Introduction to Statics

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Unit 9
Moment About a Line

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Unit 9
Moment About a Line

Frame 9-1

***Introduction**

Often when we compute the moment of a vector about a point we are particularly interested in the portion of the moment which is related to a particular line or axis.

For most two dimensional problems, and many three dimensional ones, the line is a coordinate axis, and the procedure used in the preceding unit is all that we need.

In some situations, however, it is desirable to compute the moment of a vector about a line which is not parallel to a coordinate axis. This unit will show you how this may be done with vector multiplication.

Go to the next frame.

*This topic is sometimes excluded from a short statics course. Check your schedule to see if your instructor requires you to study it at this time.

Correct response to preceding frame

No response

Frame 9-2

Moment About a Line

Read the first part of Page 9-1 in the notebook. You may continue to refer to your notebook while studying the next two frames.

P is any point on the line. Which term (or terms) in the vector equation represents the moment of $\overline{V}$ about P?

$\overline{M}_P$ = ________________________________

Correct response to preceding frame

$\overline{M} = \overline{a} \times \overline{V}$

Frame 9-3

Moment About a Line

Continue to refer to the notebook if necessary.

$(\overline{a} \times \overline{V}) \cdot \overline{e}$ represents: (check one answer)

- ☐ the component of $\overline{V}$ parallel to the line
- ☐ the component of $\overline{M}_p$ parallel to the line
- ☐ the magnitude of the component of $\overline{M}_p$ parallel to the line

Correct response to preceding frame

the magnitude of the component of $\overline{M}_p$ parallel to the line

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Frame 9-4

Moment About a Line

The term $[(\overline{a} \times \overline{V}) \cdot \overline{e}]$ is the magnitude of the component of $\overline{M}_p$ parallel to the line.

What is the direction of the component? ____

The full expression is $\overline{M}_L = [(\overline{a} \times \overline{V}) \cdot \overline{e}]\ \overline{e}$. What does the final $\overline{e}$ do?

____

CLOSE YOUR NOTEBOOK.

Correct response to preceding frame

along the line
$\bar{e}$ gives it direction

Frame 9-5

Moment About a Line

In the expression

$$\bar{M}_L = [(\bar{a} \times \bar{V}) \cdot \bar{e}]\ \bar{e}$$

1. $\bar{M}_L$ is a (***scalar, vector***).

2. $(\bar{a} \times \bar{V})$ is a (***scalar, vector***).

3. $[(\bar{a} \times \bar{V}) \cdot \bar{e}]$ is a (***scalar, vector***)

4. We multiply by the final $\bar{e}$ in order to make the right side of the equation come out as a (***scalar, vector***).

Correct response to preceding frame

1. $\overline{M}_L$ is a vector
2. $(\overline{a} \times \overline{V})$ is a vector
3. $[(\overline{a} \times \overline{V}) \cdot \overline{e}]$ is a scalar
4. We must multiply the scalar $[(\overline{a} \times \overline{V}) \cdot \overline{e}]$ by the unit vector $\overline{e}$ in order to have a vector result.

Frame 9-6

Moment About a Line

Match the terms to their descriptions. (Some descriptions are used twice.)

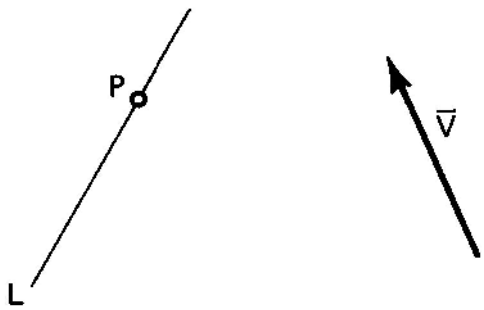
_____	(1)	$(\overline{a} \times \overline{V}) \cdot \overline{e}$	(a)	vector along line
_____	(2)	$[(\overline{a} \times \overline{V}) \cdot \overline{e}]\,\overline{e}$	(b)	moment about a point
_____	(3)	$\overline{e}$	(c)	moment about line
_____	(4)	$\overline{M}_L$	(d)	magnitude of moment about line
_____	(5)	M_L		
_____	(6)	$\overline{a} \times \overline{V}$		
_____	(7)	$\overline{M}_p$		

Correct response to preceding frame

d (1)
c (2)
a (3)
c (4)
d (5)
b (6)
b (7)

Frame 9-7

Moment About a Line



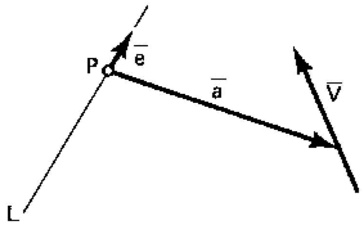
$\overline{M}_P = \overline{a} \times \overline{V}$

Draw $\overline{a}$

$\overline{M}_L = [(\overline{a} \times \overline{V}) \cdot \overline{e}]\ \overline{e}$

Draw $\overline{e}$

Correct response to preceding frame



Frame 9-8

Moment About a Line

Write the expression for the moment of a vector about a point.

$\bar{M}_P$ = ______________________________

Write the expression for the magnitude of a moment of a vector about a line.

M_L = ______________________________

Write the expression for the moment of a vector about a line.

$\bar{M}_L$ = ___________________________________

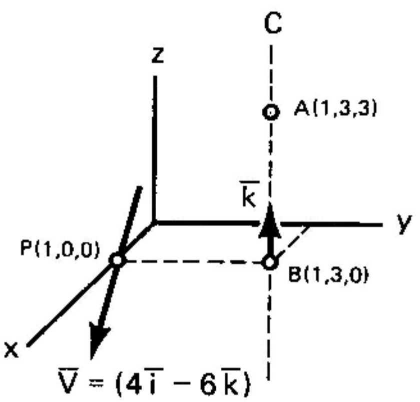
Correct response to preceding frame

$\overline{M}_P = (\overline{a} \times \overline{V})$
$M_L = (\overline{a} \times \overline{V}) \cdot \overline{e}$
$\overline{M}_L = [(\overline{a} \times \overline{V}) \cdot \overline{e}]\, \overline{e}$

Frame 9-9

Moment About a Line

In the situation below $\overline{k}$ is the unit vector along the line which passes through points A and B.



1. Find the moment of the vector $\overline{V}$ about point A.

$\overline{M}_A$ = ____

Find the moment of the vector $\overline{V}$ with respect to the line by use of the expression

$\overline{M}_L = (\overline{M}_A \cdot \overline{k})\, \overline{k}$ = ____

2. Find the moment of the vector $\overline{V}$ with respect to point B.

$\overline{M}_B$ = ____

Find the moment of the vector about the line by use of the expression

$\overline{M}_L = (\overline{M}_A \cdot \overline{k})\, \overline{k}$ = ____

Correct response to preceding frame

1. $\overline{M}_A = 18\overline{i} - 12\overline{j} + 12\overline{k}$
   $\overline{M}_L = 12\overline{k}$
2. $\overline{M}_B = 18\overline{i} + 12\overline{k}$
   $\overline{M}_L = 12\overline{k}$

Frame 9-10

Moment About a Line

Refer to the preceding frame.

1. Does $\overline{M}_A = \overline{M}_B$?

   ☐ Yes ☐ No

2. Does $\overline{M}_L$ change when we calculate it about different points on the line?

   ☐ Yes ☐ No

Correct response to preceding frame

1. No
2. No

Frame 9-11

Moment About a Line

$\overline{M}_L = [(\overline{a} \times \overline{V}) \cdot \overline{e}]\ \overline{e}$

1. The above expression is true when $\overline{a}$ is drawn from a particular point on the line to a particular point on the vector.

☐ True ☐ False

2. The above expression is true when $\overline{a}$ is drawn from any point on the line to any point on the vector.

☐ True ☐ False

Correct response to preceding frame

1. True
2. True

Frame 9-12

Problem

Find the moment of the vector $\overline{F} = \overline{i} + 2\overline{j} + 3\overline{k}$ which acts through the point A (0,2,1) about a line which passes through the point B (1,1,1) and the point C (4,5,13).

I'll give you a bit of help on this first problem.

Begin by taking the moment about some point on the line. This time the point B (1,1,1) is easiest.

$\bar{a}$ = ______________

$\bar{M}_B = \bar{a} \times \bar{F}$ = ____________________________

$$\bar{e}_{BC} = \frac{3\bar{i} + 4\bar{j} + 12\bar{k}}{13}$$

$M_{BC} = (\bar{a} \times \bar{F}) \cdot \bar{e}_{BC}$ = ______________

$\bar{M}_{BC}$ = ____________________________

Correct response to preceding frame

$$\bar{a} = -\bar{i} + \bar{j}$$

$$\bar{a} \times \bar{V} = +3\bar{i} + 3\bar{j} - 3\bar{k}$$

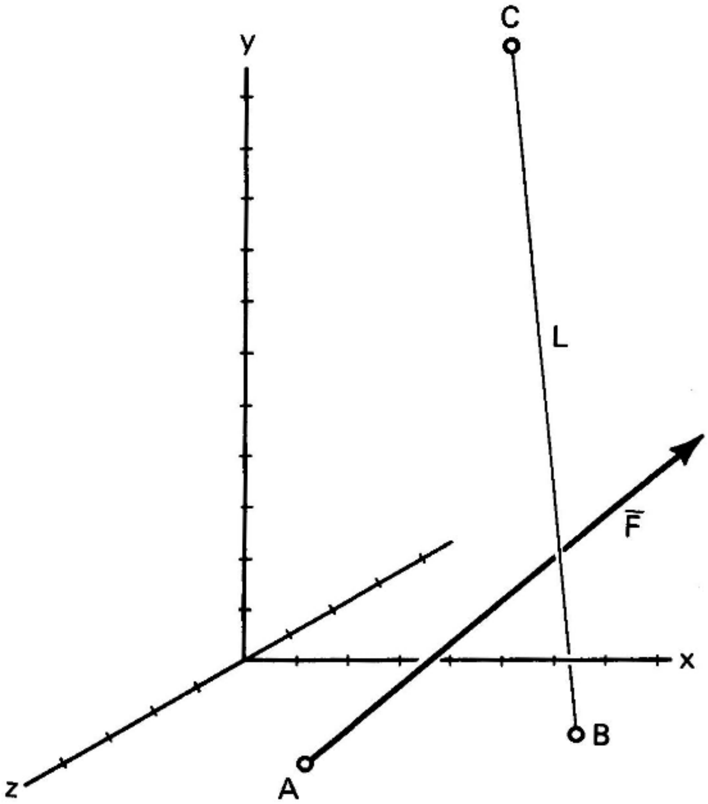
$$(\bar{a} \times \bar{V}) \cdot \bar{e} = -\frac{15}{13}$$

$$\bar{M}_L = -\frac{15}{13}\left(\frac{3\bar{i} + 4\bar{j} + 12\bar{k}}{13}\right)$$

Frame 9-13

Moment About a Line

Find the moment of the vector $\bar{F} = 3\bar{i} + \bar{j} - 7\bar{k}$ which acts through A, the point (3,-1,2), about a line which passes through B, the point (4,-3,-3), and C with coordinates (0,9,-6). (You'll have to find a unit vector along the line by yourself this time.)



Correct response to preceding frame

$\overline{M}_{BC} = -4.55\overline{i} + 13.65\overline{j} - 3.42\overline{k}$

Solution:

I took moments about C

$$\overline{M}_C = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 3 & -10 & 8 \\ 3 & 1 & -7 \end{vmatrix}$$

$$= 62\overline{i} + 45\overline{j} + 33\overline{k}$$

The unit vector along the line from C toward B is

$$\overline{e}_{BC} = \frac{4\overline{i} - 12\overline{j} + 3\overline{k}}{13}$$

and

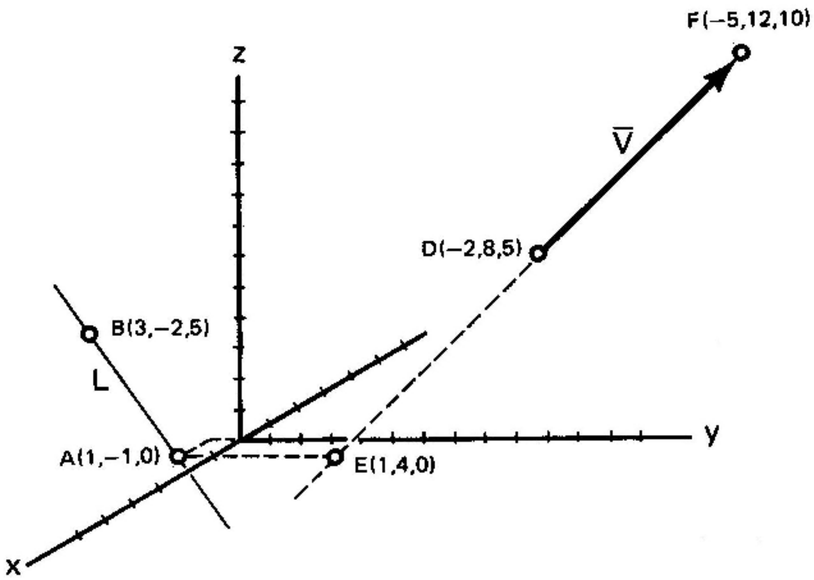
$$\overline{M}_C \cdot \overline{e}_{BC} = -\frac{193}{13} = -14.81$$

Frame 9-14

Choice of Arm

Frequently, a proper choice of the end points of the arm will make your work much easier.

In the following problem which pair of end points results in the simpler form for the arm?



- ☐ A and D
- ☐ A and E
- ☐ A and F
- ☐ B and D
- ☐ B and E
- ☐ B and F

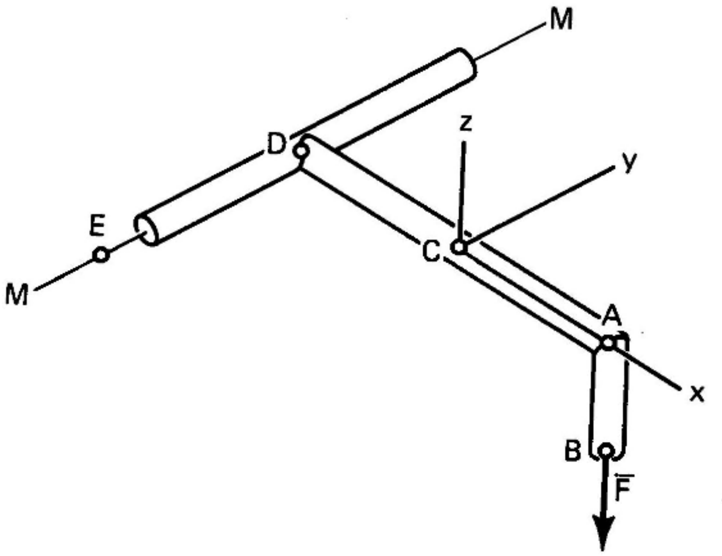
Correct response to preceding frame

A and E

Frame 9-15

Choice of Arm

Suppose that you are about to compute the moment of $\overline{\mathbf{F}}$ about line M-M.



Check the box which indicates the pair of points which will give the simplest arm.

- ☐ A and C
- ☐ B and E
- ☐ B and D
- ☐ E and C
- ☐ A and D
- ☐ E and A

Cross out the pairs of points which must not be used.

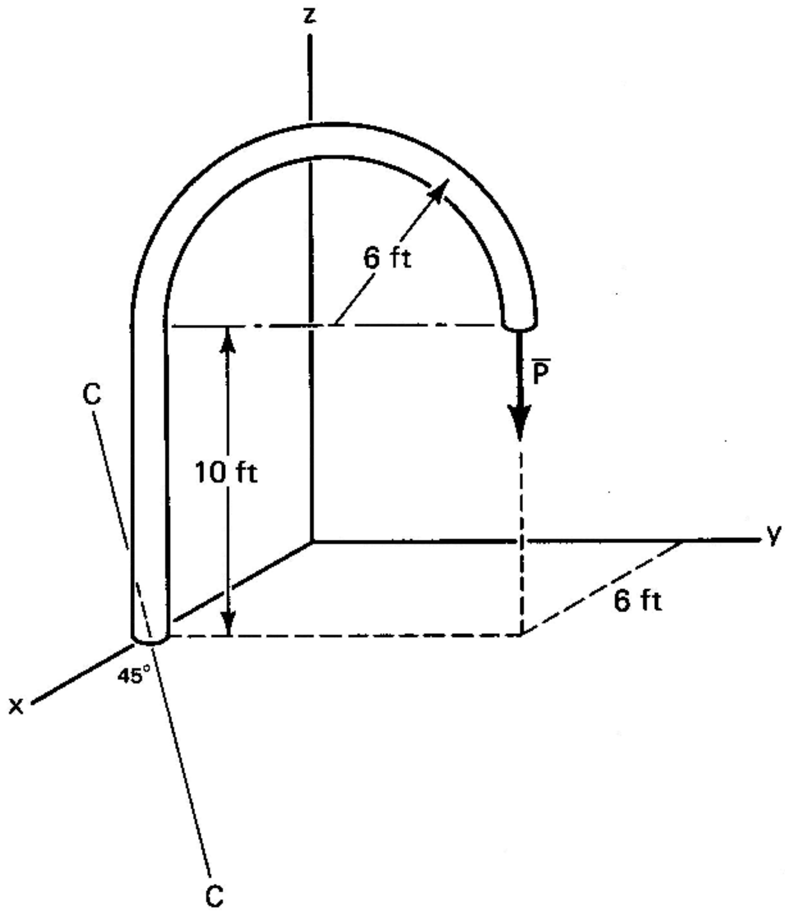
Correct response to preceding frame

A and D give the simpler arm.
You may not use [A and C] or [E and C] because C does not lie on M-M.

Frame 9-16

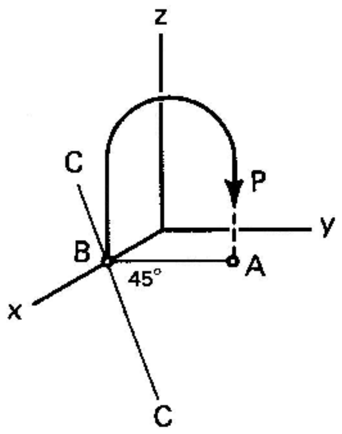
Choice of Arm

Indicate the pair of points on the drawing below which will give the simplest arm for the moment of $\overline{P}$ about line C-C.


z
6 ft
P
C
10 ft
y
6 ft
45°
x
C

Correct response to preceding frame

My choice is shown below as A-B.



Frame 9-17

Moment About a Coordinate Axis

There are two particularly important kinds of lines about which we take moments: coordinate axes, and certain kinds of physical axes such as shafts and axles.

Continue your practice by taking some moments about coordinate axes.

A force of 75 N in the direction ($0.6\bar{i} - 0.8\bar{j}$) passes through the point (7,10,3).

1. Find its moment about the origin

$\bar{M}_0$ = ____

2. Find its moment about the x axis.

$\bar{M}_x = (\bar{M}_0 \cdot \bar{i})\ \bar{i}$ = ____________________________

3. Find its moment about the y axis

$\bar{M}_y$ = _________________________

4. Find its moment about the z axis

$\bar{M}_z$ = _________________________

Correct response to preceding frame

1. $\overline{M}_o = -600\overline{i} + 555\overline{j} - 450\overline{k}$
2. $\overline{M}_x = -600\overline{i}$
3. $\overline{M}_y = +555\overline{j}$
4. $\overline{M}_z = -450\overline{k}$

<u>Solution:</u>

$$\overline{M}_o = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 7 & 10 & 3 \\ 45 & 0 & -60 \end{vmatrix}$$

$$= (-600 + 0)\overline{i} + (135 + 420)\overline{j} + (0 - 450)\overline{k}$$

$$\overline{M}_x = (\overline{M}_o \cdot \overline{i})\overline{i}$$

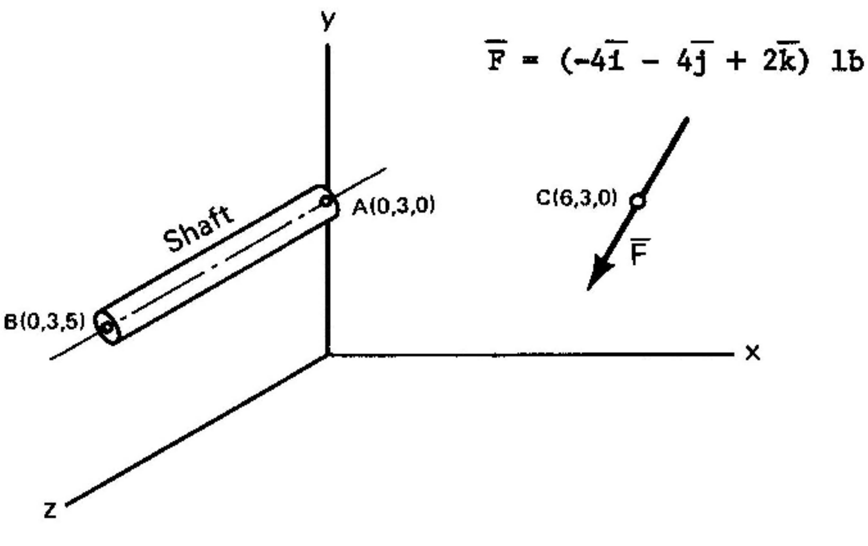
$$\overline{M}_y = (\overline{M}_o \cdot \overline{j})\overline{j}$$

$$\overline{M}_z = (\overline{M}_o \cdot \overline{k})\overline{k}$$

Frame 9-18

Moment About a Physical Axis

A shaft and a force are located in space as shown. Find the moment of the force about the shaft. Dimensions are in feet.



Correct response to preceding frame

$\overline{M}_{AB} = -24\overline{k}$ ft-lb

Solution:

$\overline{r}_{AC} = (6\overline{i} + 3\overline{j} + 0\overline{k}) - (0\overline{i} + 3\overline{j} + 0\overline{k})$

$= 6\overline{i}$

$\overline{M}_A = 6\overline{i} \times (-4\overline{i} - 4\overline{j} + 2\overline{k})$

$= -24\overline{k} - 12\overline{j}$

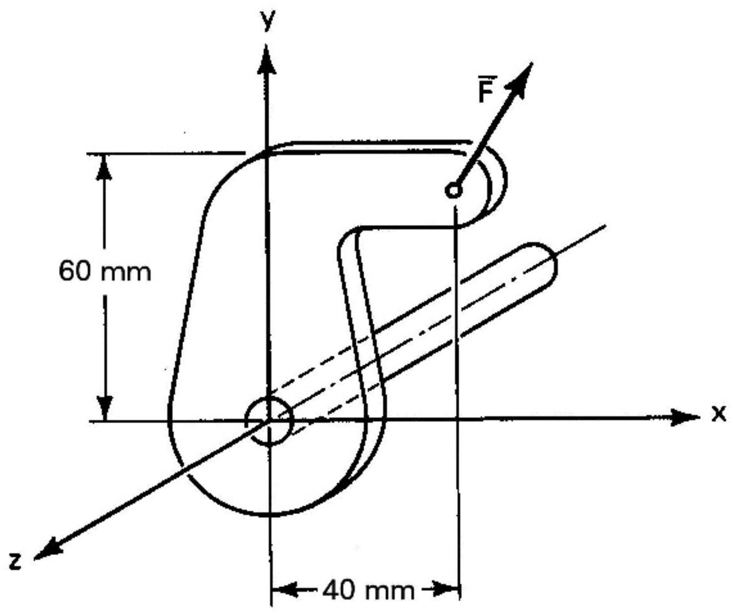
$\overline{M}_{AB} = (\overline{M}_A \cdot \overline{k})\overline{k}$

$= -24\overline{k}$

Frame 9-19

Moment About a Physical Axis

A force $\overline{F} = (200\overline{i} + 50\overline{i})$ N is applied to a bellcrank as shown. Find the moment of the force about the axis of the shaft. (In SI we do our mechanical analysis in Newtons and meters, even if a drafter uses other units on the drawing.)



Correct response to preceding frame

$\overline{M}_s = 5.00\overline{k}$ Newton-meters **Solution:**

$\overline{r} = 40\,\overline{i} + 60\,\overline{j}$ mm $= 0.04\,\overline{i} + 0.06\,\overline{j}$ meters

$\overline{M}_o = \overline{r} \times \overline{F}$

$$= \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 0.04 & 0.06 & 0 \\ 50 & 200 & 0 \end{vmatrix}$$

$= 5.00\overline{k}$

$\overline{e}_s = \overline{k}$

$\overline{M}_s = (\overline{M}_o \cdot \overline{k})\overline{k}$

Frame 9-20

Summary

You'll want to summarize this for later reference and review so complete Page 9-1 in your notebook.

Correct response to preceding frame

1. $\overline{M}_x = 5\overline{i}$
2. $\overline{M}_y = 11\overline{j}$
3. $\overline{M}_L = 7.9\overline{i} + 3.95\overline{j} - .99\overline{k}$

   or $\left(-\frac{80}{9}\right)\left(-\frac{8}{9}\overline{i} - \frac{4}{9}\overline{j} + \frac{1}{9}\overline{k}\right)$

Solution:

$\overline{r} = -3\overline{i} + \overline{j} + \overline{k}$

$$\overline{M}_o = \overline{r} \times \overline{F} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -3 & 1 & 1 \\ 5 & -3 & 2 \end{vmatrix}$$

$$= 5\overline{i} + 11\overline{j} + 4\overline{k}$$

1. $\overline{M}_x = (\overline{M}_o \cdot \overline{i})\overline{i} = 5\overline{i}$
2. $\overline{M}_y = (\overline{M}_o \cdot \overline{j})\overline{j} = 11\overline{j}$
3. $\overline{e}_L = \dfrac{1}{\sqrt{8^2 + 4^2 + 1^2}}(-8\overline{i} - 4\overline{j} + \overline{k})$

   $= -\frac{8}{9}\overline{i} - \frac{4}{9}\overline{j} + \frac{1}{9}\overline{k}$

$\overline{M}_L = (\overline{M}_o \cdot \overline{e}_L)\overline{e}_L$

$= ([5\overline{i} + 11\overline{j} + 4\overline{k}] \cdot [-\frac{8}{9}\overline{i} - \frac{4}{9}\overline{j} + \frac{1}{9}\overline{k}])\overline{e}_L$

$= \left(-\frac{40}{9} - \frac{44}{9} + \frac{4}{9}\right)\overline{e}_L$

$= \left(-\frac{80}{9}\right)\left(-\frac{8}{9}\overline{i} - \frac{4}{9}\overline{j} + \frac{1}{9}\overline{k}\right)$

Frame 9-21

Closure

By now you should know more than you ever hoped to learn about the computation of moments. The moment of any vector about any line should be a delight to solve.

Seriously, it is a skill you may find very handy in later courses. Thus endeth the day's lesson.