Introduction to Statics

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Unit 6

Equilibrium of a Particle: Concurrent Force Systems

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Unit 6
Equilibrium of a Particle: Concurrent Force Systems

Frame 6-1

Introduction

The preceding units have been devoted to helping you become familiar with the concepts of "vector" and "force as a vector". By now you should be quite competent at writing vectors as sums and as products, adding and subtracting vectors, finding resultants of vectors and breaking vectors into components.

This unit will allow you to apply your skills to the solution of some engineering problems dealing with equilibrium.

Go to the next frame.

Correct response to preceding frame

No response

Frame 6-2

Equilibrium

Read Page 6-1 of your notebook before answering the following question.

When a body is in equilibrium, which of the following statements apply?

- ☐ 1. The body has no acceleration.
- ☐ 2. The body has no motion.
- ☐ 3. The motion of the body is constant.
- ☐ 4. The resultant force on the body is zero.
- ☐ 5. The body is at rest or moving with a constant speed in a constant direction.

Correct response to preceding frame

All apply except statement 2. The body may be at rest but it is not necessarily so. Statement 3 applies since rest is a constant motion, just as zero is a number.

Frame 6-3

Equilibrium

Newton's First Law states that when a body is in equilibrium it will

____

____ .

Correct response to preceding frame

be at rest or move with a constant velocity (or equivalent response)

Frame 6-4

Equilibrium

Which of the following is not in equilibrium?

1. A car traveling along a straight road at 30 mph.

2. A stopped car.

3. A car coasting to a stop. (The driver is neither accelerating or braking.)

Case _________ is not in equilibrium because ______________________________

____

Correct response to preceding frame

Case 3 is not in equilibrium because if the car is coming to a stop its speed is changing. (or equivalent response)

Frame 6-5

Equilibrium

If the resultant force on a particle is zero the particle is in ______________________ .

Correct response to preceding frame

equilibrium

Frame 6-6

Equilibrium

The statement that a particle is in equilibrium implies that the following vector equation applies to the particle.

$\overline{R}$ = ______________________________

Correct response to preceding frame

$\bar{R} = 0$

Frame 6-7

Equilibrium

State Newton's First Law in your own words.

____

____ .

Correct response to preceding frame

Check your statement against the notebook. Did you say essentially the same thing?

Frame 6-8

Equilibrium

If a particle acted upon by several forces, $\overline{F}_1$, $\overline{F}_2$, etc., is in equilibrium, can we make the following statement?

$$\sum \overline{F} = 0$$

☐ Yes ☐ No

Why or why not? ____

____

Correct response to preceding frame

Yes. $\sum \overline{F} = 0$ since $\overline{R} = \sum \overline{F}$ and equilibrium implies $\overline{R} = 0$. (or equivalent response)

Frame 6-9

Equilibrium

If we make the statement $\sum \overline{F} = 0$ about a particle we know that the particle has

- ☐ zero velocity
- ☐ zero acceleration
- ☐ both

Correct response to preceding frame

We know that the particle has zero acceleration. (It may also have zero velocity, or it may not.)

Frame 6-10

Equilibrium

State the mathematical equation that implies equilibrium of a particle.

____________ = __________

Correct response to preceding frame

$\sum \overline{F} = 0$ or $\overline{R} = 0$

Frame 6-11

Transition

This unit is titled "Equilibrium of a Particle". So far we have talked quite a bit about "equilibrium" but not at all about "a particle." The next several frames will, therefore, be devoted to the definition of a particle. You will learn what bodies you can and cannot treat as particles and what "treating a body as a particle" implies about the forces which act on it.

To explore these exciting notions will require about ten minutes. To begin, you have only to go to the next frame.

Correct response to preceding frame

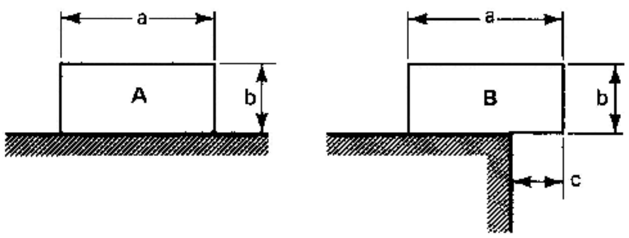
No response

Frame 6-12

Definition of a Particle

When we use the word "particle" in this unit we do not imply a body of very small size. "Particles" may, indeed, be very large. Instead we refer to those bodies whose size and shape does not affect static equilibrium.

Which of the boxes shown below can be treated as a particle?



☐ A

☐ B

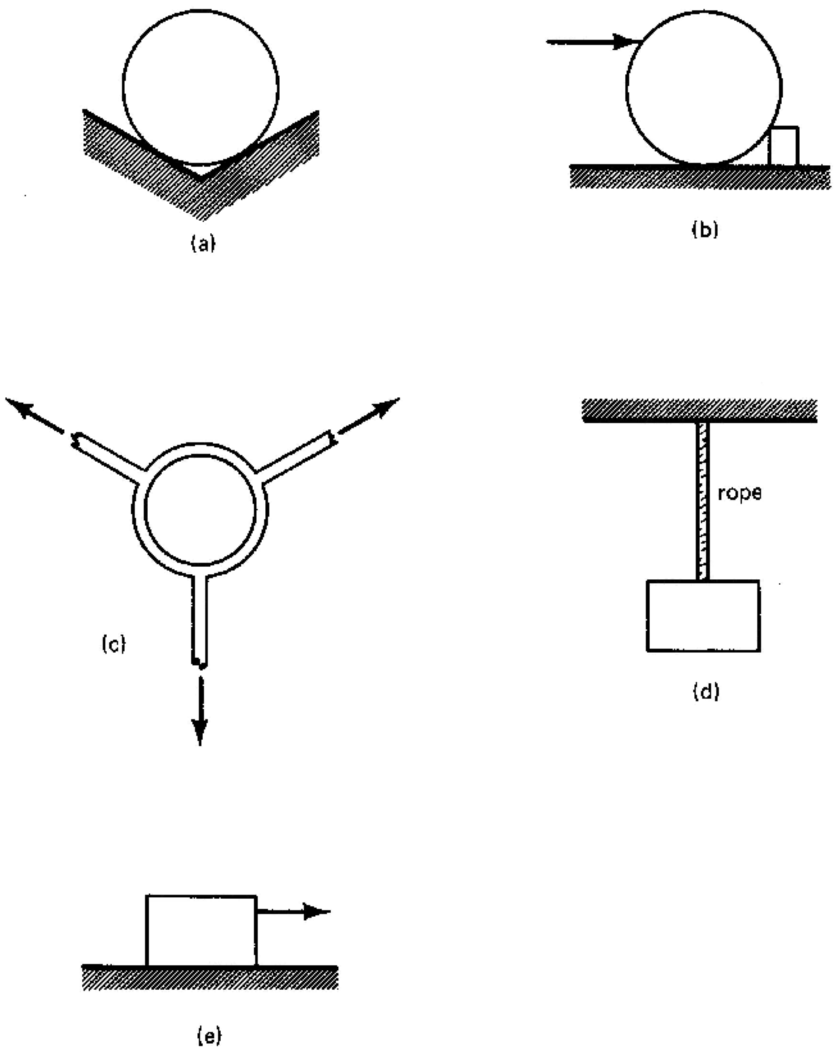
Correct response to preceding frame

Body A can be treated as a particle since its equilibrium is independent of its dimensions. Body B, however, will tip if c is too large compared to a. We must know something about its dimensions to determine its equilibrium and therefore cannot treat it as a particle.

Frame 6-13

Definition of a Particle

Which of the following bodies can be treated as a particle?



Correct response to preceding frame

Bodies a, c and d can be treated as particles. Bodies b and e are not particles since it is necessary to know some dimensions to determine whether or not each body is in equilibrium.

Frame 6-14

Review

The three characteristics of a force are its magnitude, direction and __________

____ .

Correct response to preceding frame

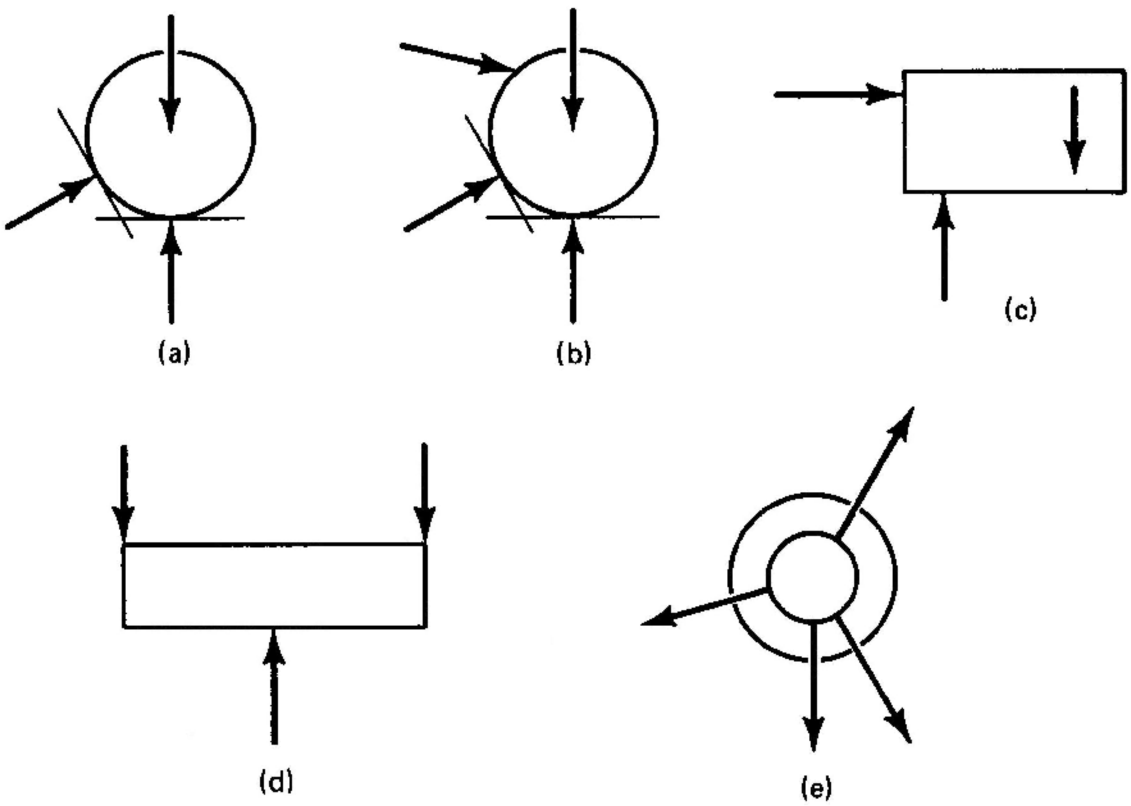
line of action or ***point of application***

Frame 6-15

Definition of a Particle

If the lines of action of all the forces acting on a rigid body intersect at a single point, the body may be considered a particle no matter what its size and shape.

Extend the lines of action of the forces shown and determine which bodies are particles.



Correct response to preceding frame

Bodies a and e are particles.

Frame 6-16

Definition of a Particle

A body which is subject to forces whose lines of action intersect at a single point is said to be subject to a concurrent force system.

May any rigid body subject to a concurrent force system be treated as a particle?

☐ Yes ☐ No

Correct response to preceding frame

Yes. Any body subject to a concurrent force system may be treated as a particle.

Frame 6-17

Review

If the lines of action of all forces in a system intersect at a point, that system is

______________________ .

Correct response to preceding frame

concurrent

Frame 6-18

Review

Complete the next section of your notebook.

Correct response to preceding frame

If you feel it necessary to check your notebook answers you will find relevant frames beginning with 6-11.

Frame 6-19

Transition

Now that we have defined "equilibrium" and "particle" we are ready at last to work some problems dealing with the equilibrium of particles.

First we shall try some examples of coplanar, or two-dimensional systems.

The next section may take you a half hour or so, if you need a break, take it now. Otherwise go to the next frame.

Correct response to preceding frame

No response

Frame 6-20

Review

1. When a particle is in equilibrium we know that all the forces on it must add up to __________ .

2. Write the above statement as an equation. ______________________________

3. Force systems acting on particles are always ______________________ .

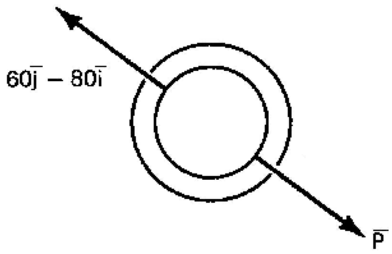
Correct response to preceding frame

1. zero
2. $\sum \overline{F} = 0$ or $\overline{R} = 0$
3. concurrent

Frame 6-21

Two Forces--Systems

For the particle shown, determine $\overline{P}$ by writing $\sum \overline{F} = 0$, and solving for $\overline{P}$.



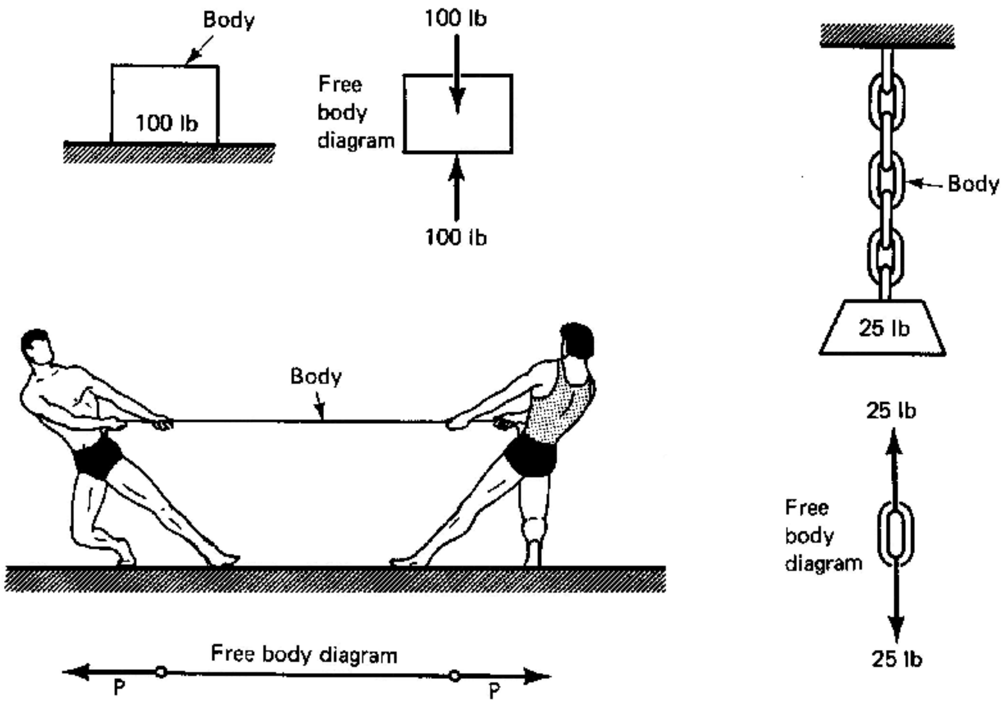
Correct response to preceding frame

$$\sum \bar{F} = \bar{P} + (60\bar{j} - 80\bar{i}) = 0$$
$$\bar{P} = 80\bar{i} - 60\bar{j}$$

Frame 6-22

Two Forces--Systems

The particles shown are all in equilibrium.



Looking at the free bodies we see that in all cases the forces acting on the body have

_____________ magnitude(s), _____________ sense(s) and ____________ line(s) of action.

Correct response to preceding frame

The forces have ***the same*** magnitude, ***opposite*** senses, and ***the same*** line of action.

Frame 6-23

Two Forces--Systems

1. Collinear means "along the same line". A two force system acting on a particle must be concurrent. Must it also be collinear?

☐ Yes ☐ No

2. Are all concurrent systems collinear?

☐ Yes ☐ No

3. May any rigid body acted upon by a collinear force system be treated as a particle?

☐ Yes ☐ No

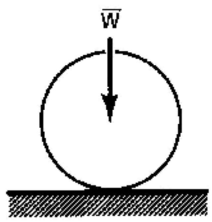
Correct response to preceding frame

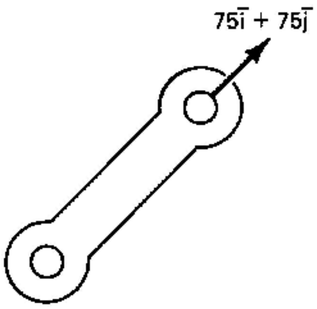
1. Yes
2. No
3. Yes

Frame 6-24

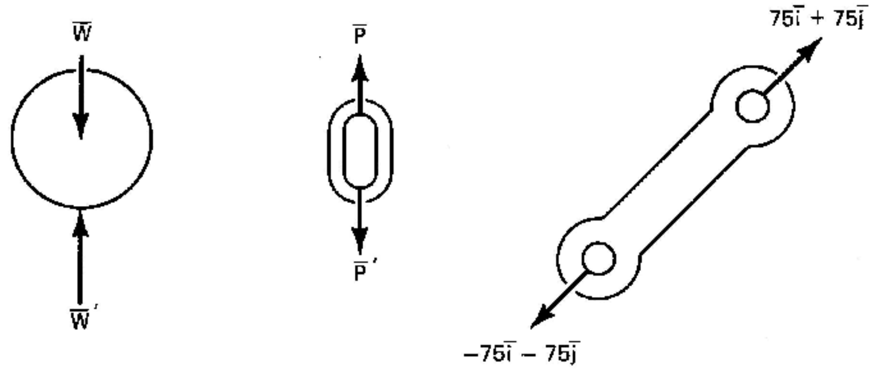
Two Force Systems

Each of the particles shown is in equilibrium, but only one of the forces acting on it is shown. Put on the other force and give its magnitude.





Correct response to preceding frame

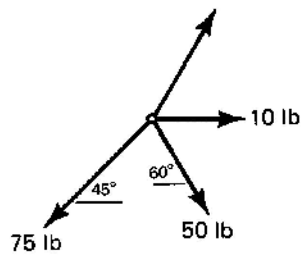


Frame 6-25

Equilibrium of Particles

When a particle is subjected to a coplanar force system the equation $\overline{R} = 0$ will give you two unknown quantities. They may be either magnitudes or directions.

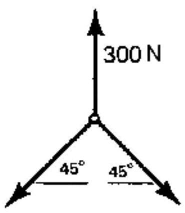
Determine what is unknown in the free bodies below. Both systems are in equilibrium.



How many magnitudes are unknown? ___________

How many directions? _______________

Can the problem be solved by $\overline{R} = 0$?



How many magnitudes are unknown? ___________

How many directions? _______________

Can the problem be solved by $\overline{R} = 0$?

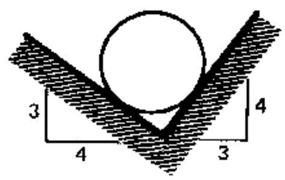
Correct response to preceding frame

1 unknown magnitude
1 unknown direction
problem can be solved

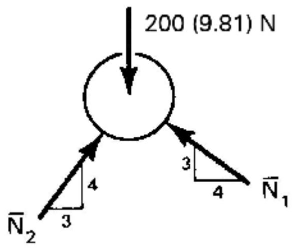
2 unknown magnitudes
0 unknown directions
problem can be solved

Frame 6-26

Equilibrium of a Particle



The cylindrical tank weighs 200 kilograms and is supported in a cradle as show. The free body shows the forces acting on it.



Writing a vector expression for each force gives the following:

$\bar{W} = -200\ (9.81)\ \bar{j}$ Newtons

$\bar{N}_1 - = N_1(-.8\bar{i} + .6\bar{j}) = -.8N_1\bar{i} + .6N_1\bar{j}$

$\bar{N}_2 - = N_2(.6\bar{i} + .8\bar{j}) = .6N_2\bar{i} + .8N_2\bar{j}$

Since the body is at rest

$$\bar{F} = 0 = \bar{W} + \bar{N}_1 + \bar{N}_2$$

$$-1962\bar{j} - .8N_1\bar{i} + .6N_1\bar{j} + .6N_2\bar{i} + .8N_2\bar{j} = 0$$

Take coefficients of $\bar{i}$ to form one equation

$$-.8N_1 + .6N_2 = 0$$

Take coefficients of $\bar{j}$ to form a second equation

$$-1962 + .6N_1 + .8N_2 = 0$$

Solve the equations and get values for N_1 and N_2.

N_1 = __________ N_2 = __________

Correct response to preceding frame

N_1 = 1,180 Newtons

N_2 = 1,570 N

<u>Solution</u>:

$-.8N_1 + .6N_2 = 0$

$N_1 = \frac{3}{4} N_2$

$-1,962 + .6(.75N_2) + .8N_2 = 0$

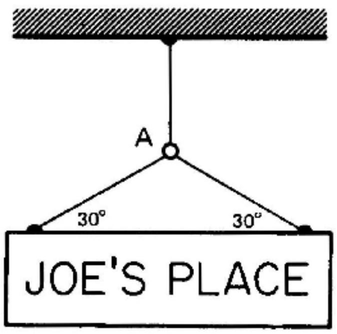
$-1,962 + .45N_2 + .8N_2 = 0$

$N_2 = \frac{1,962}{1.25} = 1,569.6$

$N_1 = \frac{3}{4}(1,569.6) = 1,177.2$

Frame 6-27

Equilibrium of Particles



The sign shown weighs 50 pounds.

First treat the entire thing as a particle.

From the free body shown in (a),

$T = 50$ lb

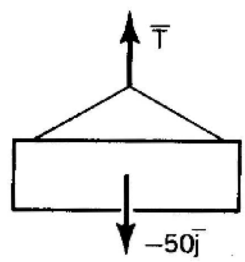
From the free body of the joint at A shown in (b), we can write the forces as vectors, thus:

$\bar{T} = 50\bar{j}$

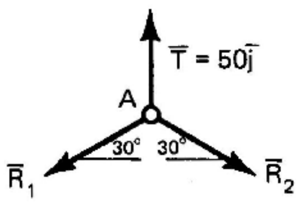
$\bar{R}_1 = R_1(-.866\bar{i} - .5\bar{j}) = -.866R_1\bar{i} - .5R_1\bar{j}$

$\bar{R}_2 = R_2(+.866\bar{i} - .5\bar{j}) = +.866R_2\bar{i} - .5R_2\bar{j}$

(a)



(b)



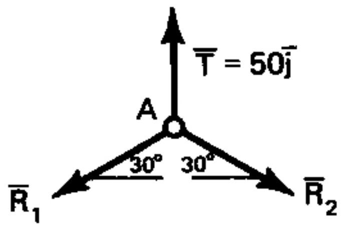
Write $\sum \bar{F} = 0 = \bar{T} + \bar{R}_1 + \bar{R}_2$ and solve for R_1 and R_2.

R_1 = ______________________ R_2 = ______________________

Correct response to preceding frame

$50 = R_1 = R_2$

Solution:



$$50\bar{j} - .866R_1\bar{i} - .5R_1\bar{j} + .866R_2\bar{i} - .5R_2\bar{j} = 0$$

taking coefficients of $\bar{i}$

$$-.866R_1 + .866R_2 = 0$$

Therefore $R_1 = R_2$

taking coefficients of $\bar{j}$

$$50 - .5R_1 - .5R_2 = 0$$

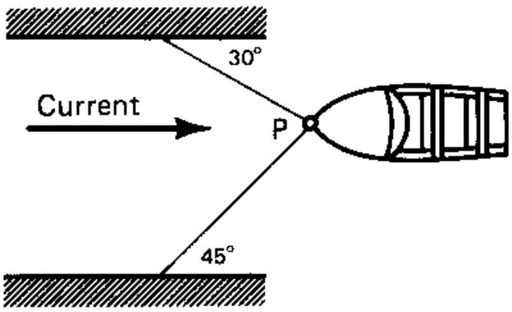
$$50 - .5R_1 - .5R_1 = 0$$

$$50 = R_1 = R_2$$

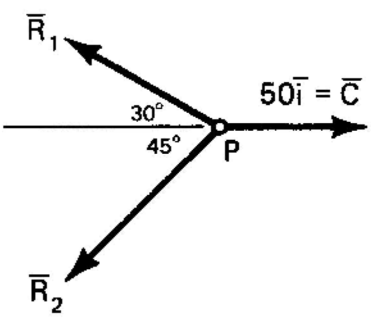
Frame 6-28

Equilibrium of Particles

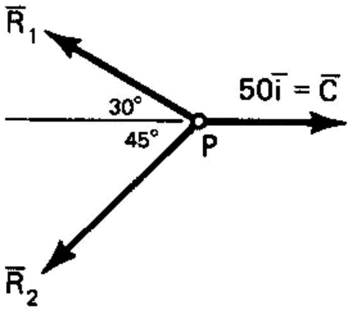
A boat is moored by two ropes as shown.



The current exerts a force of 50 Newtons against it, directed downstream. Using the free body shown of a particle at P, write each force as a vector and by taking -f= 0 solve for the magnitudes of the forces in the ropes.



Correct response to preceding frame



$R_1 = 36.6$ N

$R_2 = 25.9$ N

Solution:

$\bar{C} = 50\bar{i}$

$\bar{R}_1 = R_1(-.866\bar{i} + .5\bar{j})$

$\bar{R}_2 = R_2(-.707\bar{i} - .707\bar{j})$

$50\bar{i} - .866R_1\bar{i} + .5R_1\bar{j} - .707R_2\bar{i} - .707R_2\bar{j} = 0$

Taking $\bar{j}$ coefficients

$.5R_1 - .707R_2 = 0$

$R_2 = \frac{R_1}{1.414}$

Taking $\bar{i}$ coefficients

$50 - .866R_1 - .707R_2 = 0$

Substituting for R_2

$50 - .866R_1 - .5R_1 = 0$

$R_1 = \frac{50}{1.366} = 36.6$ N

$R_2 = \frac{36.8}{1.414} = 25.9$ N

Frame 6-29

Equilibrium of Particles

The steps in solving a problem involving the equilibrium of a particle are as follows:

1. Write all forces as vectors

2. Set $\sum \bar{F} = 0$

3. ______________________________

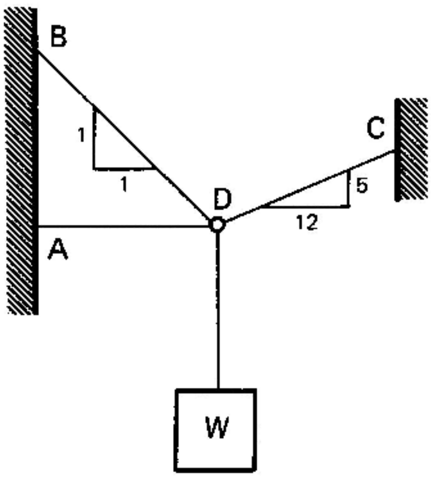
4. ______________________________

Correct response to preceding frame

1. Write all forces as vectors
2. Set $\sum \overline{F} = 0$
3. Break the vector equation into scalar equations.
4. Solve
(Your answer should be equivalent to the one given but could hardly be expected to be the same.)

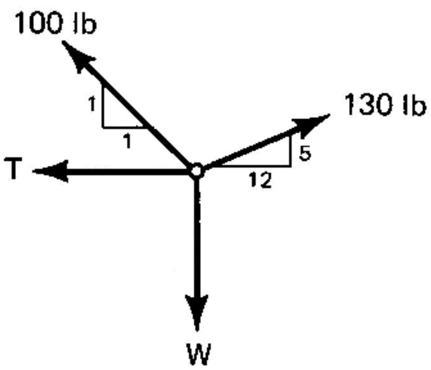
Frame 6-30

Equilibrium of Particles



The tension in BD is 100 pounds The tension in DC is 130 pounds The free body of a particle at point D is shown.

Find the weight, W, and the tension in AD.



Correct response to preceding frame

```
T = 49.3 lb
W = 120.7 lb
```

Frame 6-31

Equilibrium of a Particle

Do example 6-1 in your notebook

Correct response to preceding frame

You will be given a method for checking your answer in later frames. Please be patient.

Frame 6-32

Transition

The next topic in this unit deals with the equilibrium of a particle acted on by a three-dimensional force system. Such a problem can be solved by the analytical steps you have already learned. Unfortunately the algebra can get a trifle sticky. You should try to keep a firm grasp on your basic tools of algebra and arithmetic.

Only five frames to the next transition but they may take a while. Take a deep breath and dive in.

Correct response to preceding frame

No response

Frame 6-33

Review

The steps you took in the analytical solution of equilibrium of particles were as follows:

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

Correct response to preceding frame

1. Write each force as a vector
2. Set $\sum \overline{F} = 0$
3. Break the vector equation into simultaneous equations.
4. Solve

Frame 6-34

Review

When you wrote a vector equation for a two-dimensional equilibrium problem, how many equations could you get by equating the coefficients of each unit vector to zero?

____

How many unknowns could you solve for? ___________________

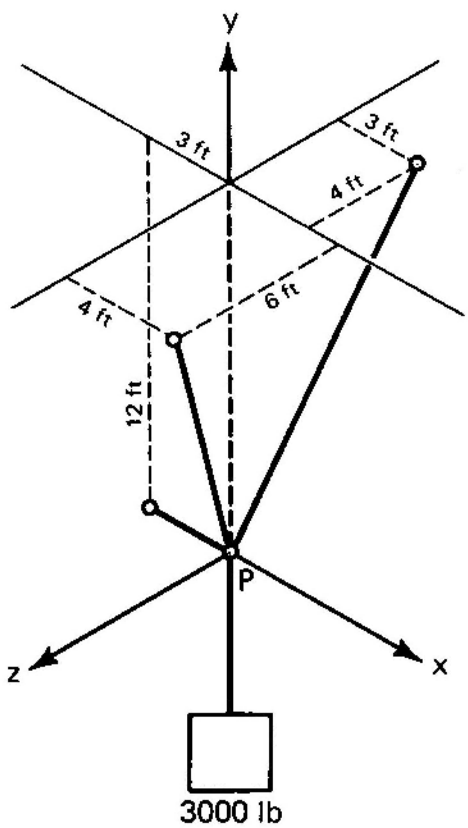
Correct response to preceding frame

two equations
two unknowns

Frame 6-35

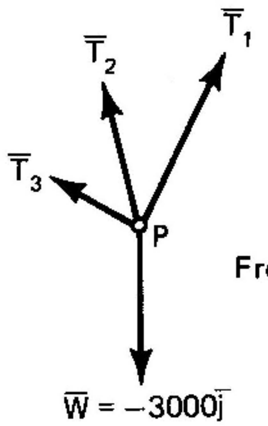
Equilibrium of a Particle in Space

In problems involving a three-dimensional concurrent force system you will use exactly the same system you used for two-dimensional systems but you will get three coefficient equations and can thus get three unknowns.



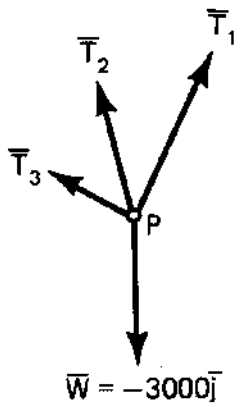
The weight is supported by three cables as shown. Two are attached to the ceiling. One is attached to the wall and is horizontal.

Find the tension in each cable.



Free Body Diagram of P

Correct response to preceding frame



$T_1 = 1950$

$T_2 = 1400$

$T_3 = 850$

Solution:

$\overline{W} = -3000\overline{j}$

$\overline{T}_1 = T_1\left(\dfrac{3\overline{i} + 12\overline{j} - 4\overline{k}}{13}\right)$

$\overline{T}_2 = T_2\left(\dfrac{4\overline{i} + 12\overline{j} + 6\overline{k}}{14}\right)$

$\overline{T}_3 = -T_3(\overline{i})$

$$0 = -3000j + \frac{3}{13}T_1\overline{i} + \frac{12}{13}T_1\overline{j} - \frac{4}{13}T_1\overline{k} + \frac{4}{14}T_2\overline{i} + \frac{12}{14}T_2\overline{j}$$

$$+ \frac{6}{14}T_2\overline{k} - T_3\overline{i}$$

Coefficient equations

$$0 = \frac{3}{13}T_1 + \frac{4}{14}T_2 - T_3$$

$$3000 = \frac{12}{13}T_1 + \frac{12}{14}T_2$$

$$0 = -\frac{4}{13}T_1 + \frac{6}{14}T_2$$

Solve simultaneous equations

$T_1 = 1950$

$T_2 = 1400$

$T_3 = 850$

Frame 6-36

Equilibrium of a Particle in Space

Solve example 6-2 in your notebook, then summarize the required steps in the solution in the space provided in the notebook.

Correct response to preceding frame

```
T₁ = 179 lb
T₂ = 148 lb
T₃ = 297 lb
Steps are listed in Frame 6-32.
```

Frame 6-37

Transition

You are now able--given time enough and sufficient patience and accuracy--to solve any problem in the equilibrium of a particle by means of vector algebra.

When you are dealing with two-dimensional problems, it is also possible to solve them graphically or trigonometrically. Since it is a good idea to check a problem by doing it by a different method, the next few frames will help you learn a graphical attack.

Unfortunately when the problem is three-dimensional you are stuck with the analytical approach--unless, of course, you want to construct a model!

It’s only nine more frames to the end of the unit and you’ll be done in about fifteen minutes. Proceed to the next frame.

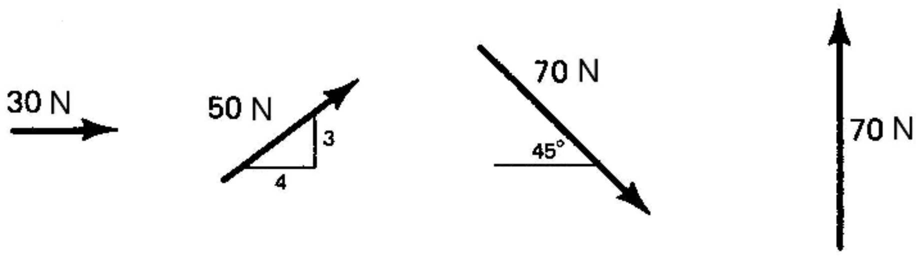
Correct response to preceding frame

No response

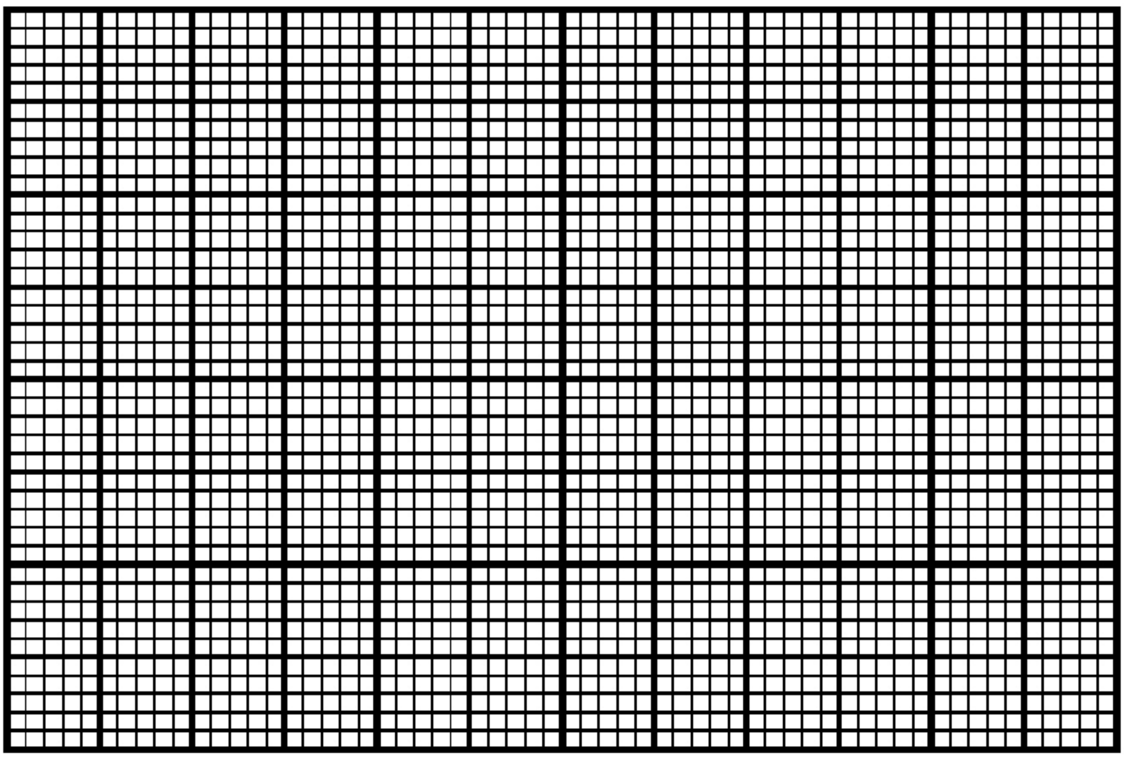
Frame 6-38

Review

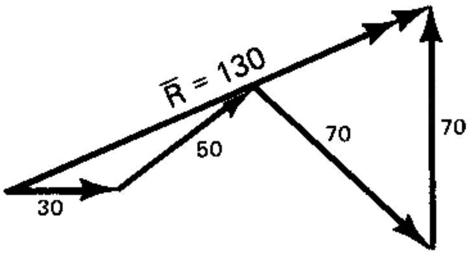
Find the resultant of the four vectors shown by adding them graphically. (The vectors are not drawn to scale.)



A sketch will, do, but be as accurate as your eyeball allows.



Correct response to preceding frame



(or equivalent response)

It does not matter in which order you choose to add the vectors. The result would be the same.

Frame 6-39

Static Equilibrium – Graphical Method

When a particle is in equilibrium the resultant of all the forces on it is zero. This means that if you solved for the resultant by graphical means, the diagram you constructed of the forces would begin and end at

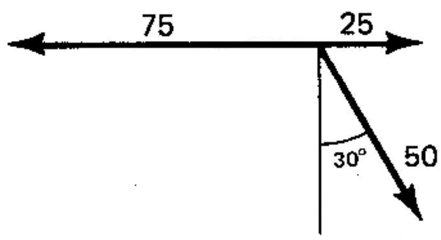
- ☐ the origin
- ☐ the same point
- ☐ different points

Correct response to preceding frame

The force diagram would begin and end at the same point. It might, or might not, be the origin of a given coordinate system.

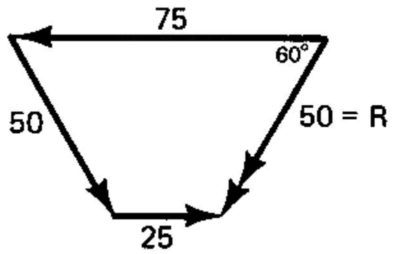
Frame 6-40

Force Diagram



When forces acting on a body are added by laying them out nose to tail, the resulting diagram is called a "force diagram". Draw a force diagram for the system shown and find the resultant of the forces.

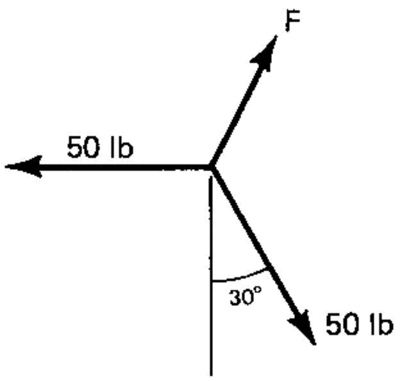
Correct response to preceding frame



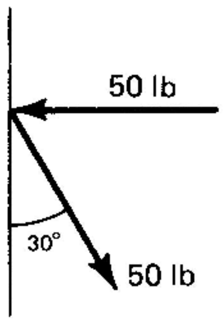
Frame 6-41

Static Equilibrium – Graphical Method

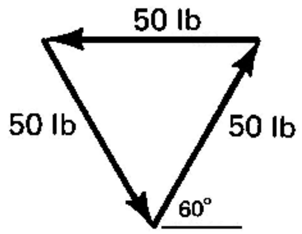
When a particle is in static equilibrium its force diagram must close.



The particle shown is in static equilibrium. Complete the force diagram and estimate (or measure) the magnitude of the unknown force and the angle it makes with the horizontal.

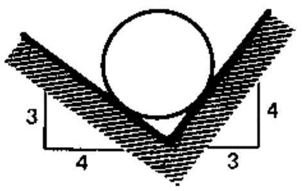


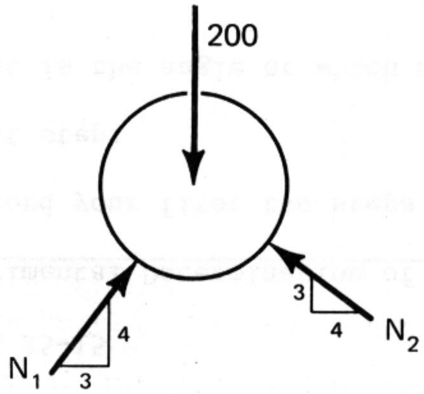
Correct response to preceding frame



Frame 6-42

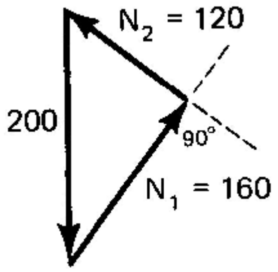
Equilibrium – Graphical Method





The cylindrical tank weighs 200 lb and its free body is shown. Sketch its force diagram and determine the unknownforces from it. Remember, the diagram must close.

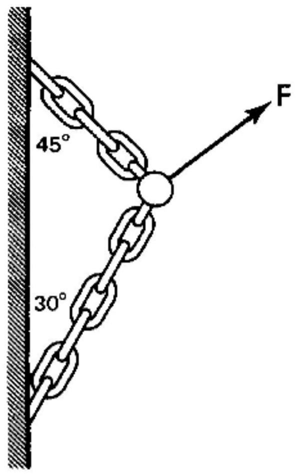
Correct response to preceding frame

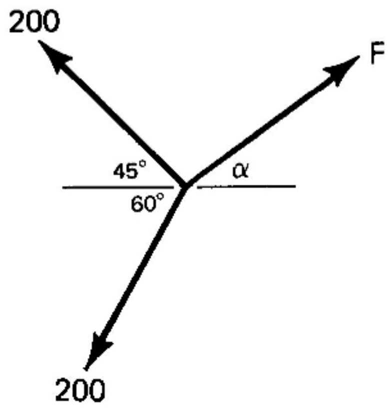


Frame 6-43

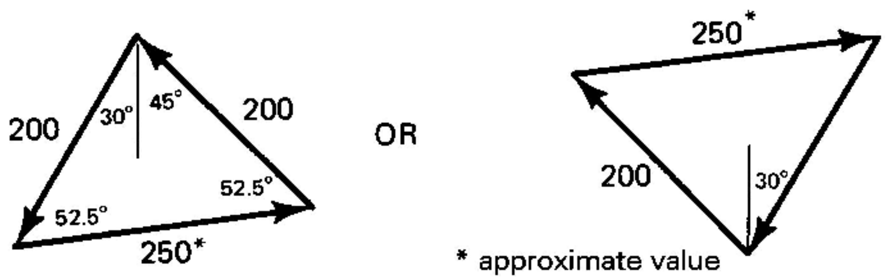
Equilibrium — Graphic Method

Determine the force $\overline{F}$ if the tension in each chain is 200 Newtons. Draw a force diagram and solve it graphically or trigonometrically as you prefer.





Correct response to preceding frame

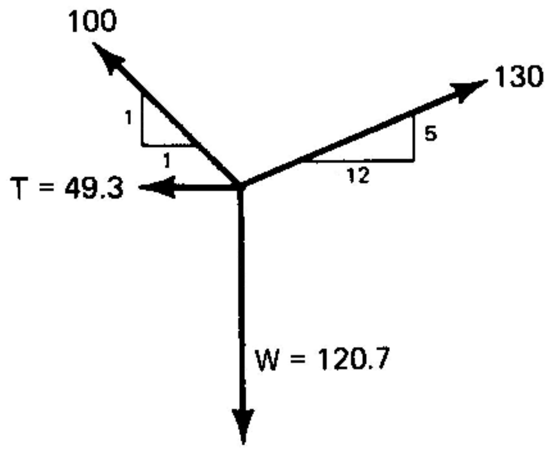


Frame 6-44

Equilibrium – Graphical Method

When many forces act on a single particle, a graphical solution may be more trouble than it is worth. However, in such a case it may prove useful to draw a force diagram which includes the forces you have found analytically. If the diagram closes your work is correct.

Here is the solution to Frame 6-30. Check it graphically.



Correct response to preceding frame

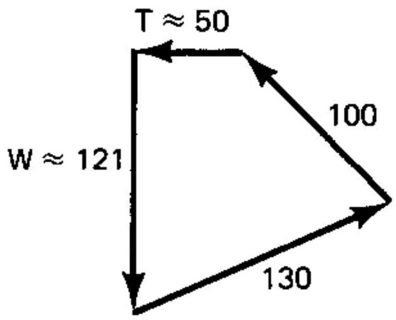


Diagram closes
T & W are correct

Frame 6-45

Equilibrium – Graphical Method

Check your answer to example 6-1 by means of a force diagram. Then turn to the next frame.

Correct response to preceding frame

Correct your analytical solution and/or your graphical check until you are convinced you are right.

Frame 6-46

Closure

In the unit just finished you have learned or reviewed Newton's First Law of motion, learned to identify "particles" in the sense of the unit, and seen thatforces acting on a particle in equilibrium form a concurrent force system. You have learned your first important "Statics Formula"

$$\sum \overline{F} = 0 \text{ or } \overline{R} = 0$$

and learned to apply it to some engineering problems. You have used an algebraic method of solution for two- and three-dimensional problems and a graphical method for problems in two dimensions.

Think a minute and be sure you have mastered all these points, then rest a bit. You've earned it!