Introduction to Statics

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Unit 7
Vector Products

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Unit 7
Vector Products

Frame 7-1

Introduction

In earlier units you have learned to handle vector addition and subtraction and to apply them to problems involving forces.

This unit will teach you a bit about vector "multiplication." Vector multiplication, while somewhat similar to scalar multiplication, is far from an identical process. In the first place, there are two kinds of products which may be obtained by multiplying vectors. They are called ***dot products*** and ***cross products*** and must be carefully separated in one's mind since they represent the results of two entirely different processes.

Then, as if it weren't bad enough to have two different ways of multiplying one vector by another, we must also deal with the result of multiplying a vector by a scalar.

Taking the three products in the order of complexity, we shall deal first with the product of a scalar and a vector, second with the dot product of two vectors, and last with the cross product of two vectors.

It is not the purpose of these programs to provide a mathematical treatment of vector multiplication. The student who wishes such a treatment will find it in any text on vector analysis. It is the purpose of the program to develop the student's ability to correctly perform vector multiplication, in order that he may use it as a tool in solving problems.

When you are ready, turn the page and begin.

Correct response to preceding frame

No response

Frame 7-2

Multiplication of a Vector by a Scalar

In the expression

$$\overline{U} = 3\overline{V}$$

$\overline{U}$ represents the product obtained by the multiplication of a vector $\overline{V}$, by the scalar, 3.

The direction of $\overline{U}$ is ____________________________________ to the direction of $\overline{V}$.

The magnitude of $\overline{U}$ is ____________________________________ the magnitude of $\overline{V}$.

$\overline{U}$ a is a (***vector* / *scalar***) (Circle one)

Correct response to preceding frame

$\bar{U}$ is parallel to $\bar{V}$.
$\bar{U}$ has three times the magnitude of $\bar{V}$.
$\bar{U}$ is a vector.

Frame 7-3

Multiplication of a Vector by a Scalar

$\bar{R} = 6\bar{i} + 3\bar{k}$

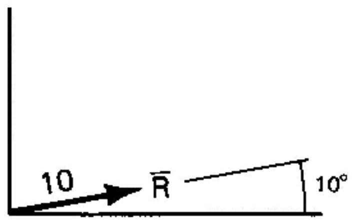
$5\bar{R} =$ ______ $\bar{i}$ + _______ $\bar{j}$

Correct response to preceding frame

$5\overline{R} = 30\overline{i} + 15\overline{k}$

Frame 7-4

Multiplication of a Vector by a Scalar

1. 

2. $\overline{B} = 5\overline{i} + 6\overline{j} + 4\overline{k}$

   $\frac{1}{2}\overline{B} =$ ____________________

3. $\overline{V} = 15(.6\overline{i} + .8\overline{j})$

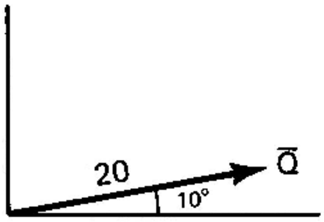
   $4\overline{V} =$ ____________________

1. Draw the vector

   $\overline{Q} = 2\overline{R}$

Correct response to preceding frame

1.



2. $\frac{1}{2}\bar{B} = 2.5\bar{i} + 3\bar{j} + 2\bar{k}$

3. $4\bar{V} = 60(.6\bar{i} + .8\bar{j})$

Frame 7-5

Multiplication of a Vector by a Scalar

About the only thing to look out for in this kind of multiplication is notation. Never indicate the multiplication of a scalar by a vector with either a dot, or a cross, since in vector algebra those signs mean something else entirely. You may use parentheses if you feel the need of a sign, though often no sign is required.

Write the product of the scalar, 3, and the vector, $\bar{Q}$, two different ways.

____

Correct response to preceding frame

Some permissible ways are: $3\overline{Q}$ or $(3)(\overline{Q})$ or $3(\overline{Q})$ and, in computer language, I guess, $3*\overline{Q}$

Forbidden are: $3 \cdot \overline{Q}$ and $3 \times \overline{Q}$

Frame 7-6

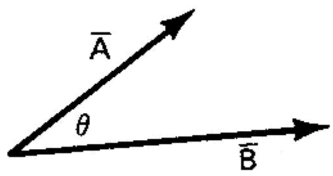
Transition

So much for the product of a scalar and a vector. You could probably have managed it very nicely just by intuition.

Our next topic, the dot product, is not intuitive but is defined as follows:

$$Q = \overline{A} \cdot \overline{B} = AB \cos \theta$$

when the vectors $\overline{A}$ and $\overline{B}$ are as shown below.



Go to the next frame.

Correct response to preceding frame

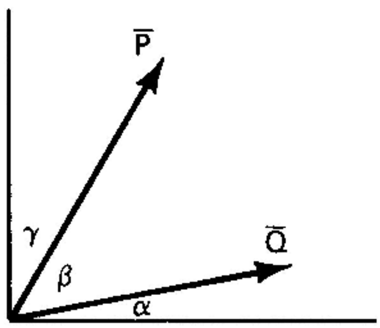
No response

Frame 7-7

Dot Product - Definition

Definition:

The dot product of any two vectors is a scalar and is defined as the product of the scalar magnitudes of the vectors and the cosine of the angle between them.



Write an expression for $\overline{P} \cdot \overline{Q}$.

$\overline{P} \cdot \overline{Q} =$ ________________________________

Correct response to preceding frame

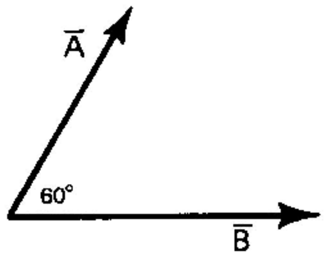
$\overline{P} \cdot \overline{Q} = PQ \cos \beta$

Frame 7-8

Dot Product

In taking dot products always use the cosine of the included angle formed by drawing the vectors from a common origin.

In the figure $\overline{A}$ has a magnitude of 20 units and $\overline{B}$ has a magnitude of 15 units.



1. Evaluate $\overline{A} \cdot \overline{B}$ = ____________
2. Evaluate $\overline{B} \cdot \overline{A}$ = ____________

Correct response to preceding frame

1. $\overline{A} \cdot \overline{B} = 150$ Solution: $\overline{A} \cdot \overline{B} = (20)\ (15)\ \cos 60°$
2. $\overline{B} \cdot \overline{A} = 150$ Solution: $\overline{A} \cdot \overline{B} = (15)\ (20)\ \cos 60°$

Frame 7-9

Dot Product

Write an expression for the dot product of the two vectors $\overline{A}$ and $\overline{B}$ separated by the angle θ.

$\overline{A} \cdot \overline{B} =$ ________________________________

Is the dot product of two vectors a scalar or a vector? ____________________

Correct response to preceding frame

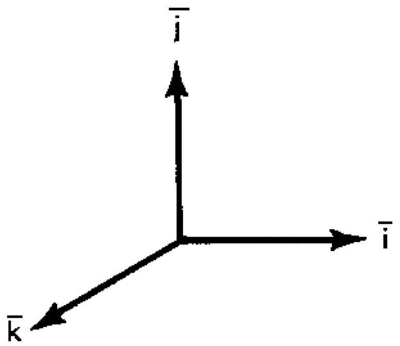
$\overline{A} \cdot \overline{B} = AB \cos \theta$
The dot product is always a scalar.

Frame 7-10

Dot Product

It is frequently convenient to evaluate a dot product without first drawing the vectors to find θ.

To do this easily we need to look at the dot products of unit vectors in a cartesian system.



$\overline{i} \cdot \overline{i} = (1)\ (1) \cos 0° =$ ________

$\overline{i} \cdot \overline{j} = (1)\ (1) \cos 90° =$ _______

$\overline{i} \cdot \overline{k} =$ _______

The dot product of two parallel unit vectors is __________

The dot product of two perpendicular vectors is ________

Correct response to preceding frame

$\bar{i} \cdot \bar{i} = 1$
$\bar{i} \cdot \bar{j} = 0$
$\bar{i} \cdot \bar{k} = 0$
The dot product of two parallel unit vectors is one.
The dot product of two perpendicular vectors is zero.

Frame 7-11

Dot Product

The dot product is distributive with respect to addition, that is

$$\bar{w} \cdot (\bar{u} + \bar{v}) = (\bar{w} \cdot \bar{u}) + (\bar{w} \cdot \bar{v})$$

For example

Given $\bar{A} = 3\bar{i}$ and $\bar{B} = 12\bar{i} + 6\bar{j} + 4\bar{k}$

$$\begin{aligned} \bar{A} \cdot \bar{B} &= (3\bar{i} \cdot 12\bar{i}) + (3\bar{i} \cdot 6\bar{j}) + (3\bar{i} \cdot 4\bar{k}) \\ &= 36\ \bar{i} \cdot \bar{i} + 18\ \bar{i} \cdot \bar{j} + 12\ \bar{k} \cdot \bar{k} \\ &= 36(1) + 18(0) + 12(0) = 36 \end{aligned}$$

Now find these dot products in a similar fashion.

$\bar{C} = 7\bar{j}$

1. $\bar{C} \cdot \bar{B} =$ ____________________

$\bar{D} = -9\bar{k}$

2. $\bar{D} \cdot \bar{B} =$ ____________________

$\bar{E} = \bar{A} + \bar{C} + \bar{D} = 3\bar{i} + 7\bar{j} - 8\bar{k}$

3. $\bar{E} \cdot \bar{B} =$ ____________________

Correct response to preceding frame

1. $\overline{C} \cdot \overline{B} = 42$
2. $\overline{D} \cdot \overline{B} = -36$
3. $\overline{E} \cdot \overline{B} = 46$

Frame 7-12

Dot Product

Evaluate the following dot products

1. $\overline{A} = -6\overline{j}$
   $\overline{B} = 9\overline{j} + 12\overline{k}$

   $\overline{A} \cdot \overline{B} =$ ________________________________

2. $\overline{C} = 12\overline{i} + 3\overline{j}$
   $\overline{D} = 3\overline{i} - 4\overline{j} + 12\overline{k}$

   $\overline{C} \cdot \overline{D} =$ ________________________________

3. $\overline{A} = 7\overline{i} + 8\overline{j} - 3\overline{k}$
   $\overline{B} = -12\overline{i} + 10\overline{j} - 14\overline{k}$

   $\overline{A} \cdot \overline{B} =$ ________________________________

4. $\overline{P} = a_1\overline{i} + a_2\overline{j} + a_3\overline{k}$
   $\overline{Q} = b_1\overline{i} + b_2\overline{j} + b_3\overline{k}$

   $\overline{P} \cdot \overline{Q} =$ ________________________________

Correct response to preceding frame

1. $\overline{A} \cdot \overline{B} = -54$
2. $\overline{C} \cdot \overline{D} = 24$
3. $\overline{E} \cdot \overline{F} = 38$
4. $\overline{P} \cdot \overline{Q} = a_1b_1 + a_2b_2 + a_3b_3$

Frame 7-13

Review

Complete Page 7-1 in your notebook.

Correct response to preceding frame

$\overline{R} \cdot \overline{S} = 9$

Frame 7-14

Transition

In the earlier frames you learned to multiply vectors by taking the "dot product". Now you will learn to use a second method of vector multiplication -- the "cross product".

The cross product of two vectors is a vector, consequently you will have to learn how to find three things about it:

1. magnitude
2. line of action
3. sense

The next part of this unit will help you learn the rules for finding cross-products. This is a dandy place to take a break. The next section will take about 20 minutes, when you are ready to begin again, go to the next frame.

Correct response to preceding frame

No response

Frame 7-15

Vocabulary

The cross product of two vectors, $\overline{A}$ and $\overline{B}$, is written $\overline{A} \; \overline{x} \; \overline{B}$ and is read "A cross B".

1. Write an expression for the cross product of vectors $\overline{C}$ and $\overline{D}$ and read it aloud.

____

2. Write an expression for the dot product of vectors $\overline{C}$ and $\overline{D}$ and figure out how you would expect it to be read.

____

Correct response to preceding frame

1. $\overline{C} \times \overline{D}$ You should have said "C cross D"
2. $\overline{C} \cdot \overline{D}$ is read "C dot D"

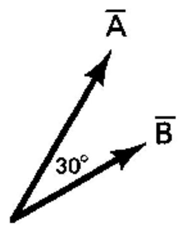
Frame 7-16

Cross-Product – Definition of Magnitude

The cross product of two vectors* A *and* B *is defined as a vector whose magnitude is* $|\overline{A}||\overline{B}| \sin \theta$ *where* θ *is the included angle between the vectors when they are drawn from a common origin.

Find the magnitude of the cross products of the following vectors:

1.



Magnitude of $\overline{A}$ = 12 units
Magnitude of $\overline{B}$ = 5 units

Magnitude of $\overline{A} \times \overline{B}$ = ______________________________

2. $\overline{C} = 6\overline{i}$
$\overline{D} = 12\overline{j}$

Magnitude of $\overline{C} \times \overline{D}$ = ______________________________

3. $\overline{E} = 6\overline{i}$
$\overline{F} = 6\overline{i}$

Magnitude of $\overline{E} \times \overline{F}$ = ______________________________

Correct response to preceding frame

30 units Solution: $|\bar{A}||\bar{B}| \sin\theta = (12)\ (5)\ \sin 30°$

72 units Solution: $\bar{i}$ is perpendicular to $\bar{j}$, therefore $\theta = 90°$

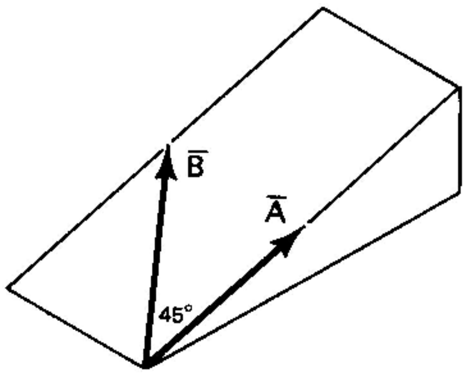
$$|\bar{C}||\bar{D}| \sin\theta = (6)\ (12)\ (1)$$

0 -- Since the vectors are parallel the sine of the angle between them is 0.

Frame 7-17

Cross-Product – Definition of Direction

$\bar{C} = \bar{A} \times \bar{B} = |\bar{A}||\bar{B}| \sin\theta\ \bar{u}$



The unit vector $\bar{u}$ which gives the direction of $\bar{C}$, the cross product of $\bar{A}$ and $\bar{B}$ is perpendicular to the plane containing vectors $\bar{A}$ and $\bar{B}$.

(Look at the line of action for now. The sense comes in the next frame.)

1. $\bar{B}$ is 12 units long and $\bar{A}$ is 10 units long.

   $\bar{A} \times \bar{B}$ is ________________ units long.

   Show the line of action of $\bar{A} \times \bar{B}$ on the sketch.

2. $\bar{C} = 10\bar{k}$ and $\bar{D} = 7\bar{i}$

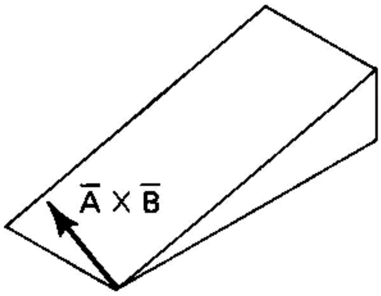
   Magnitude of $\bar{C} \times \bar{D}$ is ____________

   Line of action of $\bar{C} \times \bar{D}$ is parallel to ___________________________________

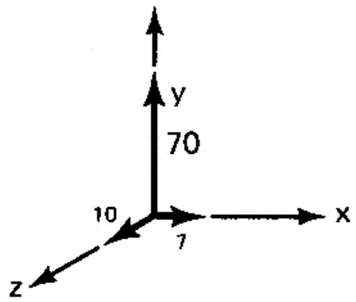
   Sketch $\bar{C}$, $\bar{D}$, and $\bar{C} \times \bar{D}$.

Correct response to preceding frame

1. $\bar{A} \times \bar{B}$ is 84.8 units long Solution: $|\bar{A}|\ |\bar{B}|\ \sin 0 = (12)\ (10)\ (.707)$
   $\bar{A}$ x $\bar{B}$ is normal to the plane containing $\bar{A}$ and $\bar{B}$.



2. $\bar{C}$ x $\bar{D}$ has a magnitude of 70 units and is parallel to the y-axis



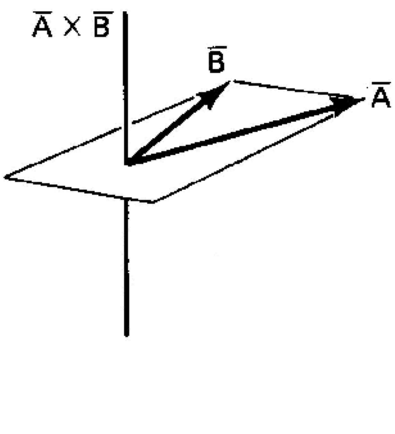
Frame 7-18

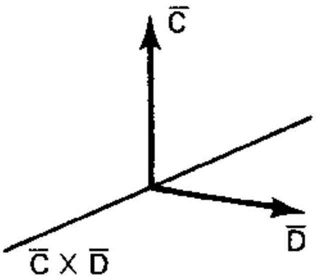
Cross-Product – Definition of Direction

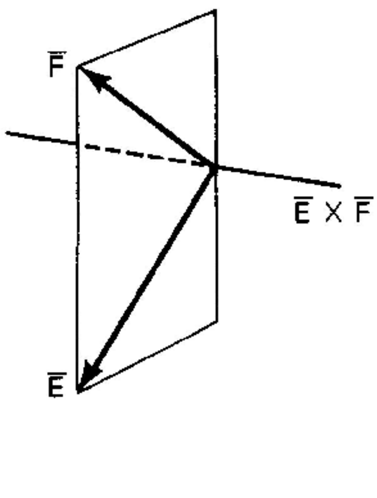
The sense or sign of the cross product is determined by the "right hand rule". This rule tells us that the vector representing $\bar{A}$ x $\bar{B}$ will point in the direction in which a right hand screw will advance if it is turned through the smaller angle from $\bar{A}$ to $\bar{B}$.

Another way of stating it is that if you curl the fingers of your right hand from $\bar{A}$ toward $\bar{B}$ your thumb will point in the direction of $\bar{A}$ x $\bar{B}$.

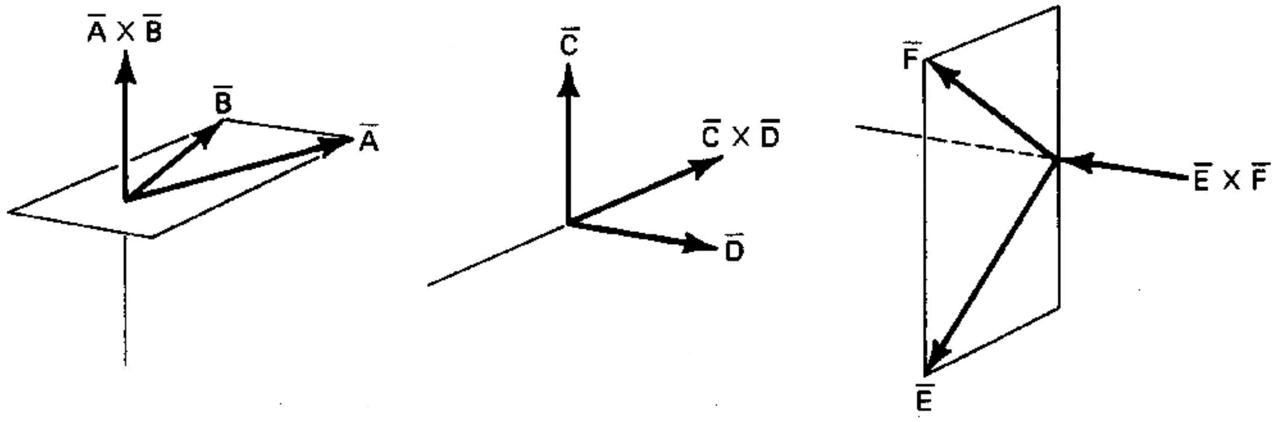
Use either or both of these descriptions to determine the directions of the cross products below. Put arrow heads on in the correct directions.







Correct response to preceding frame

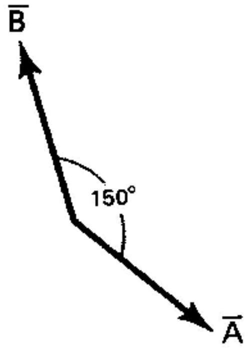


Frame 7-19

Cross-Product

Evaluate the cross-products of the following vectors. Show your results on a sketch.

1.



$\overline{A}$ and $\overline{B}$ are each 10 units long and lie in the plane of the paper.

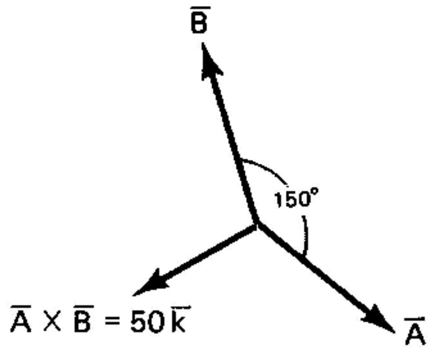
Find $\overline{A} \times \overline{B} =$ ______________________________

2. $\overline{C} = 12\overline{i}$ and $\overline{D} = -10\overline{j}$

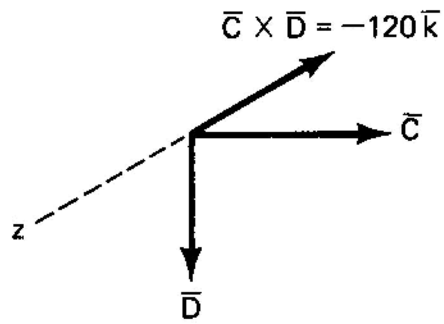
Find $\overline{C} \times \overline{D} =$ ______________________________

Correct response to preceding frame

$\overline{A} \times \overline{B} = 50\overline{k}$

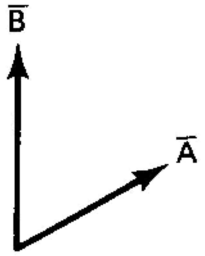


$\overline{C} \times \overline{D} = -120\overline{k}$



Frame 7-20

Cross-Product



Consider vectors $\overline{A}$ and $\overline{B}$ as shown. (You may assume them to be in the plane of the paper if you wish.)

Does $\overline{A} \times \overline{B} = \overline{B} \times \overline{A}$

☐ Yes ☐ No

Why or why not?

____.

Does $\overline{A} \cdot \overline{B} = \overline{B} \cdot \overline{A}$?

☐ Yes ☐ No

Correct response to preceding frame

No
$\bar{A} \times \bar{B} = -(\bar{B} \times \bar{A})$ The magnitudes and the lines of action are the same but the signs are opposite.
Yes $\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$

Frame 7-21

Cross-Product

The general rule for cross products may be written as follows:

$\bar{R} \times \bar{S} = |R|\ |S|\ \sin\theta\ \bar{u}$ where $\bar{u}$ is a unit vector.

1. Underline the portion of the above equation that tells you the magnitude of the cross product of $\bar{R}$ and $\bar{S}$.

2. Circle the portion of the above equation that tells you the direction of the cross product of $\bar{R}$ and $\bar{S}$.

Correct response to preceding frame

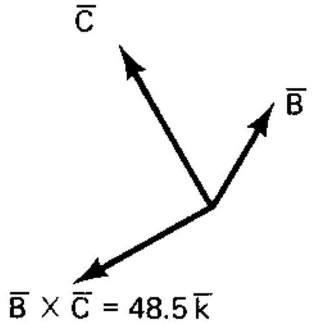
$\bar{R} \times \bar{S} = |\bar{R}|\ |\bar{S}|\ \sin\theta\ \bar{u}$

Frame 7-22

Notebook

Work the first part of Page 7-2 in your Notebook.

Correct response to preceding frame



Frame 7-23

Transition

The cross products you have just been finding deal with vectors whose geometric relationships can be easily drawn. As such drawings become more difficult you must rely on a mathematical solution and a visua1ization in your mind since you will find it very hard to make your pictures look right or convey much meaning.

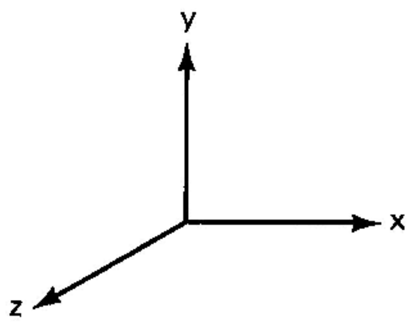
The next section of the unit will be devoted to such problems and will take about 30 minutes. When you feel up to the endeavor, go to the next frame.

Correct response to preceding frame

No response

Frame 7-24

Cross-Products of Unit Vectors



The positive directions of a system of Cartesian coordinates are shown.

Use the rule

$\overline{R} \times \overline{S} = |R|\ |S|\ \sin\theta\ \overline{u}$

to evaluate the following cross-products:

$\overline{i} \times \overline{j} =$ ______________________

$\overline{i} \times \overline{k} =$ ______________________

$\overline{i} \times \overline{i} =$ ______________________

Correct response to preceding frame

$\bar{i} \times \bar{j} = (1)(1)(\sin 90°)\bar{k} = \bar{k}$
$\bar{i} \times \bar{k} = (1)(1)(\sin 90°)-\bar{j} = -\bar{j}$
$\bar{i} \times \bar{i} = (1)(1)(\sin 0°)\bar{u} = 0$

Frame 7-25

Cross-Products

Axes (1, 2, 3) form a right hand system.

$\bar{e}_1$ is a unit vector in direction 1, $\bar{e}_2$ is a unit vector in direction 2 and $\bar{e}_3$ is a unit vector in direction 3.

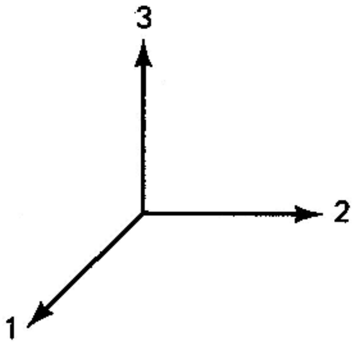
1. Draw the axes

2. $\bar{e}_1 \times \bar{e}_2 =$ ____________________________________

3. If one curls the fingers on one's right hand from the first factor of the cross-product toward the second factor the _______________ will stick out in the direction of the cross-product.

Correct response to preceding frame

1. Some variation obtained by rotation of



2. $\bar{\mathbf{e}}_3$
3. thumb

Frame 7-26

Cross-Products

($\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$) is a right hand system. Which of the following cross products are in the positive direction?

- ☐ (a) $\bar{\mathbf{i}} \times \bar{\mathbf{j}}$
- ☐ (b) $\bar{\mathbf{j}} \times \bar{\mathbf{i}}$
- ☐ (c) $\bar{\mathbf{j}} \times \bar{\mathbf{k}}$
- ☐ (d) $k \times \bar{\mathbf{j}}$
- ☐ (e) $\bar{\mathbf{k}} \times \bar{\mathbf{i}}$
- ☐ (f) $\bar{\mathbf{i}} \times \bar{\mathbf{k}}$

Correct response to preceding frame

a, c, e are positive

Frame 7-27

Cross-Products

1. (α, β, γ) constitute a right hand orthogonal system. Determine the sign of the following cross products. (It may help if you draw the system.)

(a) $\bar{e}_\alpha \times \bar{e}_\beta$ ☐ + ☐ -
(b) $\bar{e}_\alpha \times \bar{e}_\beta$ ☐ + ☐ -
(c) $\bar{e}_\alpha \times \bar{e}_\beta$ ☐ + ☐ -
(d) $\bar{e}_\alpha \times \bar{e}_\beta$ ☐ + ☐ -

2. If one takes a cross product "forward" through the description of a right hand

system, the product will be ______ in sign. If one takes a cross product "backward" it

will be ____.

Correct response to preceding frame

1. (a) +
   (b) -
   (c) -
   (d) +
2. positive
   negative

Frame 7-28

Cross-Products

(R, θ, K) is a right hand orthogonal coordinate system.

1. $\bar{\mathbf{e}}_\theta \times \bar{\mathbf{e}}_R =$ ______________________

2. $\bar{\mathbf{e}}_\theta \times \bar{\mathbf{e}}_K =$ ______________________

Correct response to preceding frame

1. $-\bar{\mathbf{e}}_k$
2. $\bar{\mathbf{e}}_R$

Frame 7-29

Products of Unit Vectors

Find the following products.

1. $\bar{\mathbf{i}} \times \bar{\mathbf{k}} =$ __________
2. $\bar{\mathbf{i}} \cdot \bar{\mathbf{k}} =$ __________
3. $\bar{\mathbf{j}} \times \bar{\mathbf{k}} =$ __________
4. $\bar{\mathbf{k}} \times \bar{\mathbf{j}} =$ __________
5. $\bar{\mathbf{i}} \times \bar{\mathbf{j}} =$ __________
6. $\bar{\mathbf{j}} \cdot \bar{\mathbf{i}} =$ __________
7. $\bar{\mathbf{k}} \cdot \bar{\mathbf{k}} =$ __________
8. $\bar{\mathbf{k}} \times \bar{\mathbf{k}} =$ __________

Correct response to preceding frame

1. $\bar{i} \times \bar{k} = -\bar{j}$
2. $\bar{i} \cdot \bar{k} = 0$
3. $\bar{j} \times \bar{k} = \bar{i}$
4. $\bar{k} \times \bar{j} = -\bar{i}$
5. $\bar{i} \times \bar{j} = \bar{k}$
6. $\bar{j} \cdot \bar{i} = 0$
7. $\bar{k} \cdot \bar{k} = 1$
8. $\bar{k} \times \bar{k} = 0$

Frame 7-30

Dot and Cross-Products

Write C beside the phrases which describe cross products. Write D beside those which describe dot products.

1. ______ vector

2. ______ scalar

3. ______ equals 0 if the vectors are perpendicular

4. ______ equals 0 if the vectors are parallel

Correct response to preceding frame

1. C
2. D
3. D
4. C

Frame 7-31

Review

Complete the next section of Page 7-2 in your notebook.

Correct response to preceding frame

No response

Frame 7-32

Cross-Products

Cross multiplication is distributive with respect to addition, that is,

$$\bar{w} \times (\bar{u} + \bar{v}) = \bar{w} \times \bar{u} + \bar{w} \times \bar{v}$$

Use the general rule

$$\bar{R} \times \bar{S} = |R|\ |S|\ \sin\theta\, \bar{u}$$

to take the following cross products:

1. $6\bar{i} \times 2\bar{j} =$ ________________
2. $6\bar{i} \times 3\bar{k} =$ ________________
3. $6\bar{i} \times 4\bar{i} =$ ________________
4. $6\bar{i} \times (2\bar{j} + 3\bar{k} + 4\bar{i}) =$ ______________________

Correct response to preceding frame

1. $6\bar{i} \times 2\bar{j} = 12\bar{k}$
2. $6\bar{i} \times 3\bar{k} = -18\bar{j}$
3. $6\bar{i} \times 4\bar{i} = 0$
4. 6i } (2j + 3k + 4i) = 12k -18j

Frame 7-33

Cross-Products

Evaluate the following cross products:

1. $3\bar{i}\ \bar{x}\ [12\bar{i} + 4\bar{j} - 10\bar{k}] =$ ____________________

2. $4\bar{j}\ \bar{x}\ [12\bar{i} + 4\bar{j} - 10\bar{k}] =$ ____________________

3. $[3\bar{i} + 4\bar{j}]\ \bar{x}\ [12\bar{i} + 4\bar{j} - 10\bar{k}] =$ ______________________________

Correct response to preceding frame

1. $3\bar{i} \times [12\bar{i} + 4\bar{j} - 10\bar{k}] = 12\bar{k} + 30\bar{j}$
2. $4\bar{j} \times [12\bar{i} + 4\bar{j} - 10\bar{k}] = -48\bar{k} - 40\bar{i}$
3. $[3\bar{i} + 4\bar{j}] \times [12\bar{i} + 4\bar{j} - 10\bar{k}] = -40\bar{i} + 30\bar{j} - 36\bar{k}$

Frame 7-34

Cross-Product

Evaluate the following cross product:

$$(3\bar{i} + 4\bar{j} - 5\bar{k}) \times (7\bar{i} - 12\bar{j} + 6\bar{k})$$

Correct response to preceding frame

$-36\bar{i} - 53\bar{j} - 64\bar{k}$

Frame 7-35

Cross-Product

Did you think the last frame was fun? (No response)

You may be interested to learn that there is a more efficient way to do such problems.

You'll find it discussed on Page 7-3 of your notebook. Read it now.

Correct response to preceding frame

No response

Frame 7-36

Cross-Product by Determinants

Refer to your notebook as necessary.

$$\bar{A} = 3\bar{i} - 4\bar{j} - 7\bar{k}$$

$$\bar{B} = -2\bar{i} - 6\bar{j} - \bar{k}$$

Form the determinants for the following cross-products:

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ & & \\ & & \end{vmatrix}$$

$$\bar{B} \times \bar{A} = \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}$$

Correct response to preceding frame

$$\overline{A} \times \overline{B} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ +3 & -4 & -7 \\ -2 & -6 & -1 \end{vmatrix}$$

$$\overline{B} \times \overline{A} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -2 & -6 & -1 \\ +3 & -4 & -7 \end{vmatrix}$$

Frame 7-37

Cross-Product by Determinants

Most "scientific" pocket calculators have ways of entering and solving determinants.

Since calculators vary, we will not deal here with the details. Consult your calculator manual for instructions if you plan to take that route.

Go to the next frame.

Correct response to preceding frame

No response

Frame 7-38

Cross-Product by Determinants

Using your favorite method, complete the solution of the following determin-

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ +3 & -4 & -7 \\ -2 & -6 & -1 \end{vmatrix}$$

$$= \bar{i}(4 - 42) + \bar{j}(14 - \quad) + \bar{k}(\qquad) = \underline{\qquad\qquad}$$

$$\bar{B} \times \bar{A} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -2 & 6 & -1 \\ +3 & -4 & -2 \end{vmatrix}$$

$$= \bar{i}(\qquad) + \bar{j}(\qquad) + \bar{k}(\qquad) = \underline{\qquad\qquad}$$

Correct response to preceding frame

$$\bar{A} \times \bar{B} = \bar{i}(4 - 42) + \bar{j}(14 + 3) + \bar{k}(-18 - 8)$$
$$= -38\bar{i} + 17\bar{j} - 26\bar{k}$$
$$\bar{B} \times \bar{A} = \bar{i}(-12 - 4) + \bar{j}(-3 - 4) + \bar{k}(8 - 18)$$
$$= -16\bar{i} - 7\bar{j} - 10\bar{k}$$

Frame 7-39

Cross-Product

Solve the following problem:

$$\bar{A} = 12\bar{i} - 5\bar{j} + 3\bar{k}$$

$$\bar{B} = 9\bar{i} + 10\bar{j} + 2\bar{k}$$

$$\bar{C} = 6\bar{i} + 8\bar{j} + 3\bar{k}$$

$$(\bar{A} - \bar{B}) \times \bar{C} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Correct response to preceding frame

$(\overline{A} - \overline{B}) \times \overline{C} = -53\overline{i} - 3\overline{j} + 114\overline{k}$

Solution:

$\overline{A} - \overline{B} = 3\overline{i} - 15\overline{j} + \overline{k}$

$$(\overline{A} - \overline{B}) \times \overline{C} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 3 & -15 & 1 \\ 6 & 8 & 3 \end{vmatrix}$$

$$= \overline{i}(-45 - 8) + \overline{j}(6 - 9) + \overline{k}(24 + 90)$$

$$= -53\overline{i} - 3\overline{j} + 114\overline{k}$$

Frame 7-40

Cross-Product

Complete Page 7-4 of your notebook.

Correct response to preceding frame

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 7 & 4 & 6 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= -2\bar{i} - 16\bar{j} + 13\bar{k}$$

$$\bar{A} \cdot \bar{B} = 14 + 12 + 24$$

$$= 50$$

$$\bar{B} \times \bar{A} = -(\bar{A} \times \bar{B})$$

$$= 2\bar{i} + 16\bar{j} - 13\bar{k}$$

$$\bar{B} \cdot \bar{A} = \bar{A} \cdot \bar{B} = 50$$

Frame 7-41

Closure

That completes the work of this unit.

You learned to compute the product of a scalar and a vector and to compute the dot product of two vectors. You have also learned to compute the cross products of two vectors and to interpret it as a vector, giving its magnitude, direction and sense.

At the moment you probably regard this as a purely mathematical operation of problematical value. Future units will show it to be a tool for solving problems which deal with rotational tendencies of bodies.