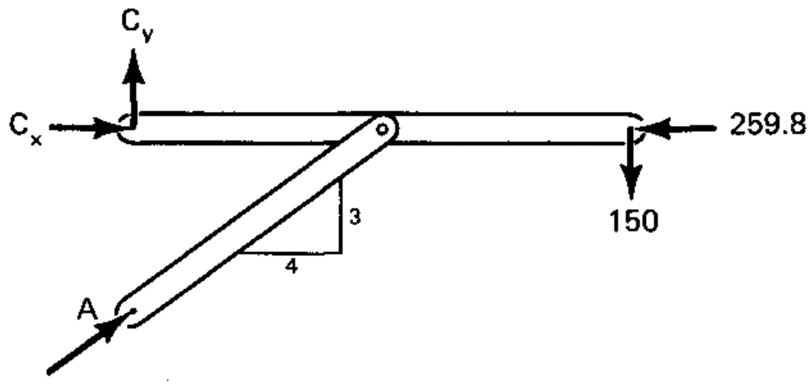
Frame 17-21

Frames

Using your answers from the preceding frames, complete the free body diagram of the frame ABCD. Label all known forces with their magnitudes.



Correct response to preceding frame



Note: The senses of the forces at C are not known but I have assumed them to be in the positive coordinate directions for convenience.

Frame 17-22

Frames

1. Using the FBD you just drew, write $\sum \overline{F} = 0$ and $\sum \overline{M}_C = 0$. Break your vector equations into coefficient equations.

2. Do you have a sufficient number of equations to allow you to solve for the remaining unknowns?

☐ Yes ☐ No

3. Why was C a good center of moments?

____

Correct response to preceding frame

1. $\Sigma\bar{F} = 0$

   $C_x\bar{i} + C_y\bar{j} - 260\bar{i} - 150\bar{j} + .6A\bar{j} + .8A\bar{i} = 0$

   $\Sigma\bar{M}_c = 0$

   $\left[8\bar{i} \times -150\bar{j}\right] - \left[3\bar{j} \times (.8A\bar{i} + .6A\bar{j})\right] = 0$

   Coefficient equations:

   From $\Sigma\bar{F} = 0$

   $C_x - 260 + .8A = 0$

   $C_y - 150 + .6A = 0$

   From $\Sigma\bar{M}_c = 0$

   $-1200 + 2.4A = 0$

2. Yes--3 equations and 3 unknowns
3. C was a good moment center because two unknown forces passed through it.

Frame 17-23

Frames

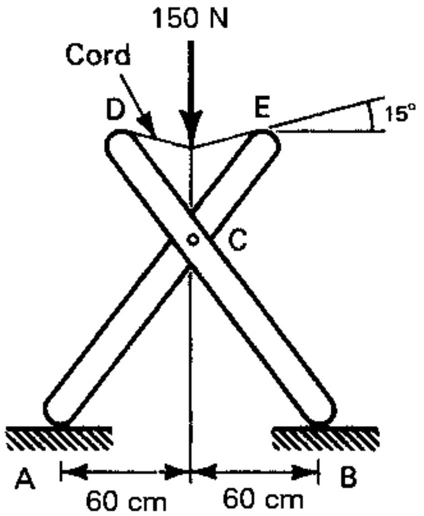
Using your preceding work, find the pin reaction at B on member AB. Write your answer as a vector.

$\bar{B} =$ ______________________________

Correct response to preceding frame

Frame 17-24

Frames

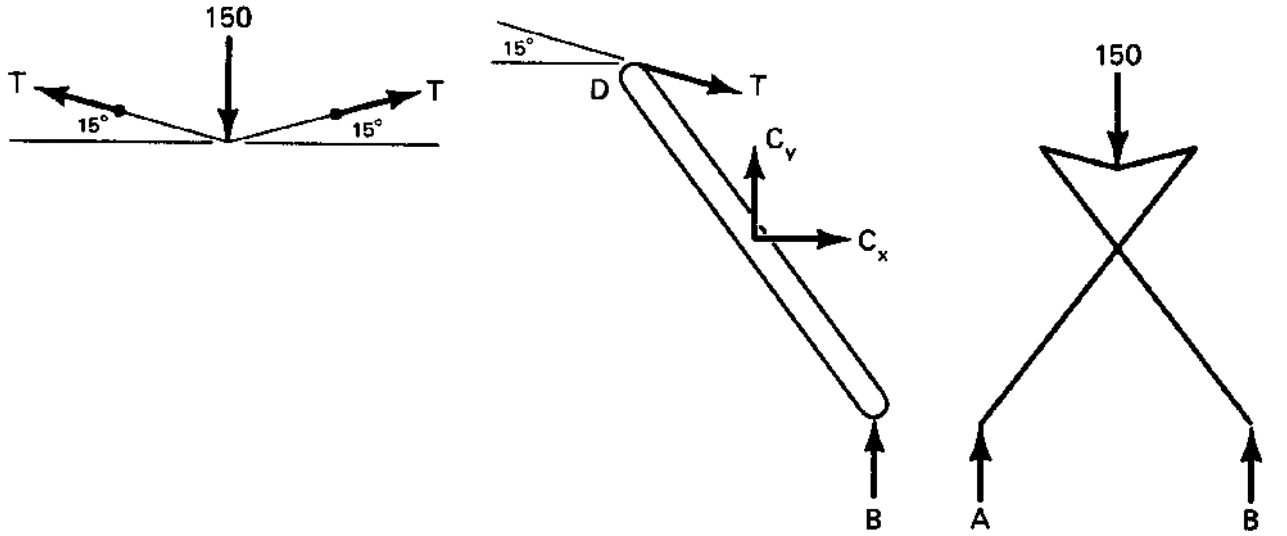


Problem:

Find the reaction of the pin C on member BCD.
All members may be considered weightless.

Draw the free bodies necessary to solve this problem. Check them against the correct response before going further.

Correct response to preceding frame



These three are sufficient to solve. ACE could be drawn but would give no new information.

Frame 17-25

Frames

Using your free bodies write the equations necessary to solve. We won't ask you to work out the algebra.

Correct response to preceding frame

From FBD (1) of previous response

$\Sigma\overline{F} = 0$

$2\,T \sin 15°\,\overline{j} - 150\,\overline{j} + T \cos 15°\,\overline{i} - T \cos 15°\,\overline{i} = 0$

$T = \dfrac{150}{2 \sin 15°}$

From FBD (3)

$\Sigma\overline{M}_A = 0$

$[1.2\overline{i} \times B\overline{j}] + [0.6\overline{i} \times (-150\overline{j})] = 0$

$1.2B - 900 = 0$

$B = 75$

From FBD (2)

$\Sigma\overline{F} = 0$

$-T \sin 15°\,\overline{j} + T \cos 15°\,\overline{i} + Bj + C_y\overline{j} + C_x\overline{i} = 0$

$-T \sin 15° + B + C_y = 0$

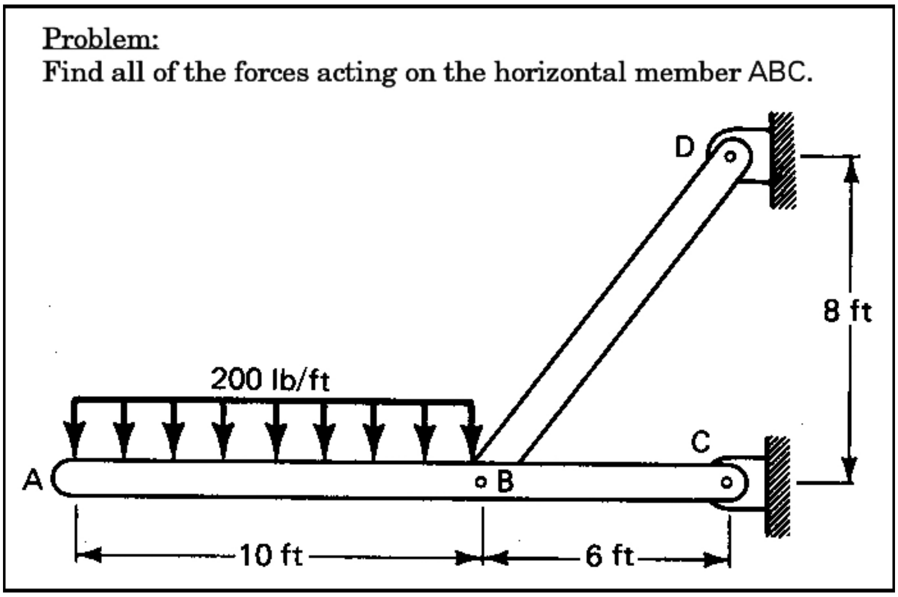
$T \cos 15° + C_x = 0$

Since B and T have already been found

C_x and C_y can now be found.

Frame 17-26

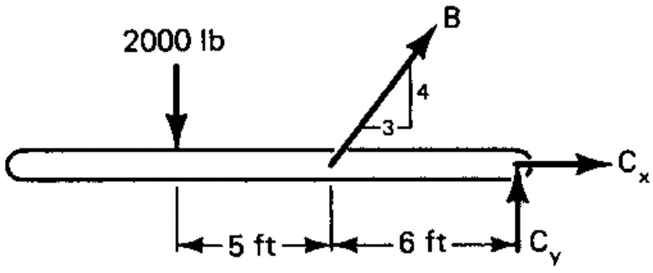
Frames

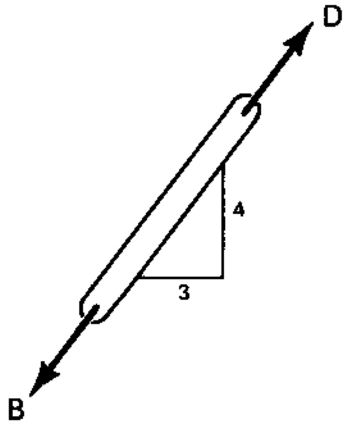


Draw the necessary FBDs and write the necessary equations to solve.

(Do not perform the calculations unless you really want to.)

Correct response to preceding frame





$\Sigma \overline{M}_C = 0$

$-11\overline{i} \times (-2000\overline{j}) - 6\overline{i} \times (.6B\overline{i} + .8B\overline{j}) = 0$

$22000 - 4.8B = 0$ (Equation 1)

$\Sigma \overline{F} = 0$

$-2000\overline{j} + .8B\overline{j} + .6B\overline{i} + C_x\overline{i} + C_y\overline{j} = 0$

$-2000 + .8B + C_y = 0$ (Equation 2)

$.6B + C_x = 0$ (Equation 3)

3 equations, 3 unknowns

Frame 17-27

Frames

You will need to have some of these in your notes, so work Problem 17-2 and Problem 17-3 in your notebook.

Correct response to preceding frame

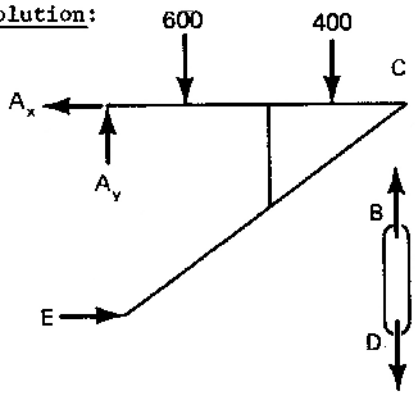
Problem 17-2

$\bar{A} = -600\bar{i} + 1000\bar{j}$ Newtons

$\bar{B} = -900\bar{j}$ Newtons

$\bar{C} = 900\bar{j} + 600\bar{i}$ Newtons

Solution:

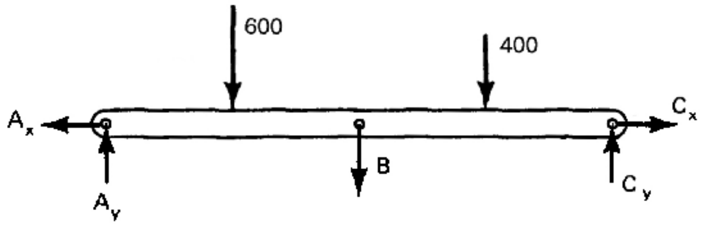


$\Sigma\bar{F} = 0$ gives $A_y = 1000$

$A_x = E$

$\Sigma\vec{M}_E = 0$ gives $A_x = 600$ in assumed direction

BD is a two force member so B = D and both act along the axis of the member



$\Sigma\bar{M}_c = 0$ gives $B = 900$ direction as assumed

$\Sigma F_y = 0$ gives $C_y = 900$

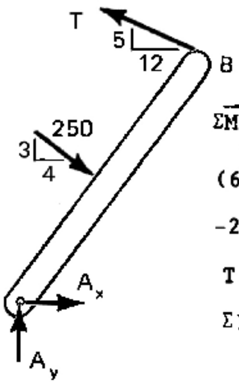
$\Sigma F_x = 0$ gives $C_x = 600$

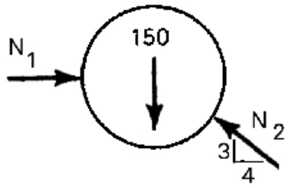
Problem 17-3

T = 129 pounds

$A_x = 80.4$ pounds

$A_y = 100$ pounds





$\Sigma F_y = 0$ gives $N_2 = 250$

$\Sigma\vec{M}_A = 0$

$$(6\bar{i} + 8\bar{j}) \times (200\bar{i} - 150\bar{j}) + (12\bar{i} + 16\bar{j}) \times T\,\frac{(-12\bar{i} + 5\bar{j})}{13} = 0$$

$$-2500\bar{k} + \frac{T}{13}(+60+192)\,\bar{k} = 0$$

$T = 129$

$\Sigma F_y = 0$ gives $A_y = 100$ lb

$\Sigma F_x = 0$ gives $A_x = 80.4$ lb

Frame 17-28

Conclusion

You have looked at the solution of nine problems involving interconnected systems of bodies -- mostly frames.

As you can see, you can end up using a lot of paper in writing a good solution.

Go to the next page.

Correct response to preceding frame

No response

Frame 17-28

Conclusion

All solutions in this unit hinged on the fact that if a system is in equilibrium, each of its parts is also in equilibrium. Analysis of the separate parts of the system, as well as of the system as a whole, provided you with sufficient information to solve.

In short you simply drew FBDs and wrote equations until you had as many equations as you had unknowns.

The process is admittedly tedious but it will solve any problem that is not statically indeterminate.