Introduction to Statics

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Unit 10
Moments of a Force System: Resultant of a Coplanar Force System

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Unit 10
Moments of a Force System: Resultant of a Coplanar Force System

Frame 10-1

Introduction

You have already learned how to find the resultant force on a particle or a body subject to concurrent forces. In this unit you will learn how to find resultants on bodies which have nonconcurrent two dimensional loads on them. In this situation you must learn to compute not only the magnitude and direction of the force but also the location of its line of action.

Go to the next frame.

Correct response to preceding frame

No response

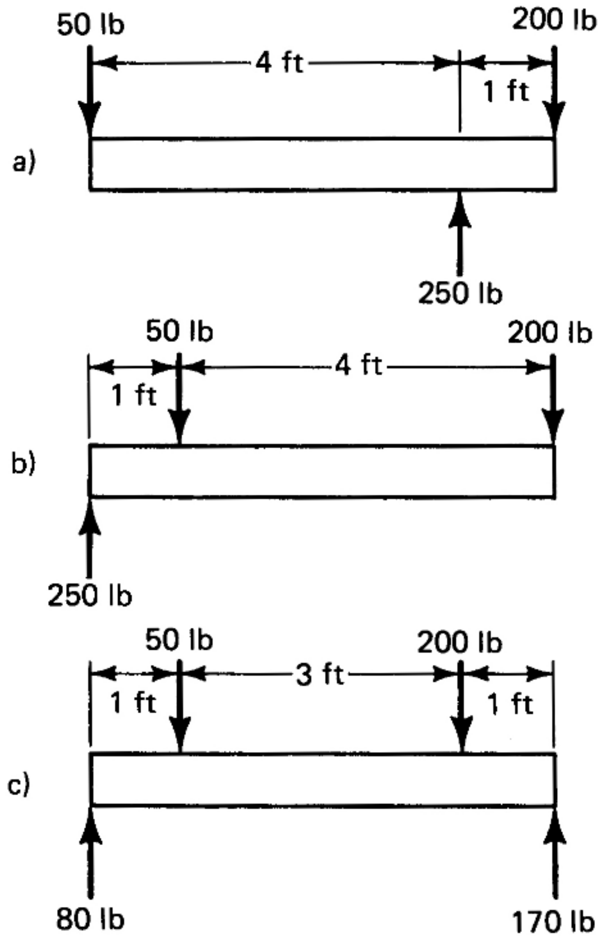
Frame 10-2

Nonconcurrent Force Systems

You already have some understanding of the conditions which determine whether a body subject to nonconcurrent forces is in equilibrium. Look at the following cases and tell in which ones

1. Sum of Forces = 0

2. The system is likely to be in equilibrium


50 lb
200 lb
4 ft
1 ft
a)
250 lb
50 lb
200 lb
1 ft
4 ft
b)
250 lb
50 lb
200 lb
1 ft
3 ft
1 ft
c)
80 lb
170 lb

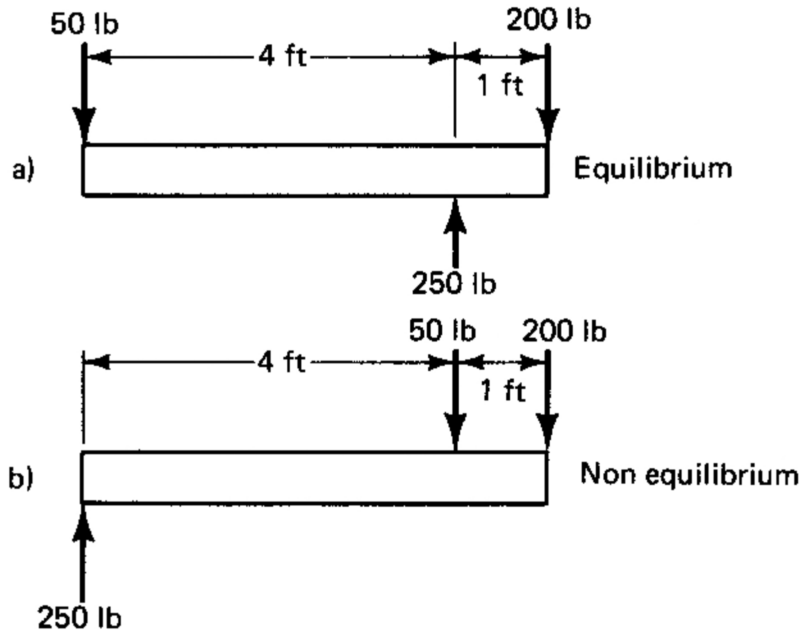
Correct response to preceding frame

1. All 3 cases $\sum \overline{F} = 0$
2. Only cases (a) and (c) are in equilibrium

Frame 10-3

Nonconcurrent Force Systems

In cases (a) and (b) below you will notice that the same loads and distances are involved.



1. Beam (a) ☐ may balance ☐ will tip

2. Beam (b) ☐ may balance ☐ will tip

Correct response to preceding frame

1. may balance
2. will tip

Frame 10-4

Introduction

Your particular job in this unit will be to learn how to place a resultant force in space so that it has the same "balancing effect" as the forces which it replaces.

The first step will be to learn to find the resultant moment of a system of forces.

Turn to the next frame.

Correct response to preceding frame

No response

Frame 10-5

Moment of a Force System

In your study of concurrent forces you learned that the resultant force was the sum of the individual forces which made up a system.

It would be reasonable to expect that the resultant moment of a system of forces is the

______________ of the _______________ of the forces which make up the system.

Since the moment of a force depends on the point about which the moment is taken, all

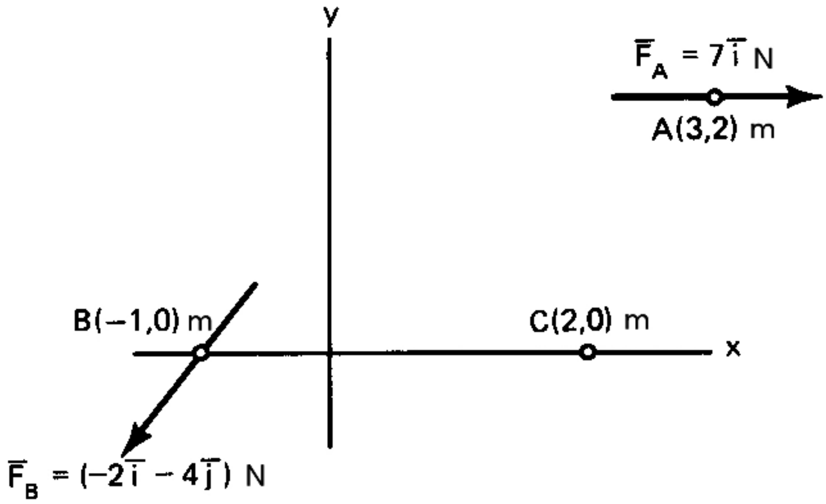
the moments must be taken about the ________________ point.

Correct response to preceding frame

sum of the moments
about the ***same*** point

Frame 10-6

Moment of a Force System



What is the moment of force $\overline{F}_A$ about point C?

$\overline{M}_1$ = ______________________________

What is the moment of force $\overline{F}_B$ about point C?

$\overline{M}_2$ = ______________________________

What is the sum of the moments about point C of the forces which make up the system shown above?

$\overline{M}_C$ = ______________________________

Correct response to preceding frame

$\overline{M}_1 = -14\overline{k}$ ft-lb

$\overline{M}_2 = 12\overline{k}$ ft-lb

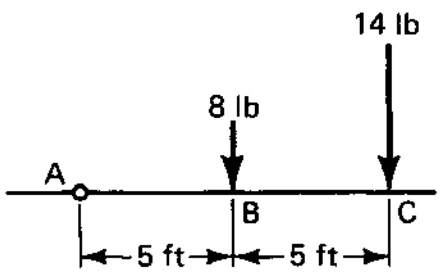
$\overline{M}_C = -2\overline{k}$ ft-lb

Frame 10-7

Moment of a Force System

The resultant moment of a system of forces about a point is equal to the sum of the moments of the forces which make up the system.

What is the resultant moment of the force system shown about point A?



$\overline{M}_A$ = ______________________________

Correct response to preceding frame

$\overline{M}_A = -180\overline{k}$ ft-lb

Solution:

The moment due to B is

$$\overline{M}_1 = (5\overline{i} \text{ ft}) \times (-8\overline{j} \text{ lb}) = -40\overline{k} \text{ ft-lb}$$

The moment due to C is

$$\overline{M}_2 = (10\overline{i} \text{ ft}) \times (-14\overline{j} \text{ lb}) = -140\overline{k} \text{ ft-lb}$$

Hence the sum

$$\overline{M}_A = -180\overline{k} \text{ ft-lb}$$

Frame 10-8

Moment of a Force System

The moment of a system of forces results from taking the (***algebraic, scalar, vector***) sum of the moments of the individual forces. (circle one)

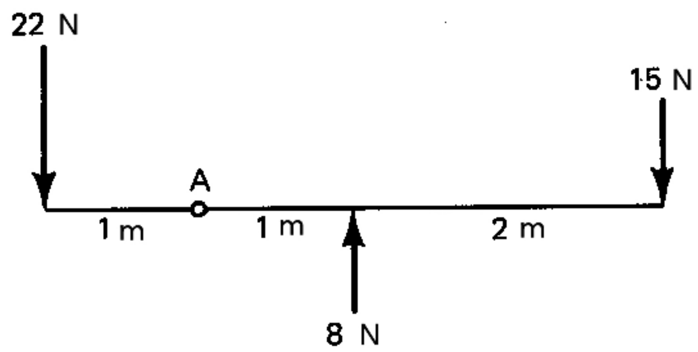
Correct response to preceding frame

The result is a ***vector*** sum, because moments are vector quantities.

Frame 10-9

Moment of a Force System

What is the moment of this system of forces about point A?



Correct response to preceding frame

$\overline{M}_A = -15\overline{k}$ N-m

<u>Solution</u>:

$$\overline{M}_A = (-1\overline{i}) \times (-22\overline{j}) + (1\overline{i}) \times (8\overline{j}) + (3\overline{i}) \times (-15\overline{j})$$

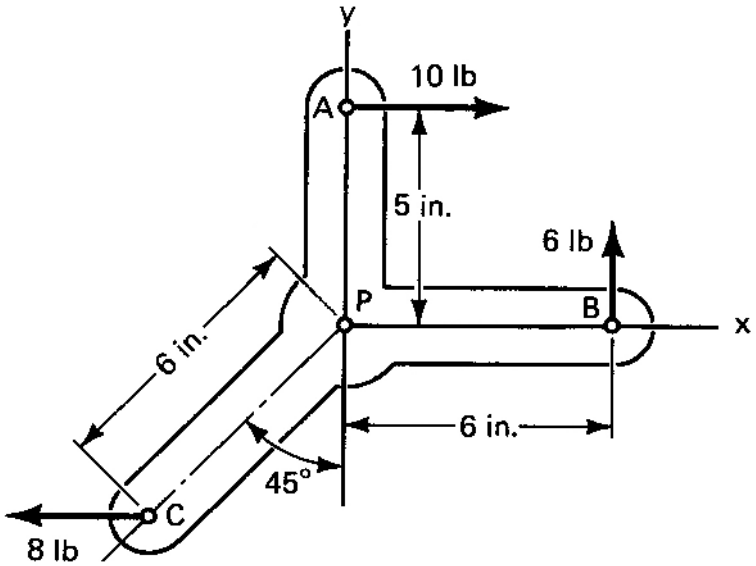
$$= (22 + 8 - 45)\overline{k}$$

Frame 10-10

Moment of a Force System

Of course, the forces may not all be in the same direction. Practice your skill on this one.

A "Bell Crank" (in the x-y plane) is subjected to this system of loads. What is the net moment about the pivot P ?



Correct response to preceding frame

$\overline{M}_p = -48\overline{k}$ in-lb

Solution:

$\overline{M}_1 = -50\overline{k}$

$\overline{M}_2 = 36\overline{k}$

$\overline{M}_3 = -\frac{48}{\sqrt{2}}\overline{k}$

$\overline{M}_p = \overline{M}_1 + \overline{M}_2 + \overline{M}_3$

Frame 10-11

Moment of a Force System

We can also find the moment of three dimensional force systems.

If force $\overline{F}_1 = 3\overline{i} + 4\overline{j} + \overline{k}$ acts through the point (0,3,3) and force $\overline{F}_2 = -\overline{i} + 3\overline{j} + 2\overline{k}$ acts through (-2,-1,-1) what is the resultant moment of the force system about the origin?

$\overline{M}_T =$ ________________________________

Correct response to preceding frame

$\overline{M}_T = -8\overline{i} + 14\overline{j} - 16\overline{k}$

<u>Solution</u>:

$$\overline{M}_1 = \overline{r}_1 \times \overline{F}_1$$

$$= (3\overline{j} + 3\overline{k}) \times (3\overline{i} + 4\overline{j} + \overline{k})$$

$$= -9\overline{i} + 9\overline{j} - 9\overline{k}$$

$$\overline{M}_2 = \overline{r}_2 \times \overline{F}_2$$

$$= (-2\overline{i} - \overline{j} - \overline{k}) \times (-\overline{i} + 3\overline{j} + 2\overline{k})$$

$$= 1\overline{i} + 5\overline{j} - 7\overline{k}$$

$$\overline{M}_T = \overline{M}_1 + \overline{M}_2$$

Frame 10-12

Transition

You've passed the first mile post (or kilometer marker) in this unit. Next you'll use the resultant moment of a force system to locate a single force which has an effect which is equivalent to the force system.

Forward!!

Correct response to preceding frame

No response

Frame 10-13

Restriction

The general case of a three dimensional force system cannot be reduced to a single force so we shall postpone our study of three dimensional systems for a few more units.

This unit will be restricted to consideration of ________________________________

____

Correct response to preceding frame

one and two dimensional systems of forces (Or equivalent response)

Frame 10-14

Review

A couple of units back you learned to find the resultant of a system of concurrent forces.

In that case you were replacing several forces with ______________________________

____

Correct response to preceding frame

one force which had the same effect (Or equivalent response)

Frame 10-15

Review

The magnitude and direction of resultant $\overline{R}$ of the system of forces $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ may be found by the equation

$$\overline{R} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Correct response to preceding frame

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 \qquad \text{or} \qquad \bar{R} = \sum_{n=1}^{3} \bar{F}_n$$

Frame 10-16

Resultant of a Force System

If we wish to replace a system of nonconcurrent forces with a single resultant we must satisfy the conditions which apply to concurrent forces so that the resultant force,

$\bar{R}$ = ______________________________

In addition, if it is to be really equivalent to our original system it must have the same

________________________ with respect to any selected point as our system of forces.

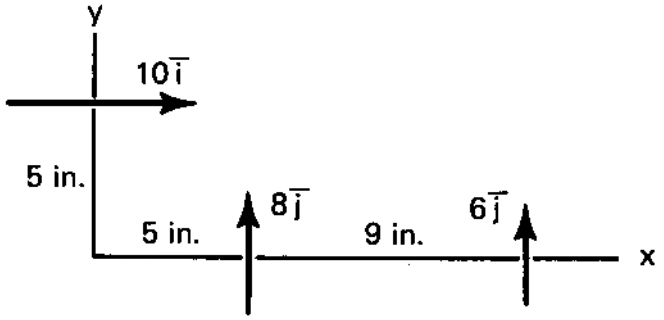
Correct response to preceding frame

$\sum \bar{F}$
resultant moment

Frame 10-17

Resultant of a Force System

Given this system of forces



What will the resultant force be?

$\bar{R}$ = ________________________________

What must the moment of $\bar{R}$ about the origin be?

$\bar{M}_0$ = ________________________________

Correct response to preceding frame

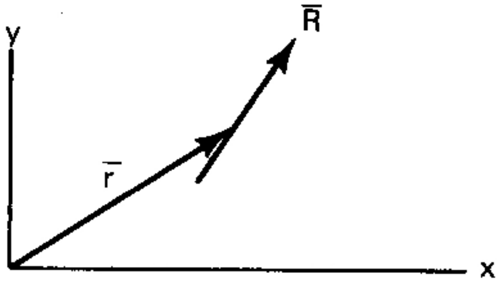
$\overline{R} = \sum\overline{F} = 10\overline{i} + 14\overline{j}$ lb

$= 17.2\,(.582\overline{i} + .814\overline{j})$

$\overline{M}_0 = 74\overline{k}$ in-lb

Frame 10-18

Resultant of a Force System



The resultant of the system which we are analyzing will be a force

$\overline{R} = (10\overline{i} + 14\overline{j})$. Since the force must have a moment $\overline{M}_0 = 74\overline{k}$ about the origin we must find a location for it such that $\overline{M}_0 = \overline{r} \times \overline{R}$.

One way that this can be done is to use the general expression for the arm $\overline{r} = x\overline{i} + y\overline{j}$.

Then

$$\overline{M}_O = \overline{r} \times \overline{R} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ x & y & 0 \\ 10 & 14 & 0 \end{vmatrix} = 74\overline{k}$$

Work out the product and find an equation for y.

$y =$ ____________________

Correct response to preceding frame

$y = 1.4x - 7.4$

Solution:

$\bar{M}_o = \bar{r} \times \bar{R}$

$74\bar{k} = (x\bar{i} + y\bar{j}) \times (10\bar{i} + 14\bar{j})$

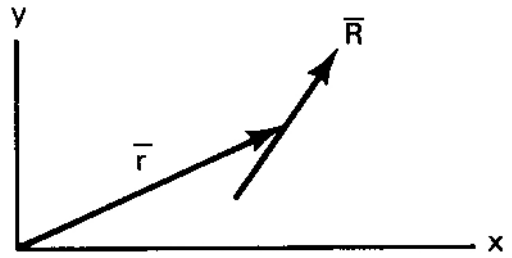
$= (14x - 10y)\bar{k}$

Hence

$10y = 14x - 74$

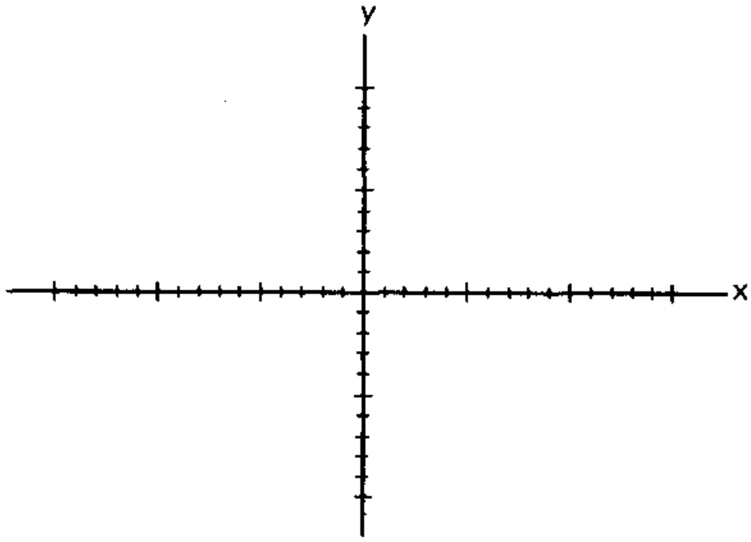
Frame 10-19

Resultant of a Force System



In the preceding frame we used a general expression $\bar{r} = x\bar{i} + y\bar{j}$ in the moment equation and ended up with $y = 1.4x - 7.4$.

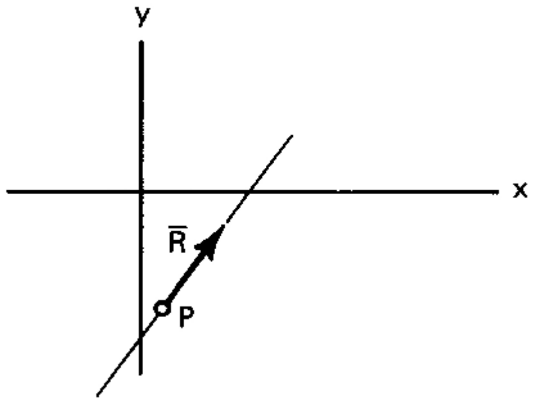
Sketch the line $\bar{r} = x\bar{i} + y\bar{j}$ on the coordinate system below.



Choose some point P on the line and sketch $\overline{R} = 10\overline{i} + 14\overline{j}$ through this point.

The line $\overline{r} = x\overline{i} + y\overline{j}$ is the ________________________________ of the resultant, $\overline{R} = 10\overline{i} + 14\overline{j}$

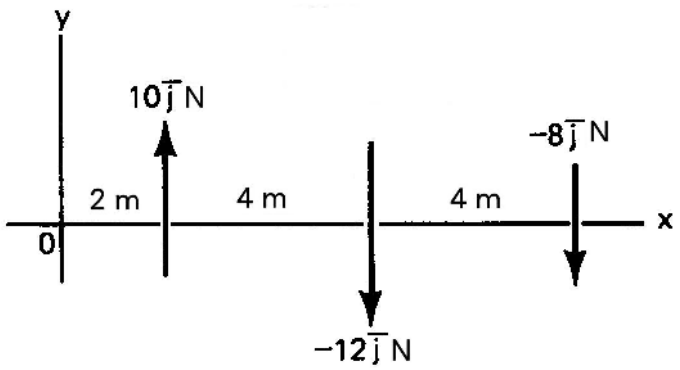
Correct response to preceding frame



line of action

Frame 10-20

Resultant of a Force System



Find $\bar{R}$ = ______________________________

and $\bar{M}_0$ = ______________________________

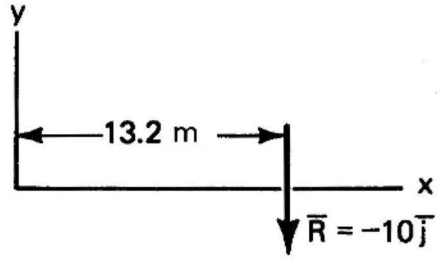
Now use $\bar{r} = x\bar{i} + y\bar{j}$ and $\bar{M}_0 = \bar{r} \times \bar{R}$ to find the line of action of $\bar{R}$. Sketch $\bar{R}$, showing its correct line of action on the drawing.

Correct response to preceding frame

$\bar{R} = -10\bar{j}$ N

$\bar{M}_o = -132\bar{k}$ N-m

x = 13.2 m



Solution:

$\bar{M}_o = \bar{r} \times \bar{R}$

$-132\bar{k} = (x\bar{i} + y\bar{j}) \times (-10\bar{j})$

$-132 = -10x$

Frame 10-21

Resultant of a Force System

The method which we have used to locate a resultant force does obviously give us the same moment about the origin. This does not completely satisfy our original requirement.

Why? ____

____

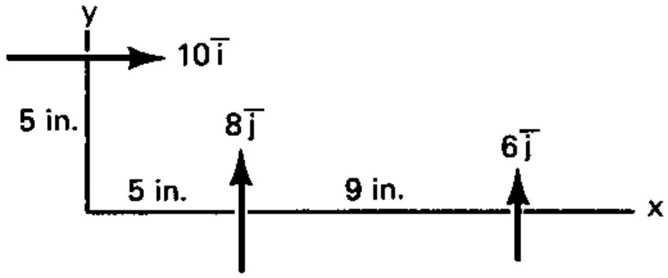
Correct response to preceding frame

Our original requirement was that it has the same moment about ANY point as the force system which it replaces. (Or equivalent response)

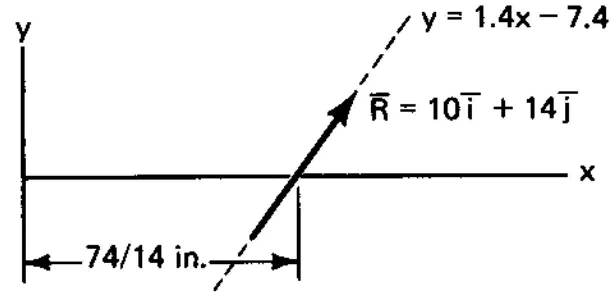
Frame 10-22

Resultant of a Force System

Let's take the first example and its answer and prove that it does in fact satisfy our conditions. We started with:

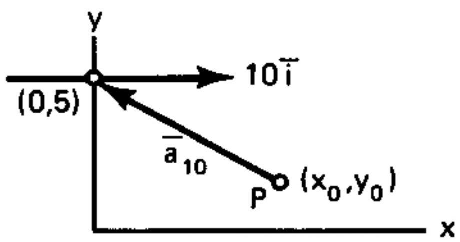


and found:



Let's consider the forces to be applied at the point where their lines of action cross the x or y axis and take moments about an arbitrary point with coordinates (x_0, y_0).

Then for the 10 lb force



$\bar{a}_{10} = (0 - x_0)\bar{i} + (5 - y_0)\bar{j}$

$\bar{M}_1 = \bar{a}_{10} \times 10\bar{i} = [(0 - x_0)\bar{i} + (5 - y_0)\bar{j}] \times 10\bar{i} = (-50 + 10y_0)\bar{k}$

Following this example find

$\bar{a}_8$ = ______________________ $\bar{M}_2$ = ______________________

$\bar{a}_6$ = ______________________ $\bar{M}_3$ = ______________________

Correct response to preceding frame

$\bar{a}_8 = (5 - x_0)\bar{i} + (0 - y_0)\bar{j}$
$\bar{M}_2 = (40 - 8x_0)\bar{k}$
$\bar{a}_6 = (14 - x_0)\bar{i} + (0 - y_0)\bar{j}$
$\bar{M}_3 = (84 - 6x_0)\bar{k}$

Frame 10-23

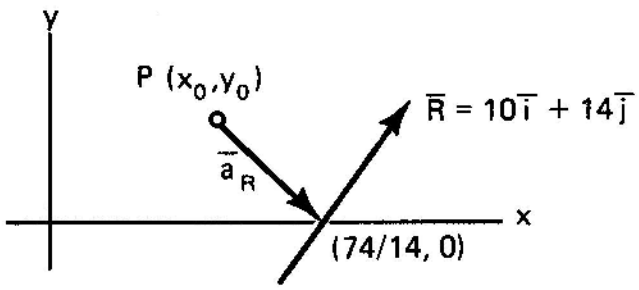
Resultant of a Force System

The total moment of this system about P is

$$\bar{M}_T = \bar{M}_{10} + \bar{M}_8 + \bar{M}_6 = (74 - 14\,x_0 + 10\,y_0)\bar{k}$$

The resultant $\bar{R}$ will have a position vector with respect to the point with coordinates (x_0, y_0)

$$\bar{a}_R = \left[\frac{74}{14} - x_0\right]\bar{i} + \left[0 - y_0\right]\bar{j}$$



Calculate the moment $\bar{M}_R = \bar{a}_R \times \bar{R} =$ ____________________

Is $\bar{M}_T = \bar{M}_R$? ☐ Yes ☐ No

What does this show? ____________________

____

Correct response to preceding frame

$\overline{M}_R = (74 - 14\ x_0 + 10\ y_0)\overline{k}$

Yes

This demonstrates that a resultant force located by this method has the same moment about any point as the resultant moment of the force system it replaces. (Or equivalent response)

Frame 10-24

Transition

You've seen how to find and locate the resultant of a force system. The next section will give you some more practice with this extremely important procedure.

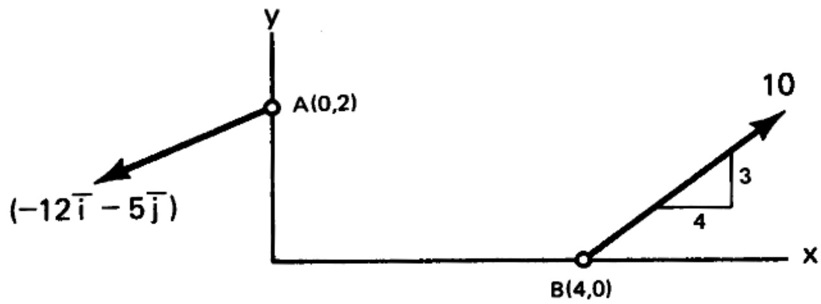
Ready? Turn the page.

Correct response to preceding frame

No response

Frame 10-25

Resultant of a Force System



Find and locate $\overline{R}$.

Correct response to preceding frame

$\overline{R} = -4\overline{i} + \overline{j}$

along line

$y = -\frac{1}{4}x + 12$

Solution:

$\overline{R} = (-12\overline{i} - 5\overline{j}) + (8\overline{i} + 6\overline{j}) = -4\overline{i} + \overline{j}$

$\overline{M}_O = 4\overline{i} \times (8\overline{i} + 6\overline{j}) + 2\overline{j} \times (-12\overline{i} - 5\overline{j}) = 48\overline{k}$

$(x\overline{i} + y\overline{j}) \times (-4\overline{i} + \overline{j}) = 48\overline{k}$

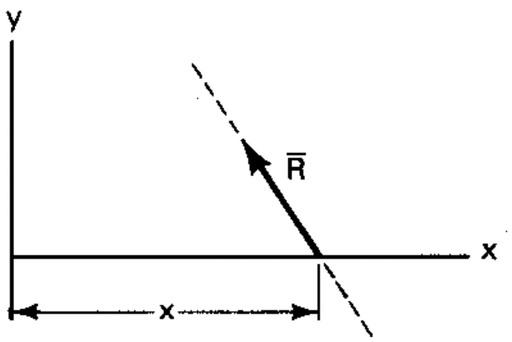
$x\overline{k} + 4y\overline{k} = 48\overline{k}$

Frame 10-26

Simplification of $\overline{a}$

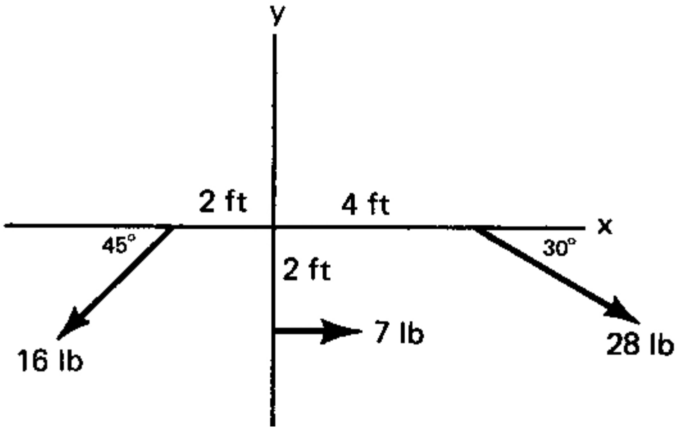
In two dimensional problems we know that the line of action of the resultant of a force system will cross at least one (and generally both) of the axes. Therefore, if we allow ourselves to locate it at one of these intersections we need to find only one component of $\overline{a}$, setting the other to zero.

For example:



$\overline{a} = x\overline{i} + 0\overline{j} = x\overline{i}$

Use this kink to help locate the resultant of this force system.


y
2 ft
4 ft
x
45°
30°
2 ft
7 lb
16 lb
28 lb

Correct response to preceding frame

$\overline{R} = 20\overline{i} - 25.3\overline{j}$

through $\begin{cases} x = .77 \text{ ft} \\ y = 0 \end{cases}$ or $\begin{cases} x = 0 \\ y = .97 \text{ ft} \end{cases}$

Frame 10-27

Problem

Find the resultant of the force system and locate its line of action.

Force	Point of Application
$\overline{A} = 3\overline{i} - \overline{j}$ lb	(3,3) ft
$\overline{B} = 3\overline{j} - \overline{i}$	(0,2)
$\overline{C} = -4\overline{i} - 4\overline{j}$	(-2,0)

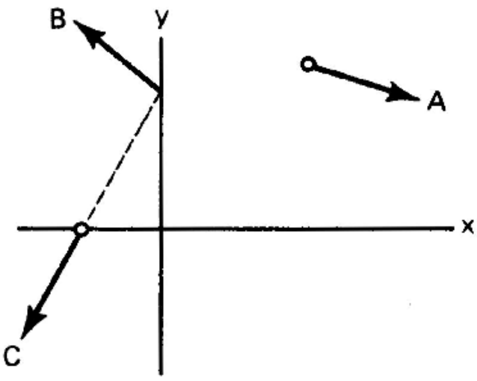
Correct response to preceding frame

$\bar{R} = -2\bar{i} - 2\bar{j}$ lb

through point (1,0)

(or along $y = x - 1$)

Solution:



$$\bar{R} = \bar{A} + \bar{B} + \bar{C}$$

$$= -2\bar{i} - 2\bar{j}$$

$$\bar{M}_o = (3\bar{i} + 3\bar{j}) \times (3\bar{i} - \bar{j}) + 2\bar{j} \times (-\bar{i} + 3\bar{j})$$

$$- 2\bar{i} \times (-4\bar{i} - 4\bar{j}) = -2\bar{k} \text{ ft-lb}$$

$$\bar{M}_o = \bar{r} \times \bar{R} = (x\bar{i} + \bar{0}\bar{j}) \times (-2\bar{i} - 2\bar{j})$$

$$-2\bar{k} = -2x\bar{k}$$

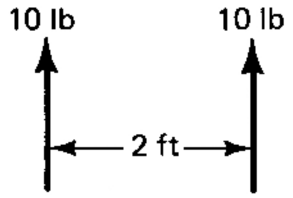
$$x = 1$$

Frame 10-28

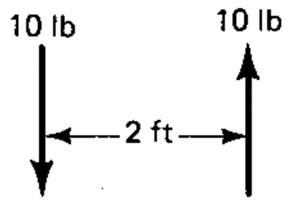
Restriction

Perhaps you've noticed that there is a special case of a two dimensional force system in which this method will not work. That is the case when R = 0 but the moment is not zero. This special case will be considered in a later unit.

Can you find the resultant of the following force systems?

1) 

☐ Yes ☐ Not now

2) 

☐ Yes ☐ Not now

Correct response to preceding frame

1. Yes
2. Not now

Frame 10-29

Review

Complete Page 10-1 in your notebook and work the problem on Page 10-2.

Correct response to preceding frame

$\overline{M}_0 = 16\overline{k}$

$\overline{M}_P = 40\overline{k}$

$\overline{R} = 9\overline{i} + 3\overline{j}$ with line of action through point [15.3, 0]

Frame 10-30

Transition

Up to this point in the unit we have been working with force systems which were expressed as vectors in a given coordinate system.

In a real life situation you would have the opportunity to select your own way of describing the system and you would have to locate your resultant at a point which had a physical meaning, such as "three feet to the right of the front edge."

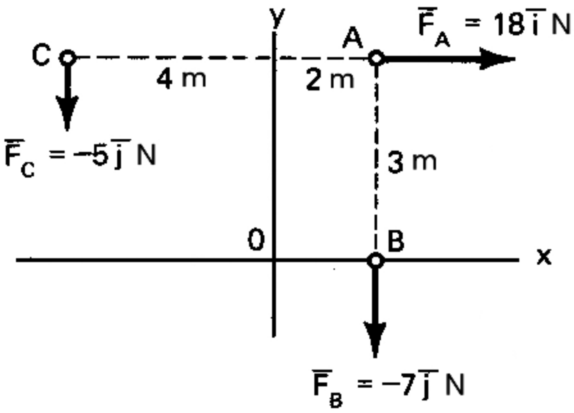
The last section of the unit will help you to see how the choice of a moment center can affect your calculations and will give you some practice at designating locations in physical terms.

Correct response to preceding frame

No response

Frame 10-31

Simplifying Moment Calculations



If you take moments about A how many of the products are not zero? ________

Calculate $\bar{M}_A$.

$\bar{M}_A$ = ____

If you take moments about the origin how many products are not zero? ________

Calculate $\bar{M}_0$.

$\bar{M}_0$ = ____

Which is easier to find? ☐ $\bar{M}_A$ ☐ $\bar{M}_0$

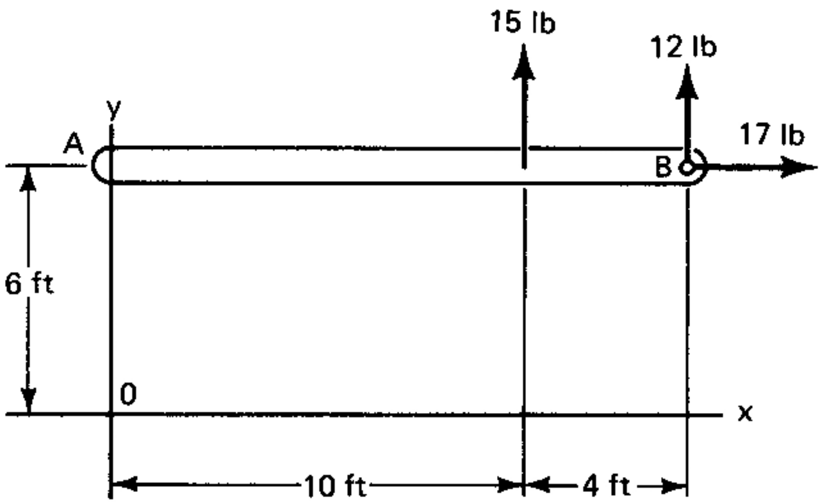
Correct response to preceding frame

One
$\overline{M}_A = 30\overline{k}$ ft-lb
Three
$\overline{M}_0 = -48\overline{k}$ ft-lb
I think $\overline{M}_A$ is the easier to find.

Frame 10-32

Simplifying Moment Calculations

Which point gives the easiest moment calculation?



A ☐ B ☐ 0 ☐

Calculate the moment of the force system about that point.

$\overline{M}_{(Your\ Choice)}$ = ______________________

Correct response to preceding frame

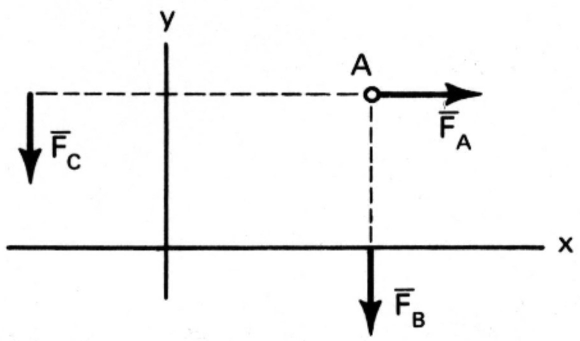
$\underline{B}$

$\overline{M}_B = -60\overline{k}$ ft-lb

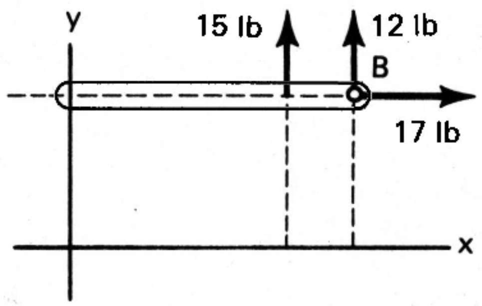
Frame 10-33

Simplifying Moment Calculations

In the two cases which you just examined,



and



the points which give the easiest calculations are points which lie on the ___________ of two forces.

Correct response to preceding frame

the ***intersection*** of two forces (Or equivalent response)

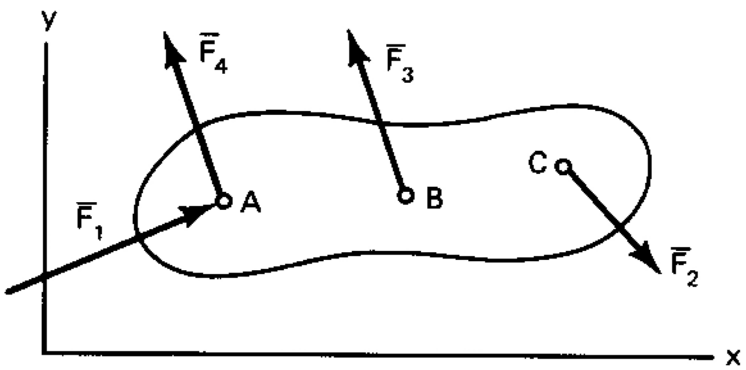
Frame 10-34

Choosing Moment Centers

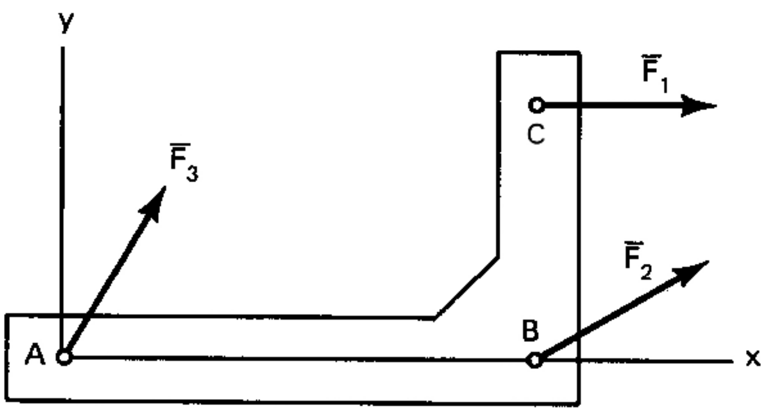
Sometimes it is difficult to find the intersection of the lines of action of two forces. Just choosing your moment center so that one force passes through it is usually helpful.

Select and identify a moment center on the sketches below:

1)



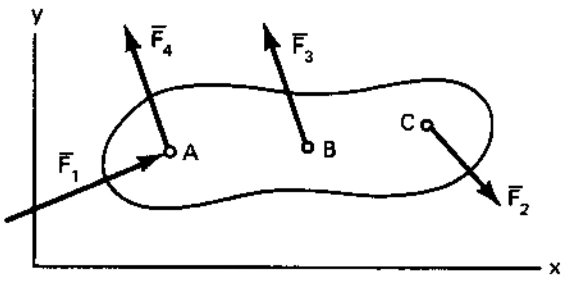
2)



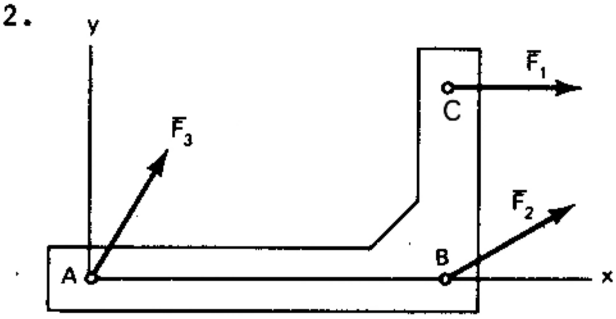
Correct response to preceding frame

The point marked "A" is my first choice. "B" and "C" my alternates.

1.



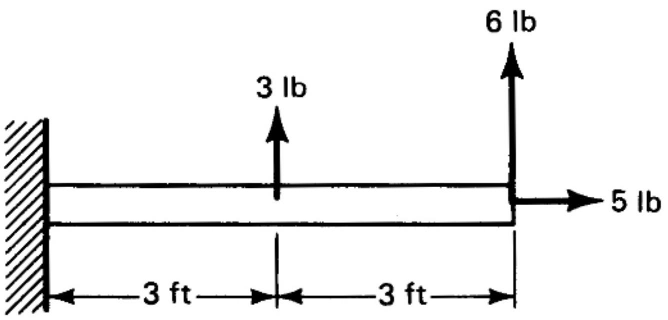
2.



Frame 10-35

Locating Resultants

Problem:
A bar 3 inches square and six feet long is subjected to the loads shown.
What is the resultant force on the bar and where does it act?



If you were on an engineering job and were asked to solve a problem somewhat like the one above, which of the answers below do you think the boss would like you to give?

(a) $\bar{R} = 5\bar{i} + 9\bar{j}$ and $y = (9/5) + 5$
(b) $\bar{R} = 5\bar{i} + 9\bar{j}$ through x = 10, y = 9
(c) The force has components of 5 lb to the right and 9 lb upward and it acts at a point 5 ft out from the wall.

Correct response to preceding frame

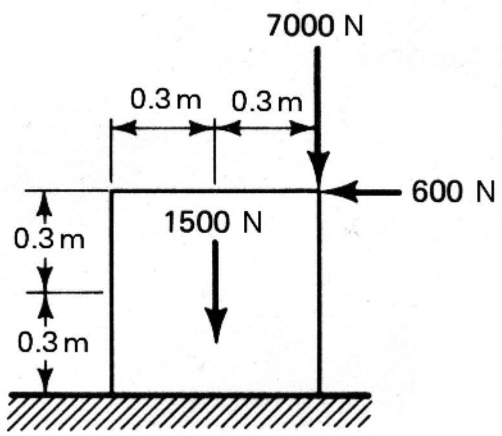
It's a good bet that if you give him answer (c) he'll pat you on the back and call you a realist.

Frame 10-36

Locating Resultants

Designers of foundations are often concerned with the point at which the line of action of the resultant load cuts the plane of contact between the foundation and the ground.

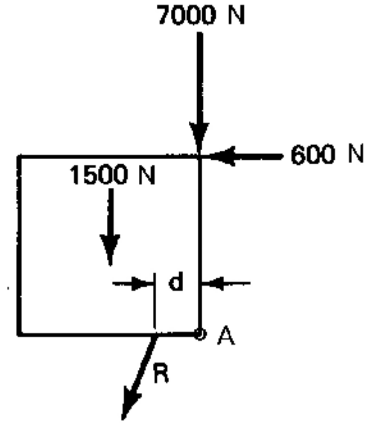
How far from the lower right hand corner does the line of action of the resultant cut the contact plane?



Correct response to preceding frame

d = 0.0953 meters to the left of the lower right corner.

Solution:



$\overline{R} = -600\overline{i} - 1500\overline{j} - 7000\overline{j}$

$= -600\overline{i} - 8500\overline{j}$

$\overline{M}_A = (-0.30\overline{i} \times -1500\overline{j}) + (0.6\overline{j} \times -600\overline{i})$

$\overline{M}_A = 810\overline{k}$

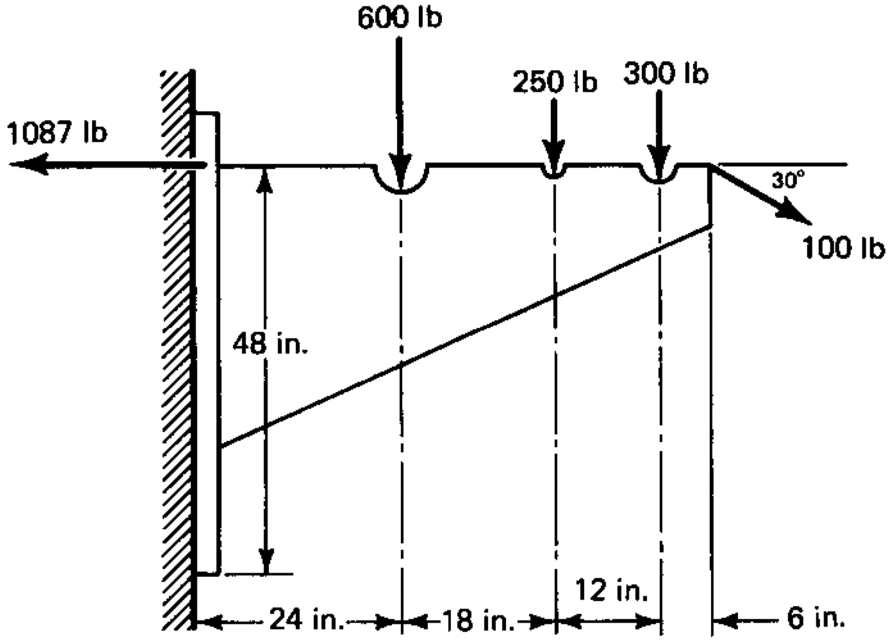
$\overline{a} = -d\overline{i}$

$810 = 8500\ d$

Frame 10-37

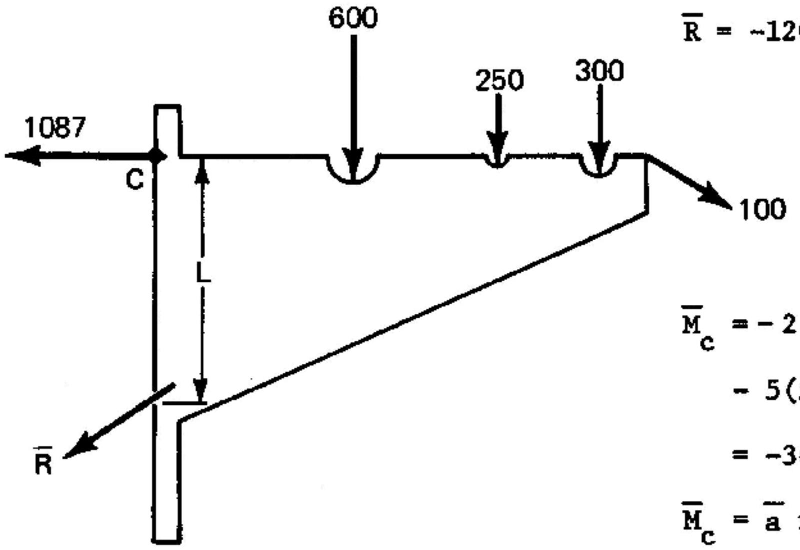
Locating Resultants

Find the resultant of the forces on the pipeline support and the point at which its line of action intersects the wall.



Correct response to preceding frame

$\bar{R}$ has a downward component of 1200 lb and a component of 1000 lb to the left. It acts at a point 3.68 ft below the horizontal load.



<u>Solution:</u>

$$\bar{R} = -1087\bar{i} - 600\bar{j} - 250\bar{j} - 300\bar{j} + 87\bar{i} - 50\bar{j}$$

$$\bar{R} = -1200\bar{j} - 1000\bar{i}$$

$$\bar{M}_c = -2(600)\bar{k} - \frac{7}{2}(250)\bar{k} - \frac{9}{2}(300)\bar{k} - 5(50)\bar{k}$$

$$= -3675\bar{k} \text{ ft-lb}$$

$$\bar{M}_c = \bar{a} \times \bar{R} = -L\bar{j} \times (-1200\bar{j} - 1000\bar{i})$$

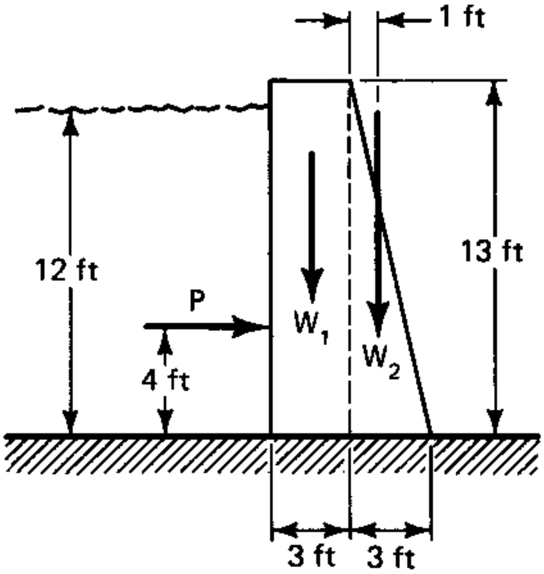
$$= -1000\, L\bar{k} \text{ lb}$$

$$L = 3.675 \text{ ft}$$

Frame 10-38

Locating Resultants

Find the point at which the line of action of the resultant force on the concrete dam intersects the base of the dam.



W_1 = 5800 lb and acts through middle of rectangular area
W_2 = 2900 lb and acts 2 ft to left of lower right corner
P = 4500 lb
(loads are on a one foot section of the dam)

Correct response to preceding frame

On the bottom, 4 ft 5 in. to the right of the left hand face

Frame 10-39

Closure

That’s it!

In this unit you have learned how to find the moment of a force system about various points, to use vector algebra to reduce most two dimensional force systems to a single force, and to describe the line of action of that force.

This mathematical approach to forces can be used to answer certain questions about the stability of structures. Its elegance can also be used to astonish dates and small cousins.