

# Introduction to Statics

.PDF Edition – Version 0.95

## Unit 16

# Free Body Diagrams of Multi-Body Systems

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# Unit 16

## Free Body Diagrams of Multi-Body Systems

Frame 16-1

### **Introduction**

This unit will give you more work on free body diagrams. In it we will deal with the methods of relating forces on several bodies which make up a system and with ways of breaking a system into several parts. These techniques are very useful in working problems involving forces in frames, machines, bridges, and buildings.

Turn to the next frame.

Correct response to preceding frame

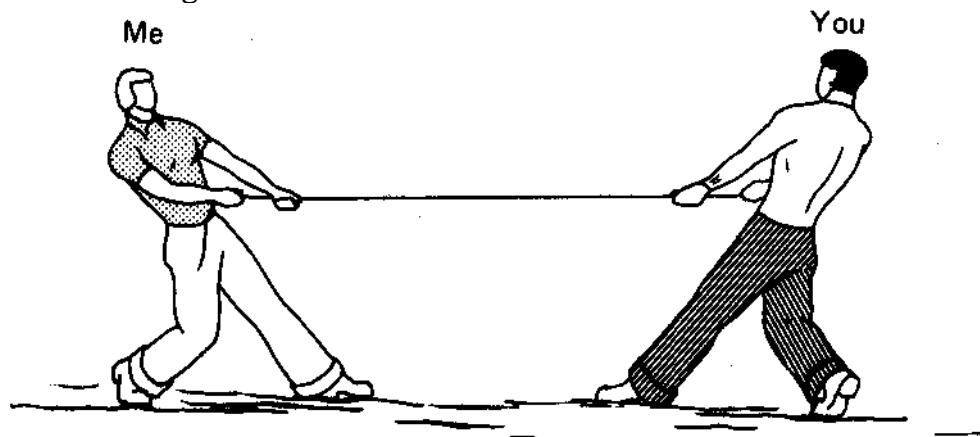
No response

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Frame 16-2

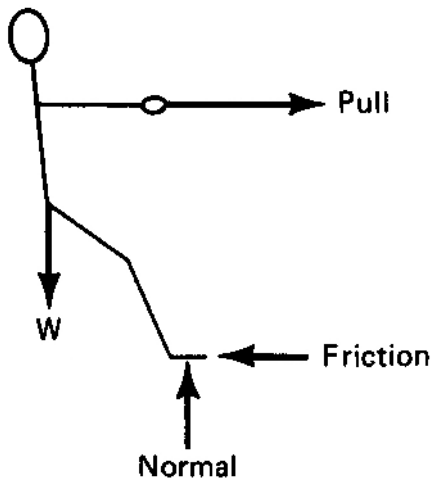
### Interaction Between Bodies

Look at the tug-of-war shown below.

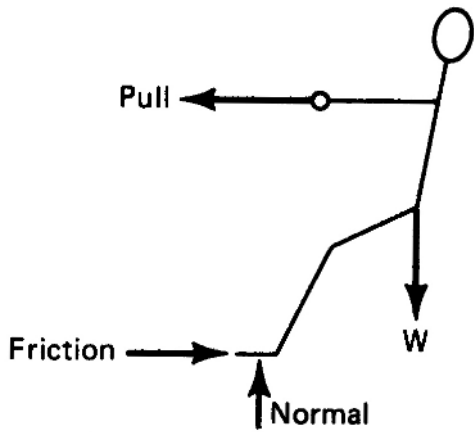


If you are trying to pull me to the right and we are not moving, what am I doing?

This is a FBD of me, draw yourself.



Correct response to preceding frame  
pulling you to the left



---

Frame 16-3

### Interaction Between Bodies

Sir Isaac Newton formulated a law which deals with the forces between bodies. His Third Law of Motion states:

***"For every action there is \_\_\_\_\_ ."***

Correct response to preceding frame

an equal and opposite reaction

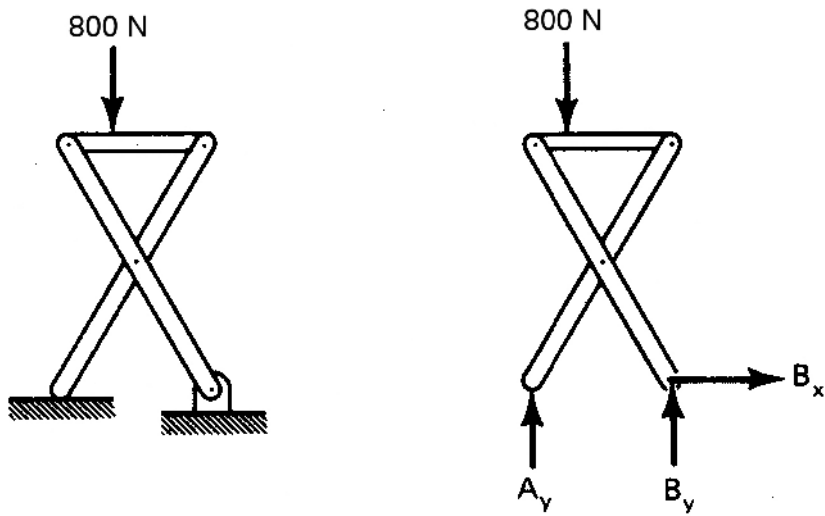
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Frame 16-4

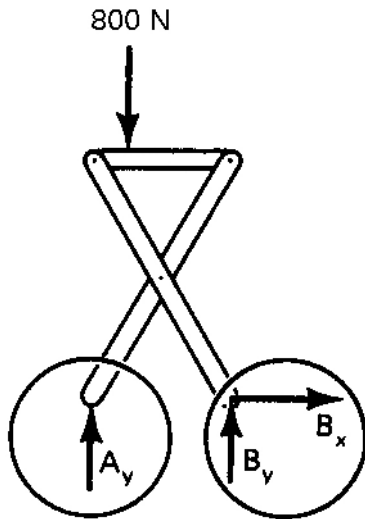
### Interaction of System With Surroundings

For some unknown reason, engineers use the word "reaction" in a special way to mean forces which act on a structure at the points where it contacts its surroundings.

A structure and its free body diagram are shown below. Circle the reactions on the FBD.



Correct response to preceding frame



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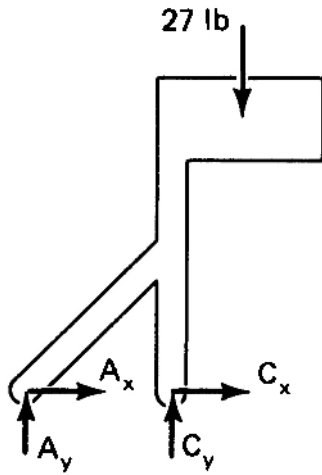
Frame 16-5

### Interaction of System With Surroundings

Complete the FBD of the system shown below.

The forces at A and C are called \_\_\_\_\_ .

Correct response to preceding frame



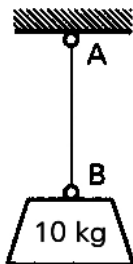
reactions

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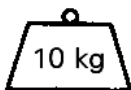
Frame 16-6

### Interaction of Bodies

A weight hangs on a cord which is in turn tied to a ring.



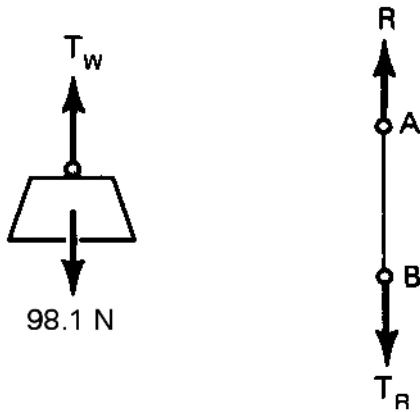
1. Complete the FBD of the weight.



2. Draw a FBD of the cord from A to B. (Make use of Newton's Third Law.)



Correct response to preceding frame



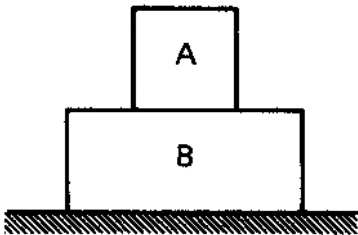
(Notice that  $T_R$  and  $T_W$  are equal and opposite, and that the weight of the string is probably negligible.)

---

Frame 16-7

### Interaction Between Bodies

Block A rests on block B and B rests on a table.



1. Draw a FBD of A.

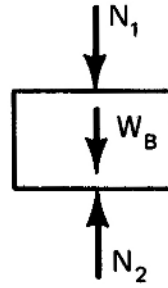
2. Draw a FBD of B.

Correct response to preceding frame

1.



2.

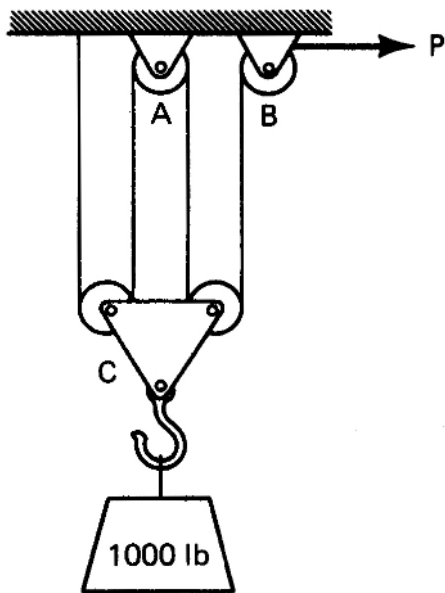


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Frame 16-8

### Interaction Between Bodies

A system of pulleys is to be used as a hoist.

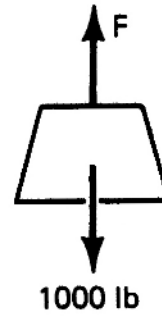
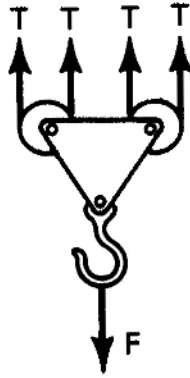
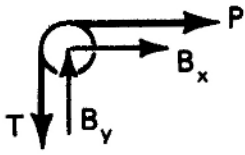


Draw a FBD of pulley  $B$ .

Draw a FBD of the hook assembly  $C$ .

Draw a FBD of the  $1000 \text{ lb}$  weight.

Correct response to preceding frame

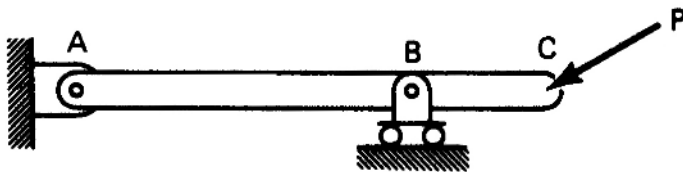


(Compared with the 1000 lb block, the pulley and hook are probably very light, and treated as weightless.)

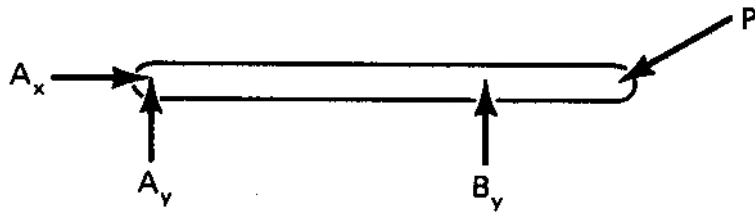
Frame 16-9

### Review

Draw a free body diagram of bar AC below.



Correct response to preceding frame



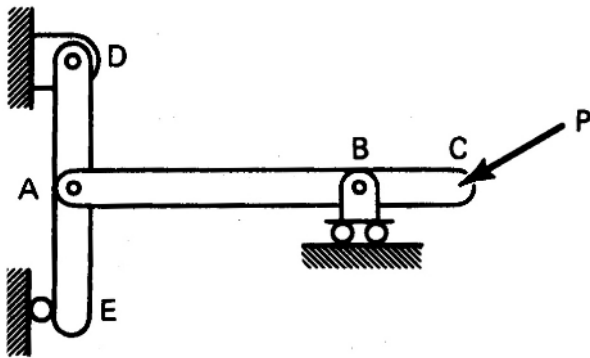
(Or equivalent response)

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Frame 16-10

**Review**

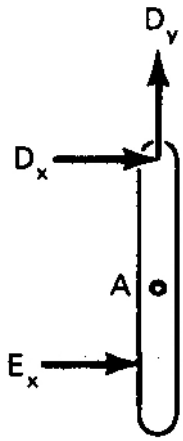
Now replace the bracket at A in the preceding frame with another bar.



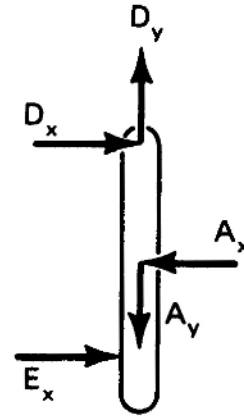
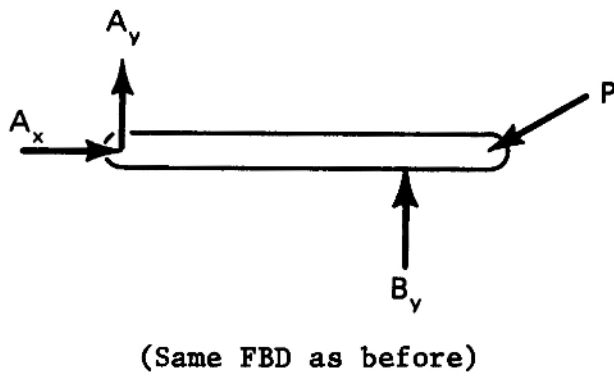
Draw a FBD of bar AC.

According to Sir Isaac, for every action there is an equal and opposite reaction. Complete the FBD of bar DE below by drawing in the force at A.





Correct response to preceding frame

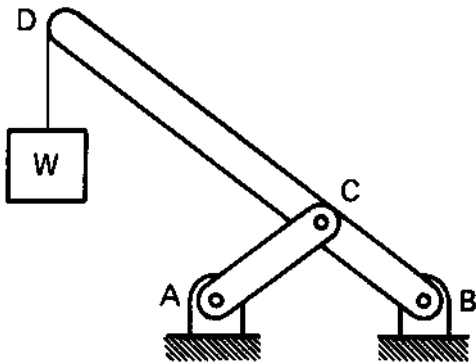


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Frame 16-11

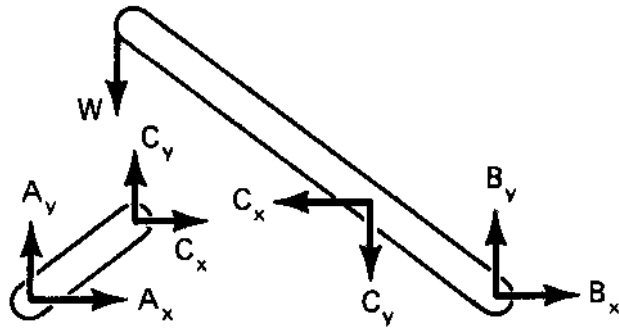
### Interaction Between Bodies

The crane boom BD is held up by a brace AC. The weights of the members are negligible.



1. Draw a FBD of the brace, showing the forces,  $A_x$ ,  $A_y$ ,  $C_x$  and  $C_y$ .
2. Draw a FBD of the boom using the notation above for C and also showing  $B_x$ ,  $B_y$  and  $W$ .

Correct response to preceding frame

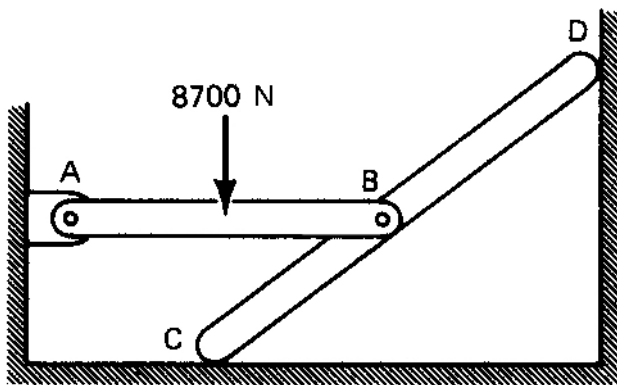


Did you remember to reverse the directions of  $C_x$  and  $C_y$  when you drew the second FBD?

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Frame 16-12

### Interactions Between Bodies



1. Draw a FBD of bar AB .

2. Draw a FBD of member CD.

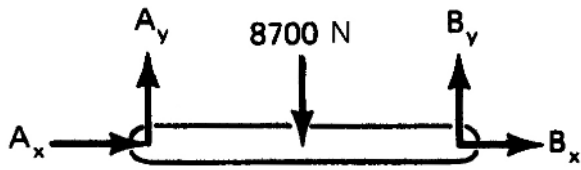
3. Interaction takes place at \_\_\_\_\_ .

4. The reactions are at \_\_\_\_\_ .



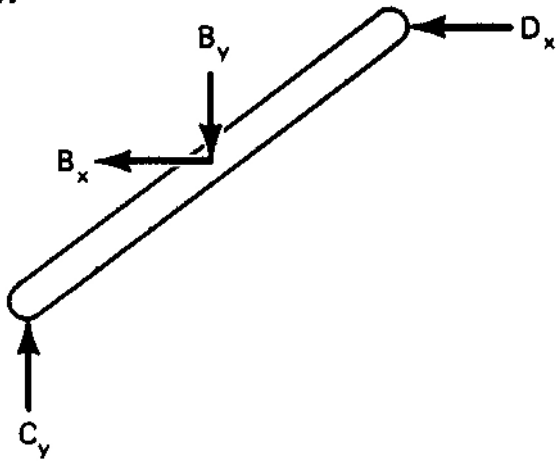
Correct response to preceding frame

1.



( $A_x$ ,  $A_y$ ,  $B_x$  or  $B_y$  could have the opposite sense.)

2.



3. B

4. A, C, and D

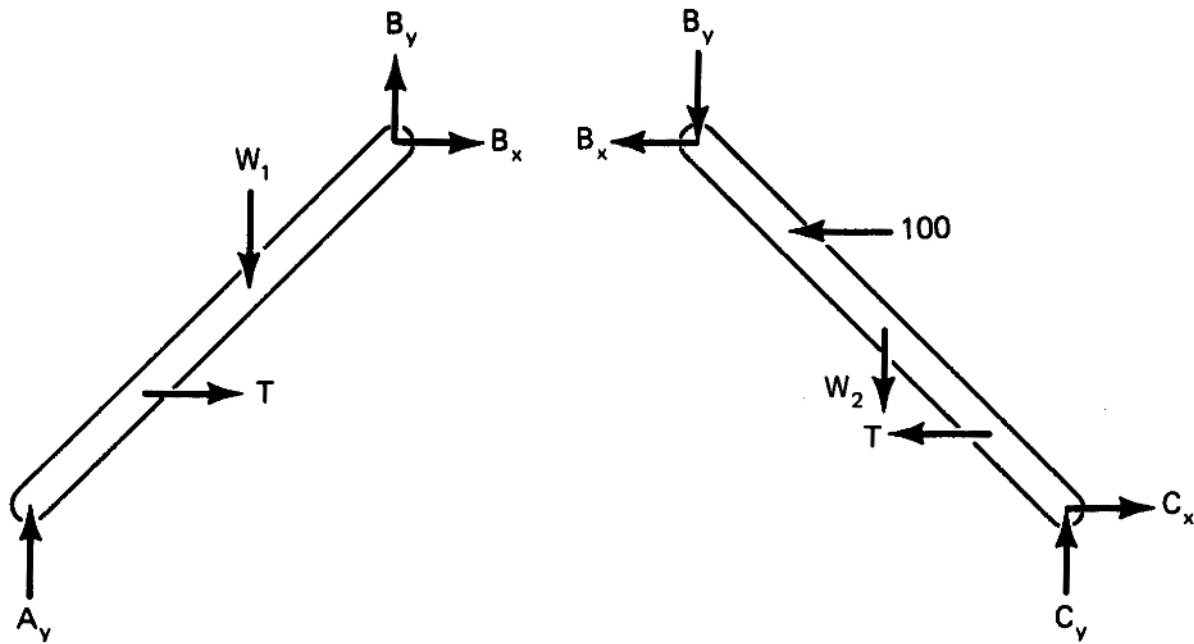
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Frame 16-13

### Notebook

Work problem 16-1 in your notebook.

Correct response to preceding frame



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Frame 16-14

### Transition

In drawing a FBD we make a sketch and show the external forces acting on the body. We've talked about external forces without explaining the difference between an internal and an external force.

Internal forces do not affect the motion of rigid bodies and may be neglected in "statics" and "dynamics." (In fact, since they don't affect the motion they must be left out. The next section will show you how this is done.)

Internal forces do affect the behavior of deformable bodies and you will have an opportunity to study them at some length in "strength of materials."

On to the next frame.

Correct response to preceding frame

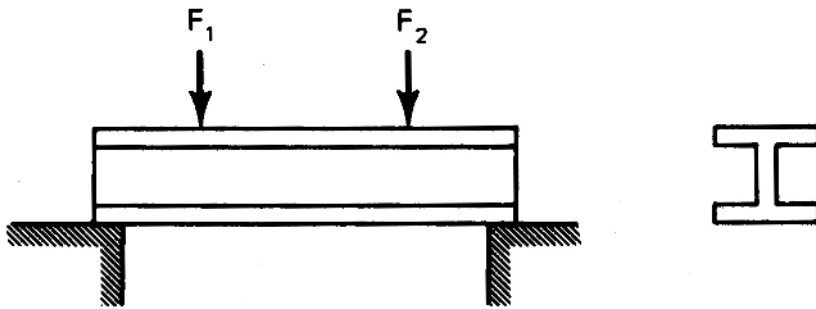
No response

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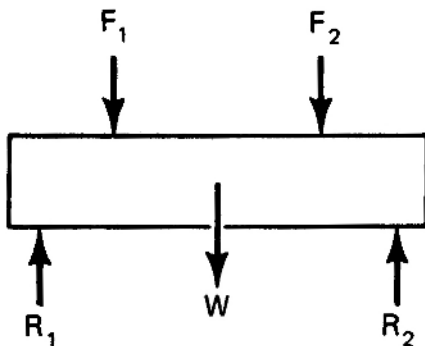
Frame 16-15

### Internal Forces

Consider a wooden "I" beam



Its FBD looks like this.



1. If I told you that the beam was made of 2 x 6 boards held together with nails would you have to change the FBD?  Yes  No

2. Do the nails exert forces on the boards?  Yes  No

Correct response to preceding frame

1. No, the FBD remains the same because the forces due to the nails don't tend to make the beam move.
  2. Yes, the nails do exert forces. They are internal forces.
- 

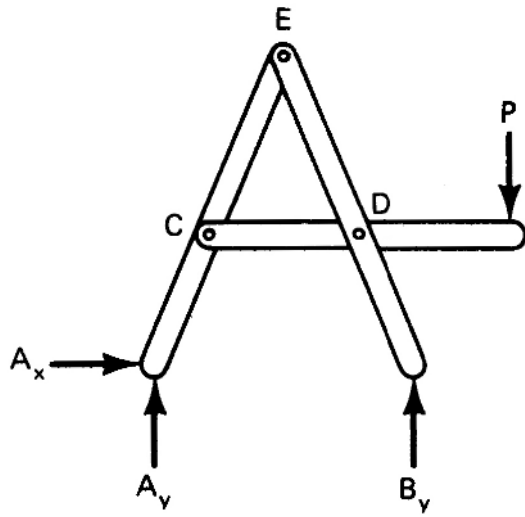
Frame 16-16

### **Internal Forces**

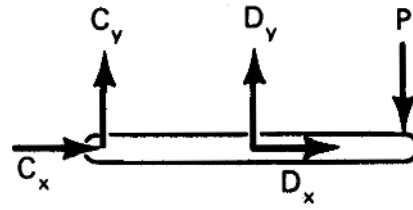
The use of the words "internal force" is often reserved for use in describing the forces between the particles which make up a body. It is more precise to say that the things we are considering now are "forces internal to the system".

Two correct FBDs are shown below.





(a)



(b)

1. What forces shown on (b) are internal to the system (a)? \_\_\_\_\_
2. Forces which are interactions between members are \_\_\_\_\_ to the system.

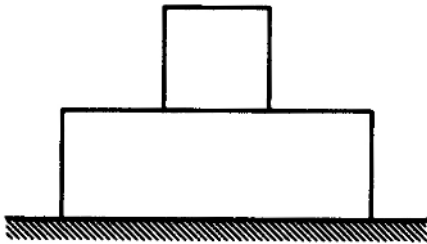
Correct response to preceding frame

1.  $C_x$ ,  $C_y$ ,  $D_x$  and  $D_y$
  2. **internal** to the system
- 

Frame 16-17

### Internal Forces

The trouble with internal forces is that they may become or may have been external forces in a slightly different problem.

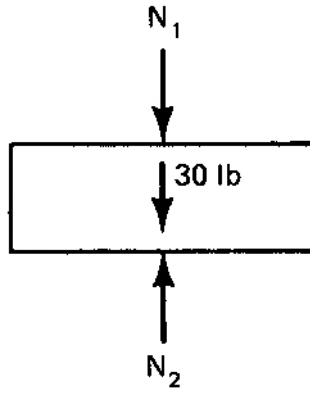


Suppose we had two boxes in a stack: we could draw the following FBDs

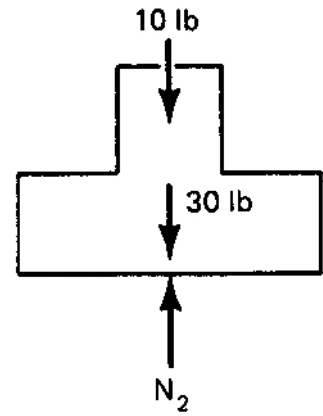
a)



b)



c)



All of the FBDs are correct.

1. In which case (or cases) is  $N_1$ , the force between the boxes, an internal force? \_\_\_\_\_

2. In which case (or cases) is  $N_1$  an external force? \_\_\_\_\_



Correct response to preceding frame

1. (c) internal
  2. (a) and (b) external
- 

Frame 16-18

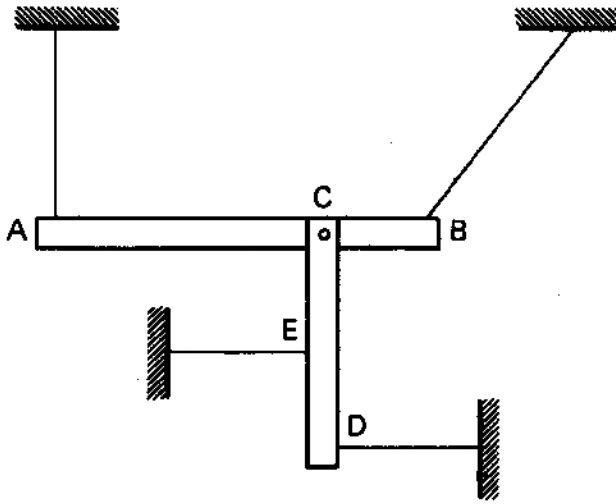
### **Internal Forces**

Complete the free body diagrams for the systems shown below:

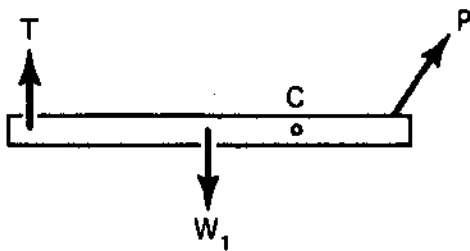




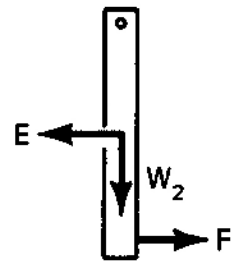




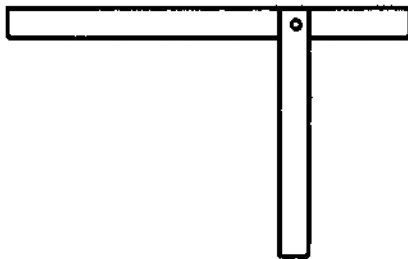
1.



2.



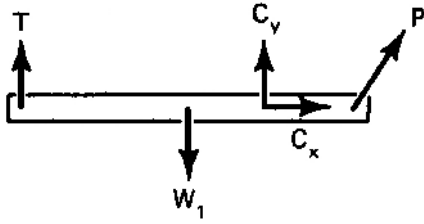
3.



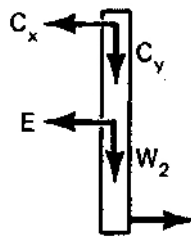
4. The forces shown at C on 1. and 2. are \_\_\_\_\_ on 3.

Correct response to preceding frame

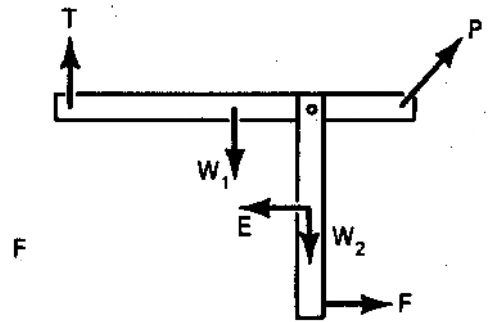
1.



2.



3.

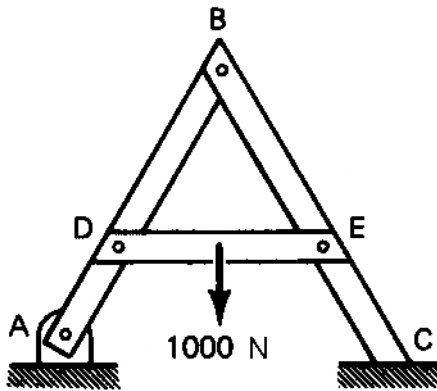


4. internal to the system

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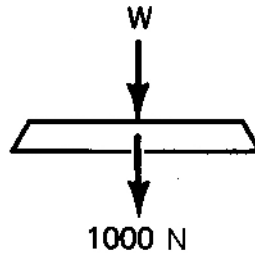
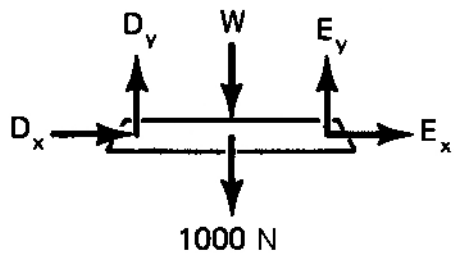
Frame 16-19

### Internal and External Forces

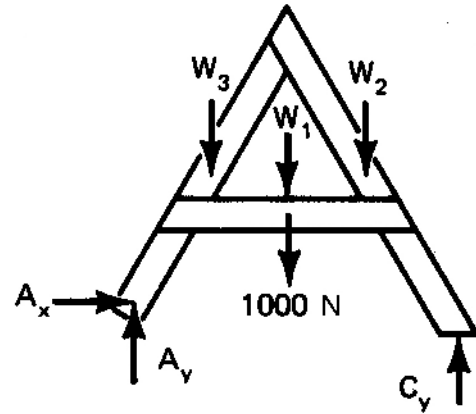
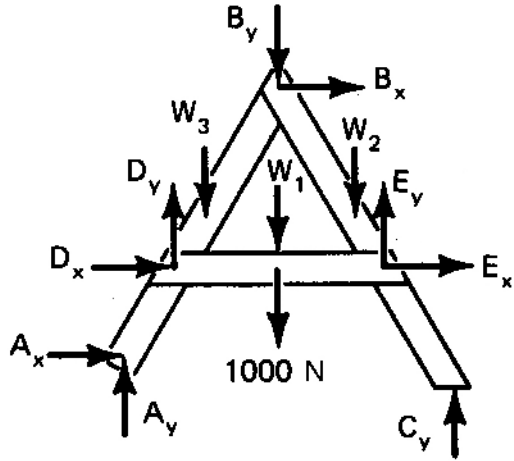


Here is a picture of a support frame.

1. Circle the correct FBD for bar DE.

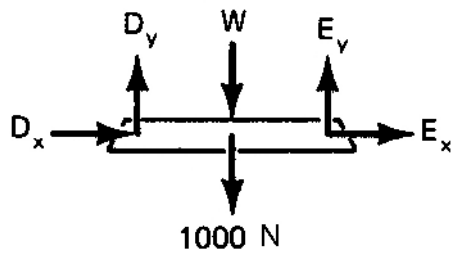


2. Circle the correct FBD for the entire structure.

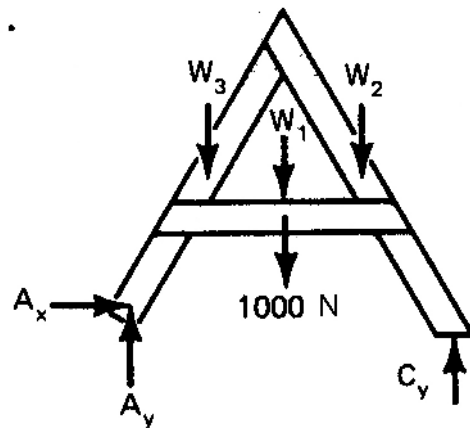


Correct response to preceding frame

1.



2.

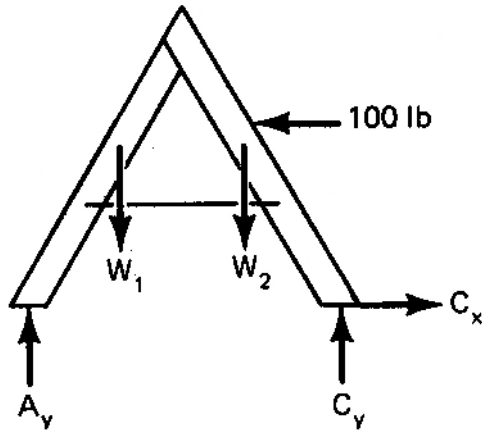


Frame 16-20

**Notebook**

Work Problem 16-2 in your notebook.

Correct response to preceding frame



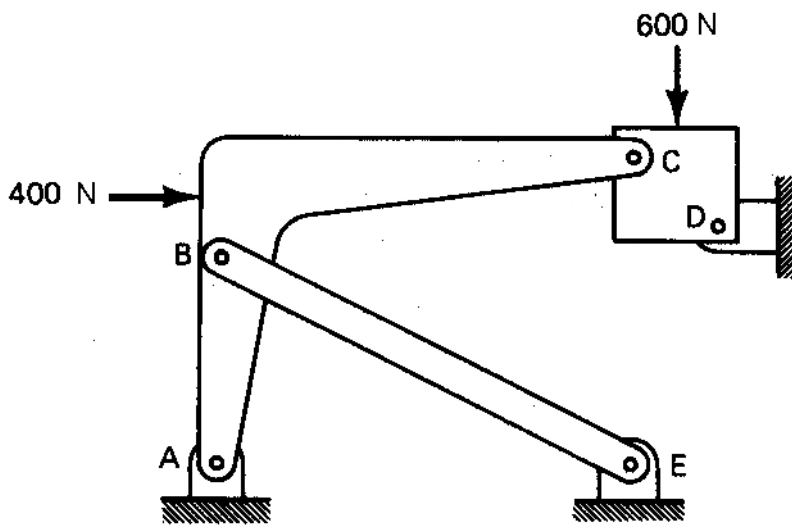
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Frame 16-21

### Complete Free Body Analysis

A complete free-body analysis of a system will include a FBD of the entire system and a FBD of each of its parts.

1. Draw the FBD needed to find the reactions on the system below.



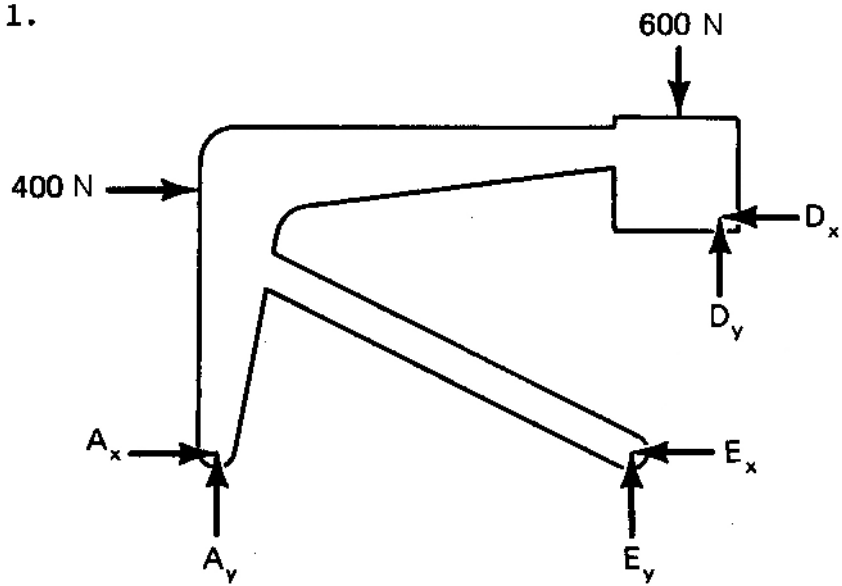
2. How many additional free bodies would you need for a complete free body analysis?

\_\_\_\_\_ additional FBDs



Correct response to preceding frame

1.



2. 3 additional FBDs

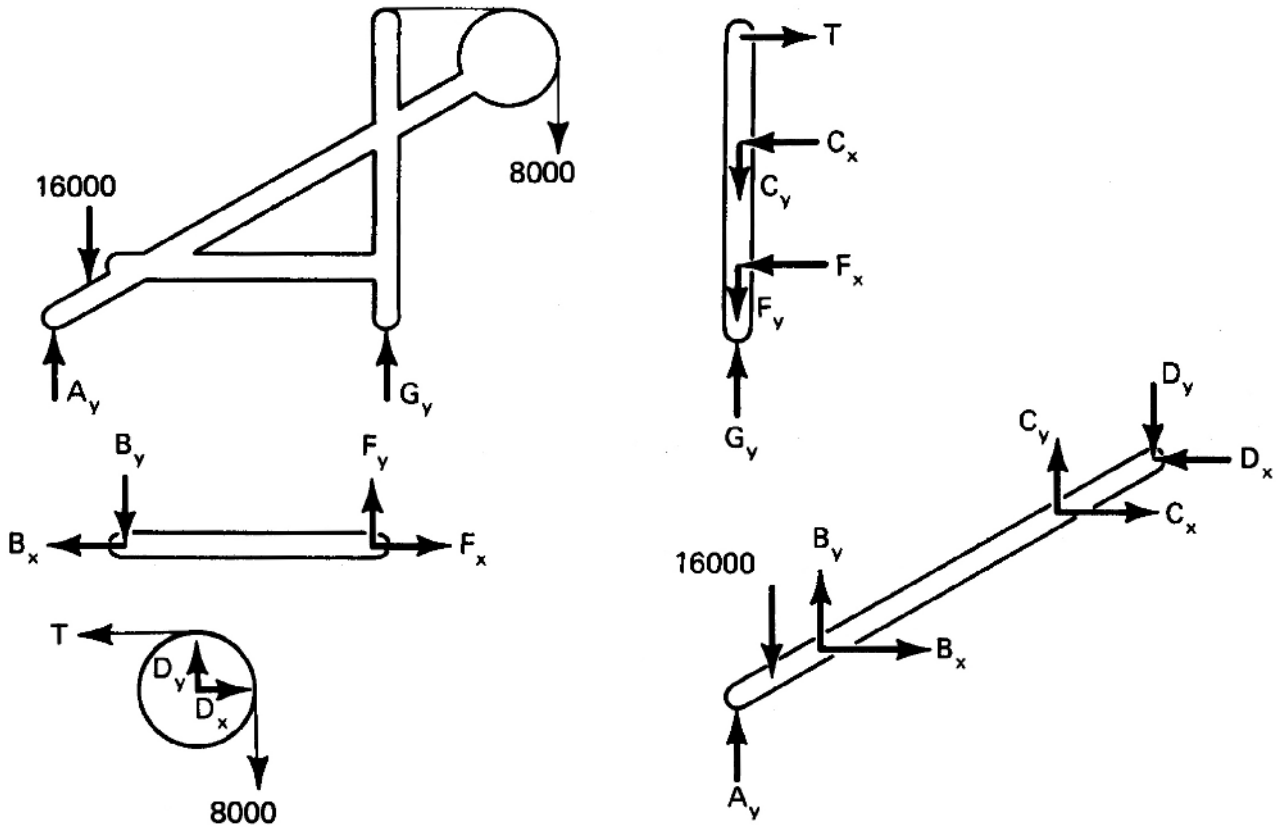
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Frame 16-22

**Notebook**

Work Problem 16-3 in your notebook.

Correct response to preceding frame



Frame 16-23

### Transition

You have already learned how to draw the FBD of a body and of a system. Obviously it is always possible to draw a diagram for every system and for every part of every system. Sometimes, however, it is possible to get by without one or two drawings. The next section will help you deal with the question "how many?".

Flip to the next page.

Correct response to preceding frame

No response

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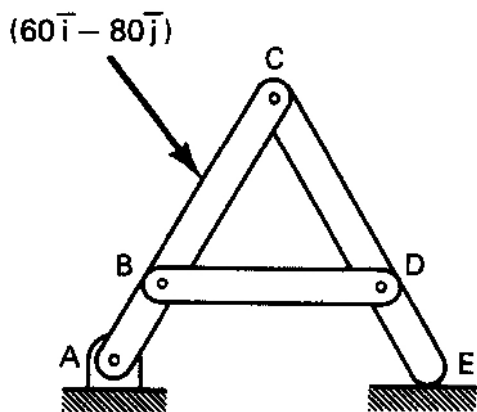
Frame 16-24

### The Magic FBD

Any problem can be solved by drawing FBDs of the parts alone, but usually this approach results in a mess of simultaneous equations.

There is one "extra" FBD which often makes the problem fall apart like magic --the FBD of the system.

Draw a FBD of the system below.



What forces will the system FBD allow you to calculate? \_\_\_\_\_

Correct response to preceding frame

The reactions  $A_x$ ,  $A_y$  and  $E$ .

---

Frame 16-25

### **Strategy**

There is, in fact, a strategy in minimizing the effort of drawing FBDs at the same time that their benefits are maximized.

1. Draw a free body diagram of the whole.
2. Ask, "What can I get from this FBD?" "Is it enough?"
3. If not, draw a FBD of a part which shows the forces you need.
4. Ask "What can I get from this FBD alone?" "Is it enough?"
5. If not, look at the two FBDs and see if, together, they provide simultaneous equations which will solve your problem.
6. If not, repeat steps 3, 4, and 5 until the problem yields to solution.

The system takes a bit of getting used to but the next several frames will help you to practice it.

Go to the next frame.

Correct response to preceding frame

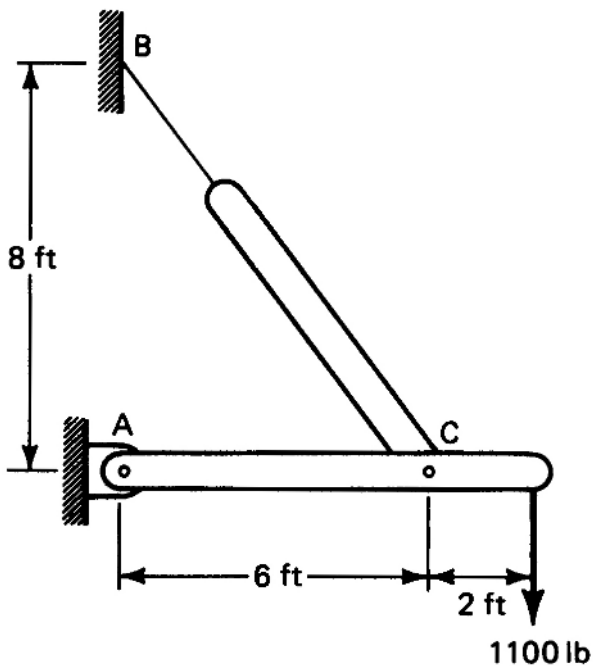
No response

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Frame 16-26

### Necessary and Sufficient FBDs

Draw a system FBD for the frame below.



1. If the problem asked you to find the forces at B does your FBD give you enough for a solution?  Yes  No
2. Could you find the forces at C from your FBD?  Yes  No
3. Could you find the forces at A?  Yes  No

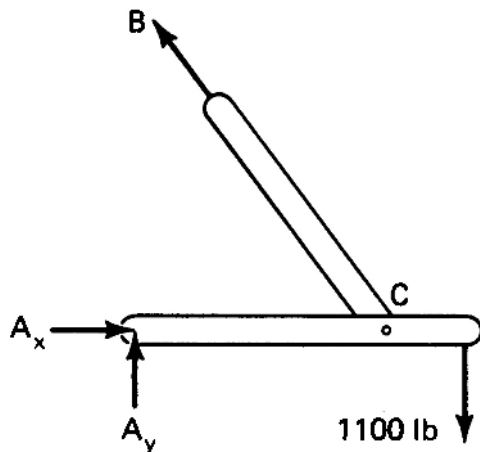
Correct response to preceding frame

1. Yes
2. No
3. Yes

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Frame 16-27

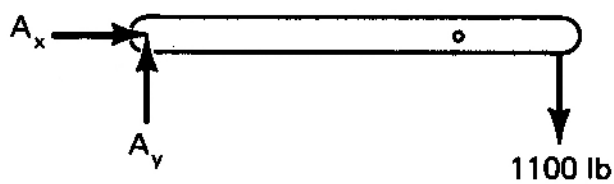
### Necessary and Sufficient FBDs



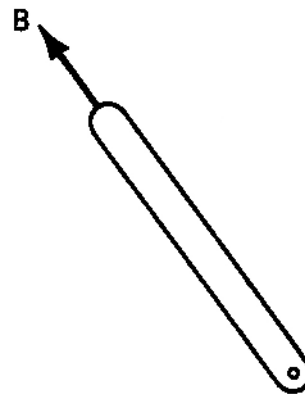
The system FBD gives us some necessary information about the forces but it is not sufficient if we need to find the forces at C.

Complete the FBDs below.

(a)



(b)

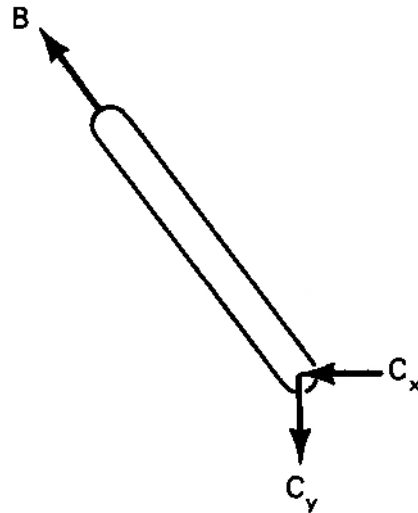
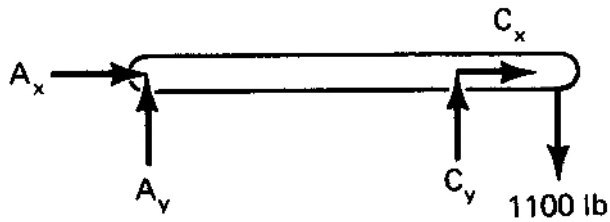


1. Does FBD (a) give you the necessary information to enable you to find the forces at C if you know the reactions?  Yes  No

2. Does FBD (b) give you the necessary information to enable you to find the forces at C if you know the reactions?  Yes  No

3. Do you need both (a) and (b) to find all the forces acting on the system?  Yes  No

Correct response to preceding frame



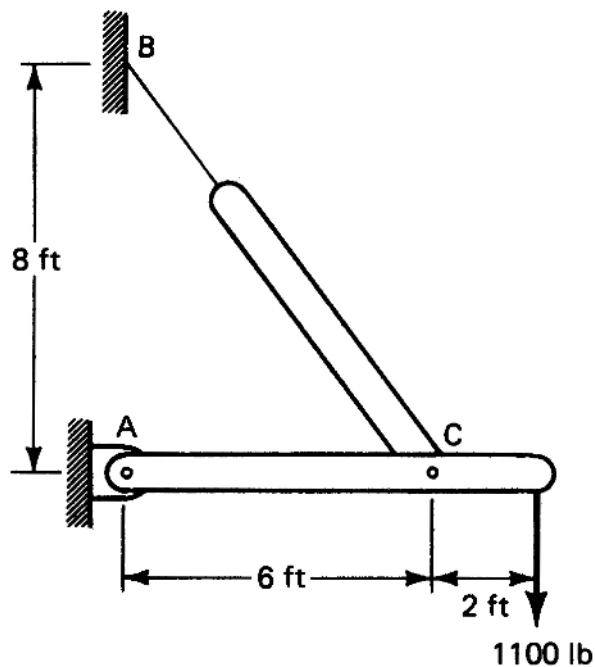
(It's OK if you have used a different sense on the forces at A and C.)

1. Yes
2. Yes
3. No

---

Frame 16-28

### Necessary and Sufficient FBDs



For the system

1. How many parts make up the system? \_\_\_\_\_
2. How many FBDs can you draw? \_\_\_\_\_
3. How many FBDs are necessary if you are to find all the forces? \_\_\_\_\_

Correct response to preceding frame

1. 2 parts
  2. 3 FBDs possible
  3. 2 FBDs needed for all forces
- 

Frame 16-29

### **Necessary and Sufficient FBDs**

If it is necessary to find all of the forces on a structure it is necessary that the number of FBDs be equal to \_\_\_\_\_ .

Which FBD should be drawn first? \_\_\_\_\_



Correct response to preceding frame

Number of FBDs must equal number of parts. (Or equivalent response)  
System FBD should be drawn first.

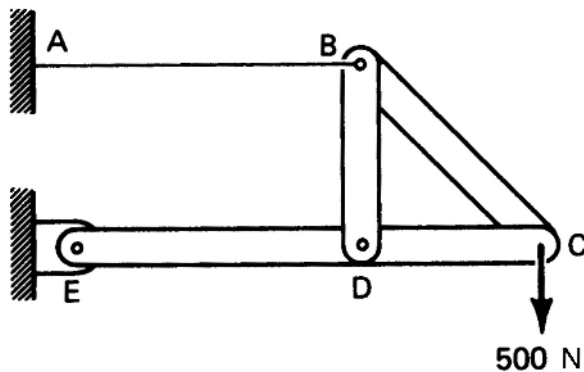
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Frame 16-30

### Necessary and Sufficient FBDs

If you need to find all the unknown forces acting on a system you must draw a number of free bodies equal in number to the parts of the system.

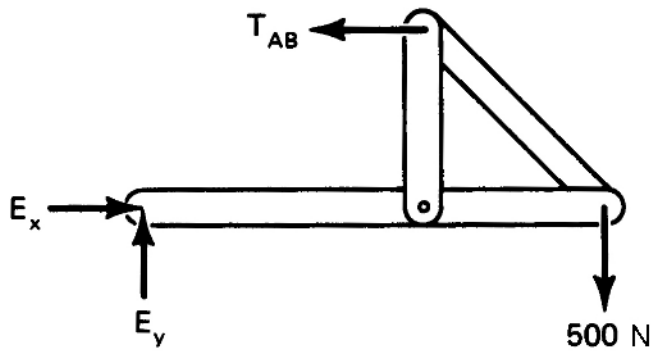
However it is not always necessary to find all of the forces acting on a system. If this is the case it is not necessary to draw so many. Rather, you will find it economical of your time to spend a minute or two in choosing the really applicable FBDs.



How many FBDs must be drawn to find the tension in cable AB?

Correct response to preceding frame

1.



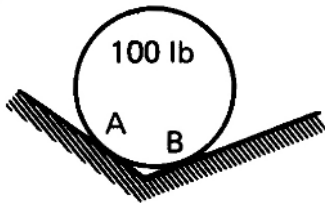
Only one is needed.

Frame 16-31

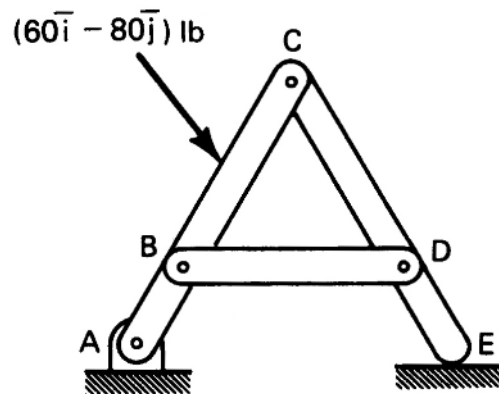
**Necessary and Sufficient FBDs**

If you are asked to find all of the unknown forces at labelled points, how many FBDs must you draw for each structure below? How many can you draw?

1.



2.



Must draw

Can draw

1. \_\_\_\_\_ FBDs      \_\_\_\_\_ FBDs

2. \_\_\_\_\_ FBDs      \_\_\_\_\_ FBDs

Correct response to preceding frame

	Must draw	Can draw
1.	1	1
2.	3	4

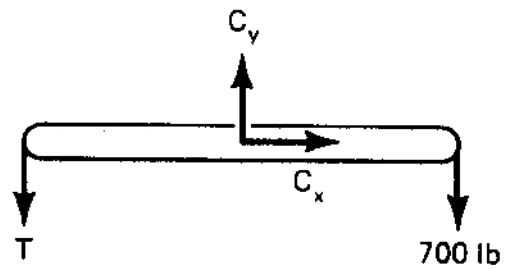
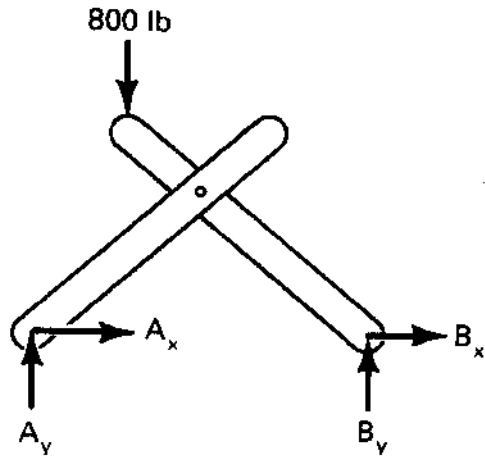
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Frame 16-32

**Notebook**

Complete Page 16-3 in your notebook.

Correct response to preceding frame



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Frame 16-33

### Closure

In this unit you have learned that some teachers can sure make a fuss over a few easy pictures. You may have also learned how to draw FBDs for systems of several members. If you have, and if you are willing to use the skill you have developed, you should do well.

If you have an enemy you would like to force into failing statics, convince him that he doesn't need to learn to draw these stupid figures. It will work almost every time. It's that important.

On the other hand, if you are coaching a friend who needs to improve his statics grade, tell him to focus on drawing FBDs in hopes of getting more partial credit on tests. He'll probably end up getting more correct equations and answers as well.