Introduction to Statics

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Unit 8
Moment About a Point

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Unit 8
Moment About a Point

Frame 8-1

Introduction

In this unit you will be introduced to a new concept called "moment." Your immediate need is for a working knowledge of the "moment of a force" and "the moment of a couple," so most of your time will be spent in wrestling with these two particular ideas, but your success in the following mechanics courses will depend in part on the mastery of moment as a general vector concept. This unit will begin with an introduction to the general concept. The later frames will introduce, and work with, the moment of a force or force system.

Go to the next frame.

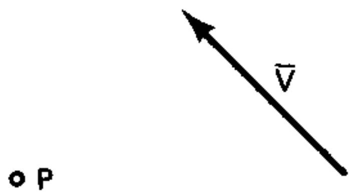
Correct response to preceding frame

No response

Frame 8-2

Moment of a Vector about a Point

The curtain for the first act rises to display a stage which apparently contains only a vector $\overline{V}$ and a point P, floating in space.



As the show progresses we shall see an engrossing plot develop in which these two are deeply involved.

To learn what happens, turn to the next frame.

Correct response to preceding frame

No response

Frame 8-3

Moment of a Vector about a Point

Let us begin by recalling a theorem from solid geometry.

"A line in space and a point which does not lie on the line, together determine (or define) ______________________________________ ."

Correct response to preceding frame a plane

a plane

Frame 8-4

Moment of a Vector about a Point

Definition (Part 1) ***The moment of a vector $\overline{V}$ about a point P is a vector through point P whose line of action is perpendicular to the plane of $\overline{V}$ and P.***

1. If $\overline{V}$ and P lie in the plane of the paper, the line of action of the moment is

    ______________________________ .

2. If $\overline{V}$ and P lie in the plane of the floor, the line of action of the moment is

    ______________________________ .

3. If $\overline{V}$ and P lie in the y-z plane, the moment is

    ______________________________ .

Correct response to preceding frame

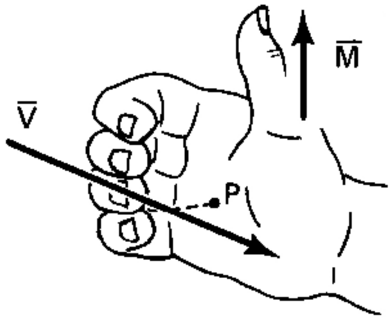
1. perpendicular to the plane of the paper
2. vertical
3. directed parallel to the x-axis

(Or equivalent responses)

Frame 8-5

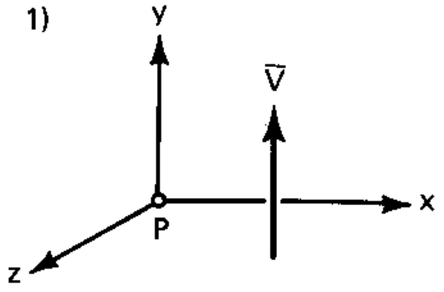
Moment of a Vector about a Point

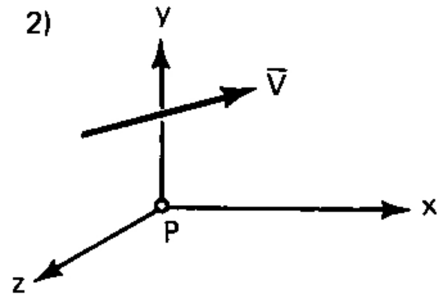
Definition (Part 2) ***The moment of a vector $\overline{V}$ about a point P is a vector whose sense (or sign) is determined by the right-hand screw rule.***

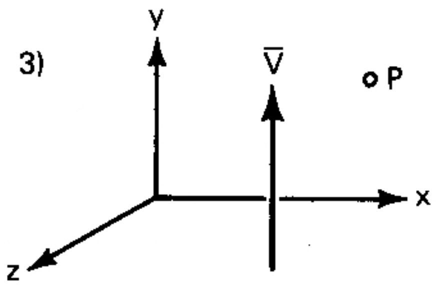


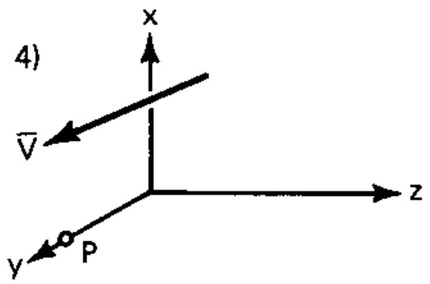
That is, if the vector $\overline{V}$ is directed along the fingers of your RIGHT hand with the point P on your palm, the sense of moment $\overline{M}$ will be along your thumb.

If the vector $\overline{V}$ and point P lie in the x-y plane give the line of action and sense of the moment in the following cases:









Correct response to preceding frame

1. +z
2. -z
3. -z
4. +z

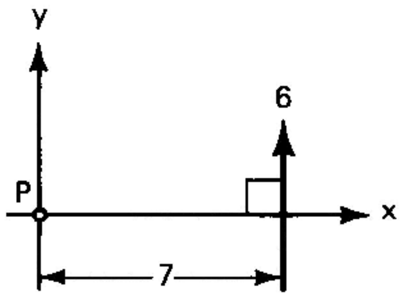
Frame 8-6

Moment of a Vector about a Point

Definition (Part 3) ***The magnitude of the moment of a vector $\bar{V}$ about a point** P **is equal to the product of the perpendicular distance from** P **to $\bar{V}$ and the magnitude of vector $\bar{V}$.***

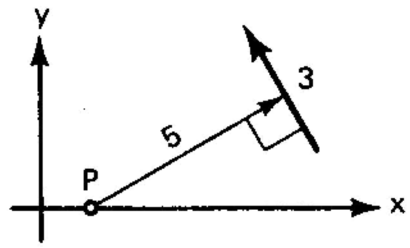
The vectors and points lie in the plane of the paper. Give the magnitude of the moment for the following cases:

1)



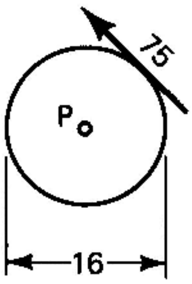
M = ____________

2)



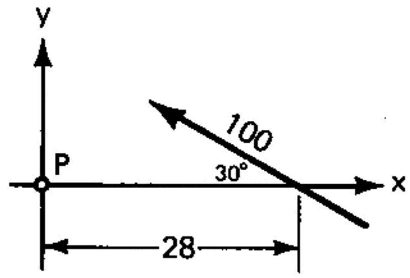
M = ____________

3)



M = ____________

4)



M = ____________

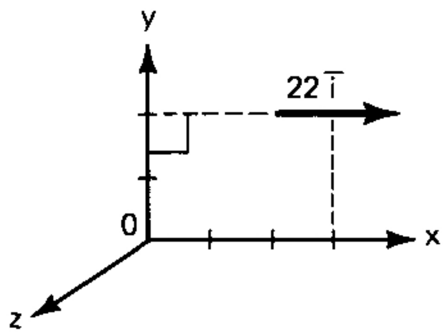
Correct response to preceding frame

1. 42
2. 15
3. 600
4. 1400

Frame 8-7

Moment of a Vector about a Point

A vector $\overline{V} = 22\overline{i}$ acts through the point (3,2,0). Find its moment about the origin.



Careful !! Give magnitude, sense and line of action.

Correct response to preceding frame

$\overline{M} = -44\overline{k}$

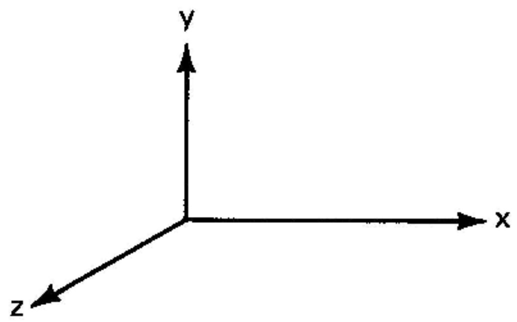
The moment is negative because "thumb goes into paper." The unit vector $\overline{k}$ is perpendicular to x-y plane and $M = 2(22)$.

Frame 8-8

Moment of a Vector about a Point

A vector $\overline{V} = 1.6\overline{i}$ acts through the point (4,-2,0). Find its moment about the origin, and show the moment on the sketch.

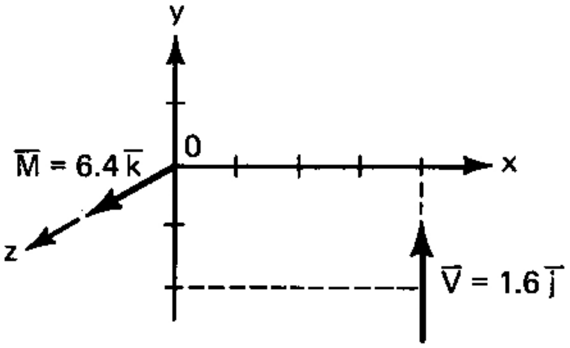
$\overline{M} =$ ________________________________



Correct response to preceding frame

$\overline{M} = + 6.4\overline{k}$ Solution:

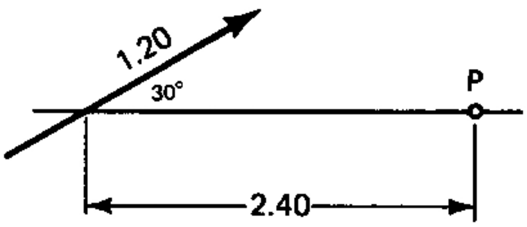
$M = 4\ (6.4)$



Frame 8-9

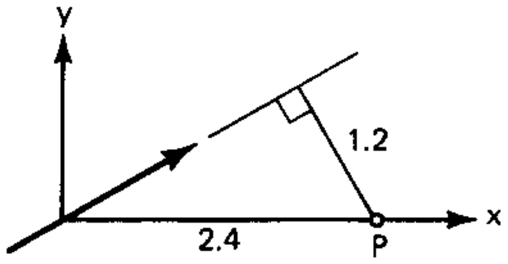
Practice

A vector and a point lie in the plane of the paper as shown. What is the magnitude of the moment of the vector about the point?



Correct response to preceding frame

$\overline{M} = -1.44\overline{k}$ (if you choose the coordinates shown)



Frame 8-10

Recapitulation

The definition of the moment of a vector about a point and our knowledge of solid geometry have shown us that we may associate four things with the moment resulting from the combination of a vector and a point:

1. The point and the vector define ________________________ .

2. The moment is a vector directed _________________________ .

3. The right hand rule gives _____________________ .

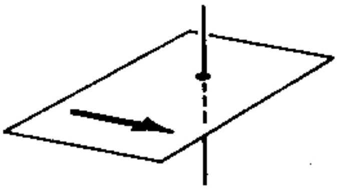
4. The distance from the point and the magnitude of the vector and its give

____ .

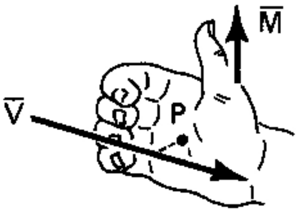
Correct response to preceding frame

1. a plane containing the point and the line

2. perpendicular to the plane through the point



3. the sense of the moment



4. the magnitude of the moment

(Or equivalent responses)

Frame 8-11

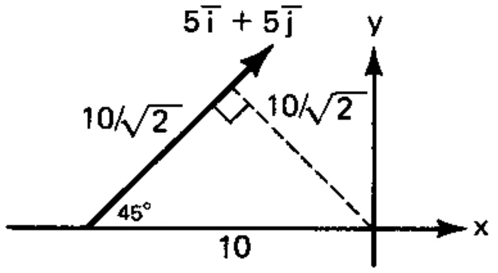
Problem

A vector $\overline{V} = 5\overline{i} + 5\overline{j}$ acts through the point (-10,0,0). Find its moment about the origin.

Correct response to preceding frame

$\overline{M} = -50\overline{k}$

Solution:



$$\overline{V} = 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\overline{i} + \frac{1}{\sqrt{2}}\overline{j}\right)$$

$$M = \left(\frac{10}{\sqrt{2}}\right)\left(5\sqrt{2}\right) = 50$$

$\overline{M}$ is in $-\overline{k}$ direction by right hand rule

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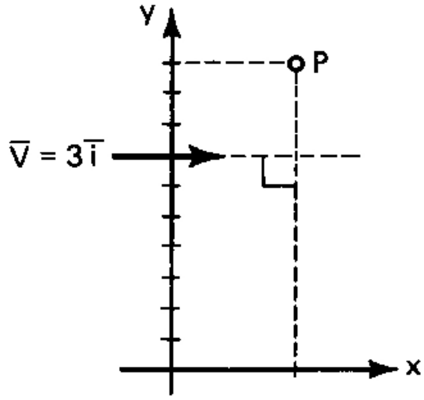
Frame 8-12

Problem

Find the moment about the point (4,10,0) of the vector $\overline{V} = 3\overline{i}$ which acts through the point (0,7,0).

Correct response to preceding frame

$\overline{M} = +9\overline{k}$

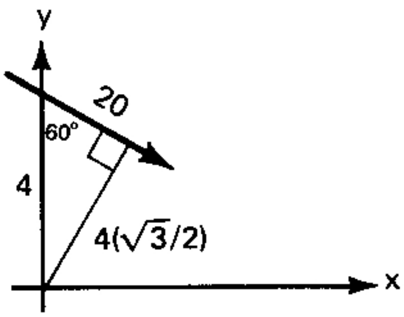


Frame 8-13

Summary

Complete Page 8-1 in your notebook.

Correct response to preceding frame



$M = (2\sqrt{3})(20)$

$\overline{M} = -40\sqrt{3}\,\overline{k}$

Frame 8-14

Transition

There is a way of finding moments with vector algebra which often simplifies calculations. Two of the parts of the definition may have already suggested it to you.

What vector algebra operation results in a vector normal to the plane of two other vectors and a sense determined by the right hand rule?

____

Correct response to preceding frame

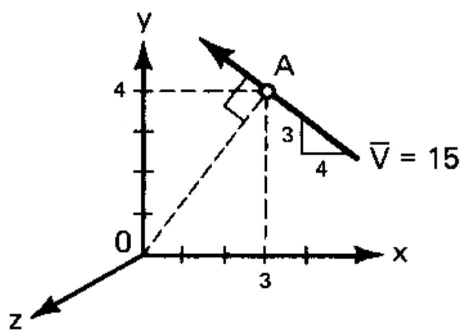
the cross product of two vectors

Frame 8-15

Moments by Vector Products

Let's look at an example of how the cross product can be used in the calculation of moments.

A vector $\overline{V}$ with a magnitude of 15 passes through the point A (3,4,0) as shown.



1. What is its moment about the origin? $\overline{M}$ = ________________________

The vector from 0 to A may be represented by the vector $3\overline{i} + 4\overline{j}$ and the vector $\overline{V}$ is

$$\bar{V} = 15\left(-\frac{4}{5}\,\bar{i} + \frac{3}{5}\,\bar{j}\right)$$

2. Calculate the product

$$\bar{N} = \bar{r} \times \bar{V} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 4 & 0 \\ -12 & 9 & 0 \end{vmatrix} = \underline{\qquad\qquad\qquad\qquad}$$

3. Does $\bar{M} = \bar{N}$? ☐ Yes ☐ No

4. Which is easier to calculate?

____

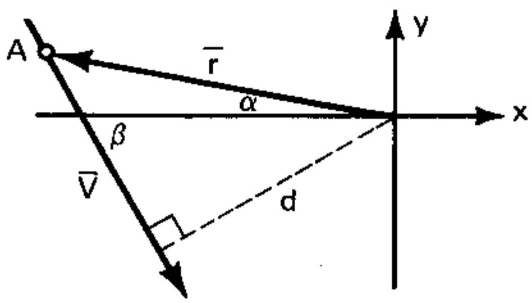
Correct response to preceding frame

1. $\overline{M} = 75\overline{i}$
2. $\overline{N} = 75\overline{i}$
3. Yes
4. $\overline{M}$ is easier if you see that the perpendicular distance OA is 5. On the other hand, you can calculate $\overline{N}$ quickly even if you have trouble visualizing the perpendicular distance.

Frame 8-16

Moments by Vector Products

A vector $\overline{V} = 23\overline{i} - 41\overline{j}$ passes through the point A (-17,+3) and we wish to calculate its moment about the origin.



We could use the information given to calculate the angles α and β and the distance d, but it is much easier to simply note that

$$\overline{r} = (-17\overline{i} + 3\overline{j})$$

$$\overline{V} = 23\overline{i} - 41\overline{j}$$

and therefore, if $\overline{M} = \overline{r} \times \overline{V}$

$$\overline{M} = \underline{\hspace{6cm}}$$

Correct response to preceding frame

$$\bar{M} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -17 & 3 & 0 \\ 23 & -41 & 0 \end{vmatrix}$$

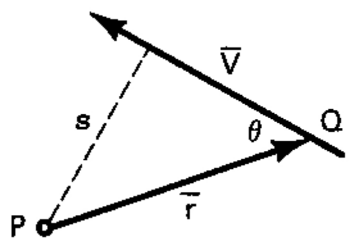
$$= [17(41) - 3(23)]\bar{k}$$

$$= 628\bar{k}$$

The reason for teaching cross products should be a bit clearer to you now.

Frame 8-17

Moment by Vector Product



The vector $\bar{r}$ from P to some point Q on $\bar{V}$ is not the perpendicular from the point to the vector, however, we see that perpendicular distance $S = r \sin\theta$, hence the magnitude of the moment.

1. $M = sV =$ ____________________________ (a)

2. What is the definition of the magnitude of the cross product $|\bar{r} \times \bar{V}|$?

$|\bar{r} \times \bar{V}| =$ ____________________________ (b)

3. Are expressions (a) and (b) the same? ☐ Yes ☐ No

Correct response to preceding frame

1. $M = (r \sin\theta) V$
2. $| \bar{r} \times \bar{V} | = r V \sin\theta$
3. Yes, they are the same since the terms r, V and $\sin\theta$ are scalars and the multiplication of scalars is commutative, (does not depend on the order of the terms.)

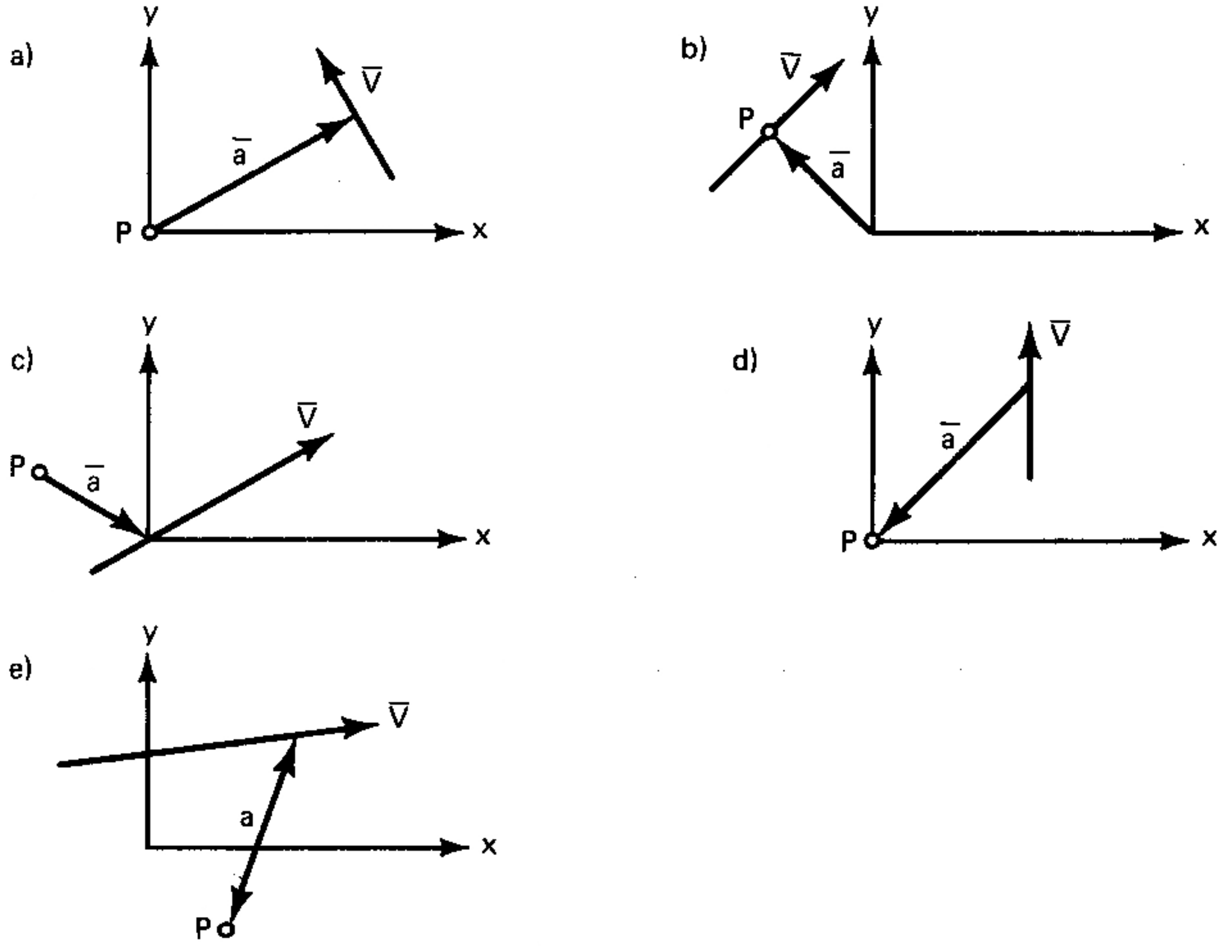
Frame 8-18

The Arm Vector

The vector from a point to a force is often called the moment-arm.

An arm vector is taken FROM the point about which the moment is to be taken TO any point on the vector whose moment is needed.

In which of the following cases is the arm vector drawn incorrectly?



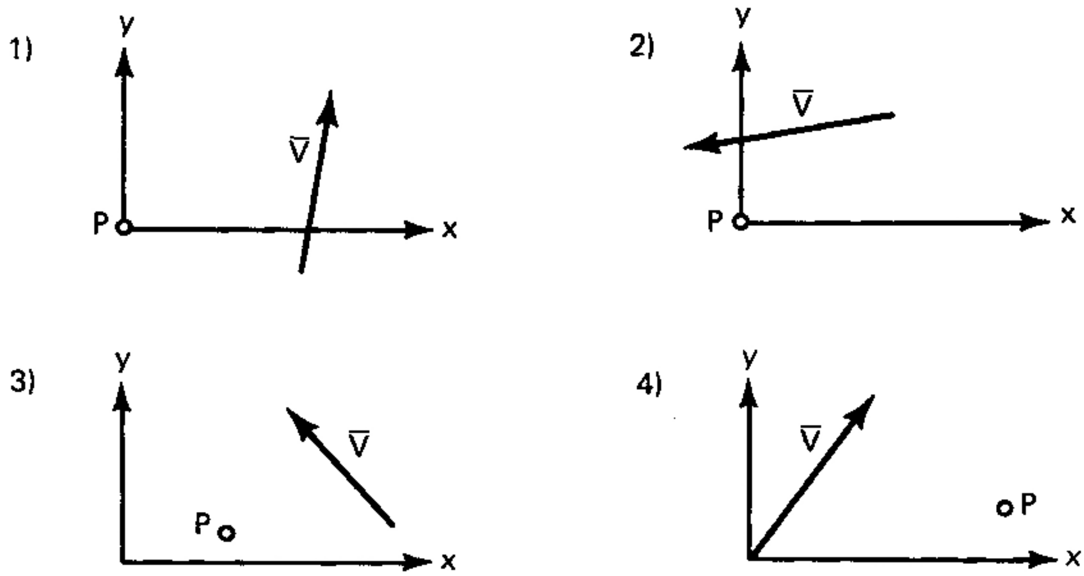
Correct response to preceding frame

b, d and e are incorrect

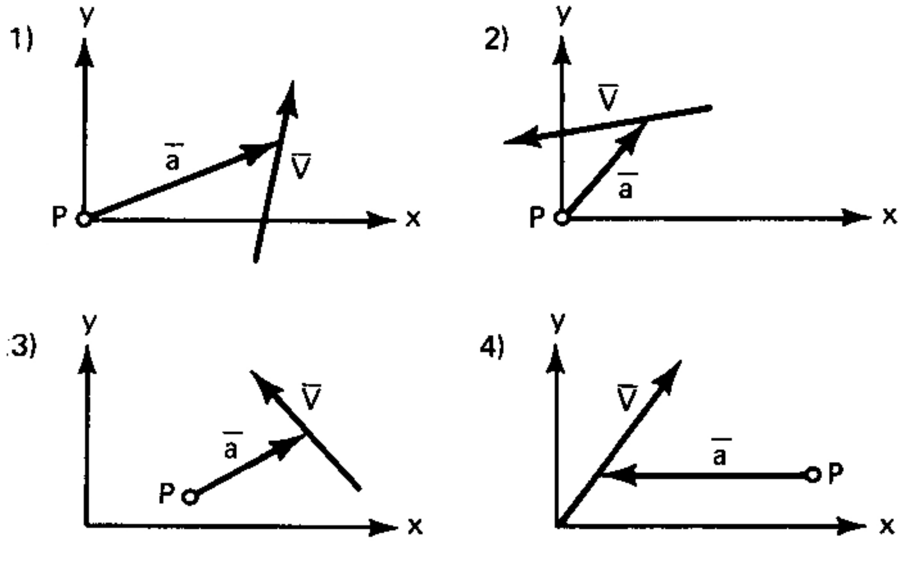
Frame 8-19

The Arm Vector

For each example below, draw an arm vector connecting P and $\overline{V}$.



Correct response to preceding frame

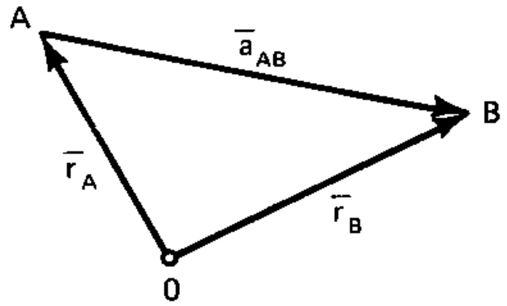


The angle chosen between a and V does not matter if we are going to take cross products, but the arrow must have its head on the proper end.

Frame 8-20

The Arm Vector

The arm vector $\overline{\mathbf{a}}_{AB}$ from A to B may be found by subtracting the vector $\overline{\mathbf{r}}_A$ from the origin to A from the vector $\overline{\mathbf{r}}_B$ which goes from the origin to B.



1. Find $\overline{\mathbf{a}}_{AB}$ if A is the point (3,7,0) and B is the point (4,4,0).

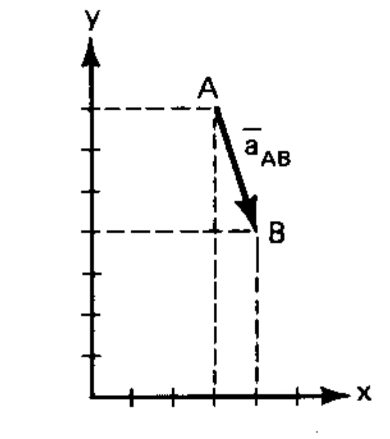
   $\overline{\mathbf{a}}_{AB}$ = ________________________________

2. Find $\overline{\mathbf{a}}_{AB}$ if $\overline{\mathbf{r}}_A = 3\overline{\mathbf{i}} - 2\overline{\mathbf{j}} + 4\overline{\mathbf{k}}$ and $\overline{\mathbf{r}}_B = 10(.6\overline{\mathbf{i}} - .8\overline{\mathbf{j}})$.

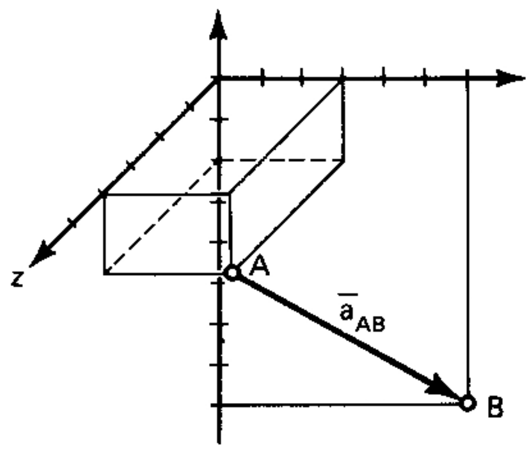
   $\overline{\mathbf{a}}_{AB}$ = ________________________________

Correct response to preceding frame

1) $\bar{a}_{AB} = \bar{i} - 3\bar{j}$



2) $\bar{a}_{AB} = 3\bar{i} - 6\bar{j} - 4\bar{k}$



(A little sketch will sometimes prevent quite embarassing little mistakes.)

Frame 8-21

Moment by Vector Products

You now have all the tools to take care of the following problem.

The vector $\bar{V} = 5\bar{i} - 12\bar{k}$ passes through the point B which has the coordinates (3,4,0). Find its moment about point A which has coordinates (2,-2,1).

1. $\bar{a}_{AB} = (B_x - A_x)\bar{i} + (B_y - A_y)\bar{j} + (B_z - A_z)\bar{k}$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

2. $\bar{M}_A^* = \bar{a}_{AB} \times \bar{V}$

$= \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

*The use of the subscript on $\bar{M}$ means "the moment about point A" or "the moment with respect to point A."

Correct response to preceding frame

1. $\overline{r}_{AB} = \overline{i} + 6\overline{j} - \overline{k}$

2. $\overline{M}_A = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 1 & 6 & -1 \\ 5 & 0 & -12 \end{vmatrix}$

$= -72\overline{i} + 7\overline{j} - 30\overline{k}$

Frame 8-22

Moment by Vector Products

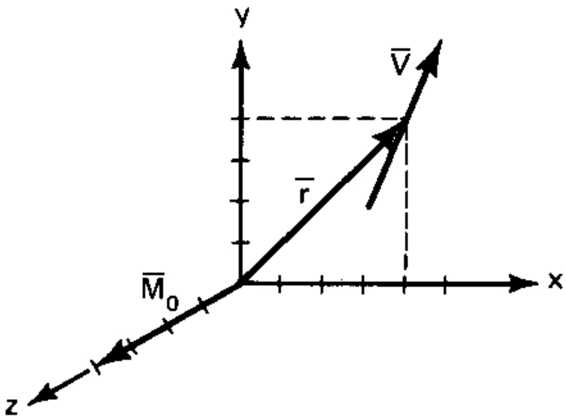
The vector $\overline{V} = 3\overline{i} + 4\overline{j}$ passes through the point (4,4,0). Find its moment about the origin, and show $\overline{V}$, $\overline{r}$ and $\overline{M}_0$ on a sketch.

Check to be sure that the "handedness" of your sketch agrees with the cross product result.

Correct response to preceding frame

$$\overline{M}_o = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 4 & 4 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

$$= 4\overline{k}$$



Frame 8-23

Summary

Summarize this section on Page 8-2 of your notebook.

Correct response to preceding frame

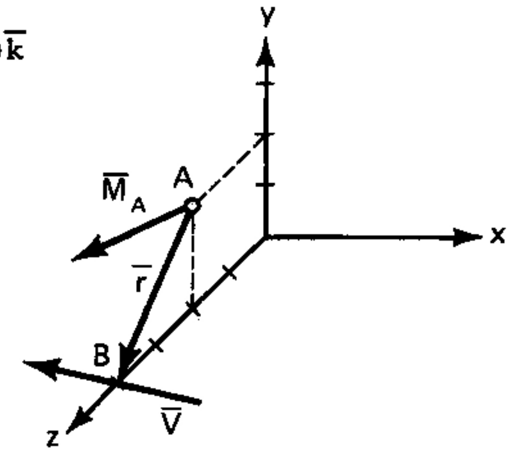
$$\overline{r}_{AB} = (B_x - A_x)\overline{i} + (B_y - A_y)\overline{j} + (B_z - A_z)\overline{k}$$

$$= -2\overline{j} + 2\overline{k}$$

$$\overline{M}_A = \overline{r} \times \overline{V}$$

$$= \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 0 & -2 & 2 \\ -3 & 4 & 5 \end{vmatrix}$$

$$= -18\overline{i} - 6\overline{j} - 6\overline{k}$$



Frame 8-24

Transition

So far this unit has dealt with a rather general concept, the moment of a vector. This material has many uses in the various fields of mechanics which you will study later, and you will now learn about the most important of these.

This unit contains only six more frames -- delightful idea, isn't it?

Go to the next frame.

Correct response to preceding frame

No response

Frame 8-25

Units

In statics the vectors whose moments we seek will be forces.

If we measure distances and forces in American engineering units what units will we use for moments?

____

If we measure distances and forces in SI units what units will we use for moments?

____

Correct response to preceding frame

In American engineering units, foot-pounds and inch-pounds are commonly used. Others like foot-tons are possible, but generally used.

In SI units are generally restricted to Newton-meters.

Frame 8-26

Moment of a Force

If you have worked around automobiles much you probably have seen specifications expressed in foot-pounds (ft-lb).

What can you think of that has these units?

____

Correct response to preceding frame

torque (both engine output and bolt tightening)

Frame 8-27

Common Speach and Moments

"Torque" is a term which is often used to denote the tendency of a force to cause rotation. Generally it is only used to indicate a "twisting" moment as opposed to a "bending" moment. This is a distinction you will learn to make when you study dynamics and mechanics of materials. However, while you are working statics problems, it would be a good idea to learn to think and talk about both of these as "the moment of the force" involved.

Go on to the next frame.

Correct response to preceding frame

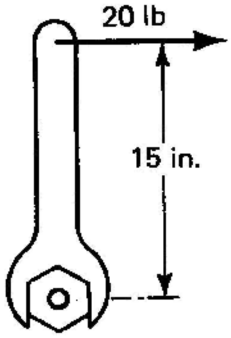
No response

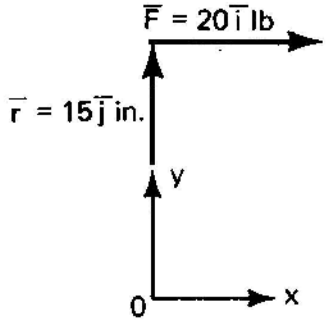
Frame 8-28

Moment of a Force

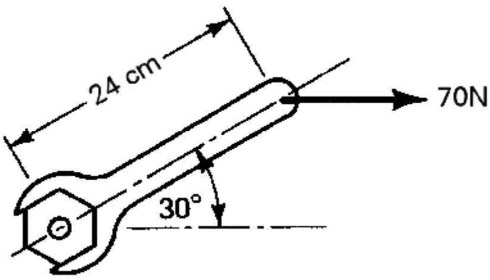
In each of the situations below a moment is being applied to the bolt with a wrench.

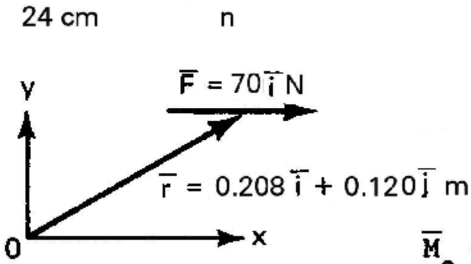
Compute the moments.



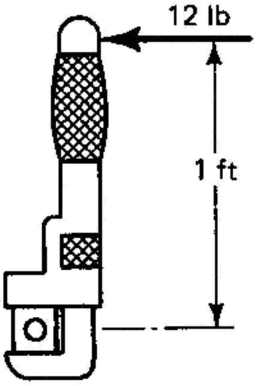


$\overline{M}_o$ = ____________





$\overline{M}_o$ = ____________



$\overline{M}_o$ = ____________

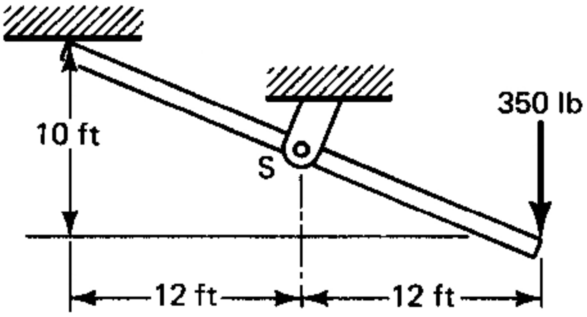
Correct response to preceding frame

1. $\overline{M}_0 = -300\overline{k}$ in-lb
2. $\overline{M}_0 = -8.4\overline{k}$ N-m
3. $\overline{M}_0 = 12\overline{k}$ ft-lb (or $144\overline{k}$ in-lb)

Frame 8-29

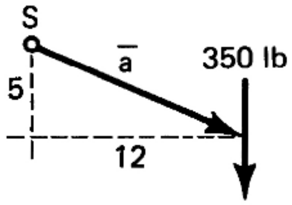
Moment of a Force

A force of 350 pound acts on the end of a 26 foot beam as shown. What is the moment of the force about point S?



Correct response to preceding frame

$\overline{M}_S = (12\overline{i} - 5\overline{j}) \times (-350\overline{j}) = -4200\overline{k}$ ft-lb



Frame 8-30

Closure

As you have seen, there is little new in taking moments of forces if you remember the following ideas:

1. Draw a sketch with r going from the point about which you want moments to some point on the force

2. Use the cross product

3. Use appropriate units

Soon you will learn how to use these calculations to determine forces on bodies which tend to tip or rotate.