Introduction to Statics

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Unit 5
Components of Forces

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Unit 5
Components of Forces

Frame 5-1

Introduction

The preceding units have shown a certain pre-occupation with combining vectors to form new or "resultant" vectors. This unit will reverse the process and will deal with taking vectors apart into "component" vectors. This something which may be done with any sort of vector quantity, including vectors as mathematical abstracts.

In particular you will learn to break a vector into its rectangular components and into specified non-rectangular components. This is a skill which you will eventually use to solve many sorts of problems. We will do a few graphical solutions, so keep your ruler handy.

Since this is a mechanics course, in most of the problems we will apply the concepts and techniques to the vector quantity, force.

The first section of the unit will deal with rectangular components. When you are ready, go to the next frame.

Correct response to preceding frame

No response

Frame 5-2

Components

Read the first paragraph on Page 5-1 of your notebook.

Consider the vector,

$$\bar{R} = 7\bar{i} + 12\bar{j} + 3\bar{k}$$

$7\bar{i}$ is the component in the direction of the x-axis.

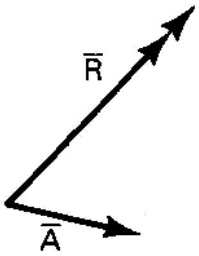
What is the z component of $\bar{R}$? _____

Correct response to preceding frame

$3\bar{k}$

Frame 5-3

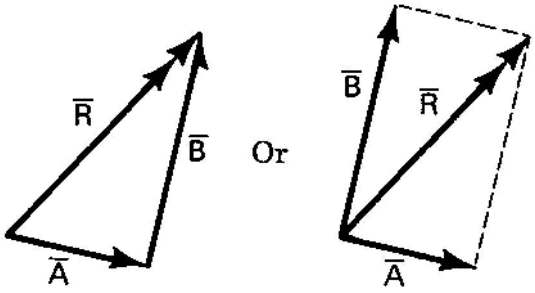
Components



$\bar{R}$ is the resultant of two components, $\bar{A}$ and $\bar{B}$. Draw $\bar{B}$.

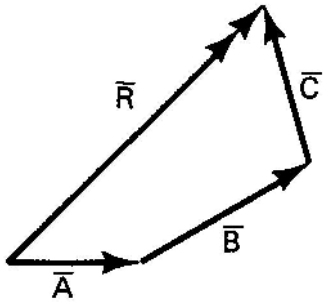
(When showing components and resultants it is a good idea to put two arrowheads on a resultant, one on a component.)

Correct response to preceding frame



Frame 5-4

Components



1. Are $\overline{A}$, $\overline{B}$ and $\overline{C}$ all components of $\overline{R}$?

☐ Yes ☐ No

2. What is the largest number of components a single force can have? _____

3. What is the smallest number of components a single force can have?

_____

Correct response to preceding frame

1. Yes
2. There is no upper limit on the number of components any vector may have.
3. Two components are the smallest number necessary, if one rejects the argument that the force is its own component.

Frame 5-5

Rectangular Components

Write expressions for the horizontal and vertical components of the vector

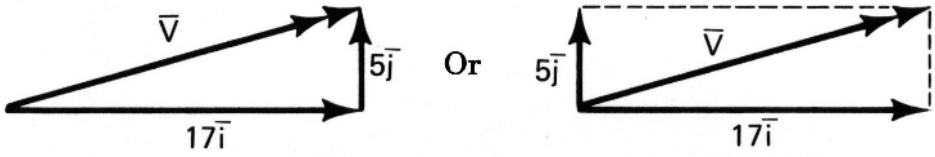
$$\bar{V} = 17\bar{i} + 5\bar{j}$$

1. Horizontal Component _____

2. Vertical Component _____

3. Show $\bar{V}$ and its rectangular components on a sketch.

Correct response to preceding frame

1. $17\bar{i}$
2. $5\bar{j}$
3. 

Frame 5-6

Rectangular Components

Components are often indicated by subscripts.

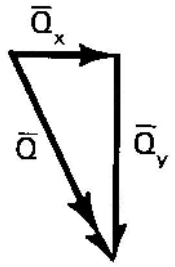
$\bar{Q} = 3\bar{i} - 6\bar{j}$

$\bar{Q}_x =$ _____

Draw a sketch of $\bar{Q}$ and its components in the x and y directions.

Correct response to preceding frame

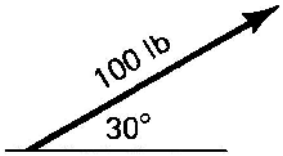
$\bar{Q}_x = 3\bar{i}$



(Or Equivalent Response)

Frame 5-7

Rectangular Components



The horizontal component of the force shown is _____

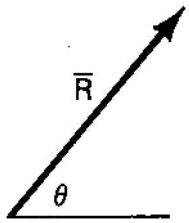
The vertical component is _____

Correct response to preceding frame

$\bar{F}_x = 86.6\bar{i}$ lb
$\bar{F}_y = 50.0\bar{j}$ lb

Frame 5-8

Rectangular Components

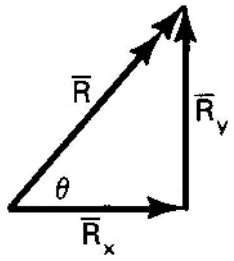


Draw the horizontal and vertical components of $\bar{R}$ on the sketch.

$\bar{R}_x = R$ _____ $\bar{i}$

$\bar{R}_y =$ _____

Correct response to preceding frame

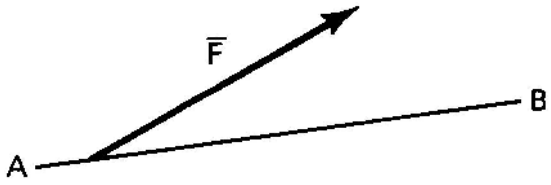


$\overline{R}_x = R \cos \theta \, \overline{i}$

$\overline{R}_y = R \sin \theta \, \overline{j}$

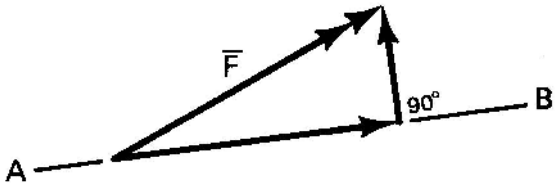
Frame 5-9

Rectangular Components



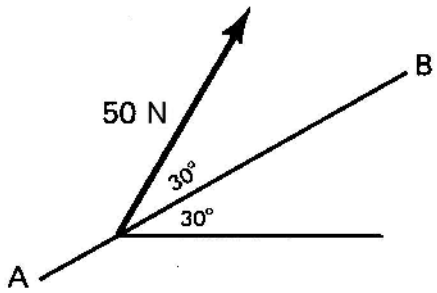
Draw rectangular components for $\overline{F}$ such that one component is along line AB.

Correct response to preceding frame



Frame 5-10

Rectangular Components



1. Sketch rectangular components for the 50 N force so that one component acts along the line AB.

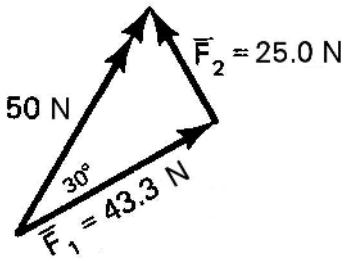
Find the magnitude of each component and show the magnitudes on the sketch.

2. Sketch rectangular components for the 50 N force, making one component vertical. Show their magnitudes on your sketch.

3. How many pairs of rectangular components may a single force have?

Correct response to preceding frame

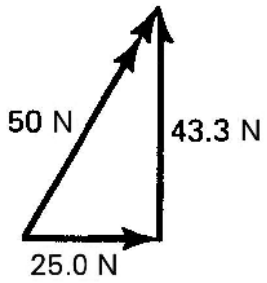
1.



$F_1 = 50 \cos 30° = 43.3$

$F_2 = 50 \sin 30° = 25.0$

2.



$F_x = 50 \cos 60° = 25.0$

$F_y = 50 \sin 60° = 43.3$

3. A single force may have an infinite number of pairs of rectangular components.

Frame 5-11

Transition

You should now have a fair idea of the basic ground rules for breaking forces into rectangular components.

The next several frames will afford you a bit more practice in doing so, and will show you how to compute components by proportion.

Go to the next frame.

Correct response to preceding frame

No response

Frame 5-12

Rectangular Components by Proportion

Given a 260 N force

$$\overline{P} = 260\left(\frac{5\overline{i} - 12\overline{j}}{13}\right)$$

$\overline{P}_x$ = _____

$\overline{P}_y$ = _____

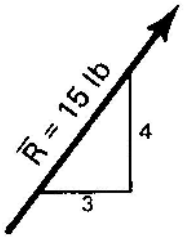
Correct response to preceding frame

$\bar{P}_x = 100\bar{i}$

$\bar{P}_y = -240\bar{j}$

Frame 5-13

Rectangular Components by Proportion



1. Write $\bar{R}$ as the product of a magnitude and a unit vector.

   $\bar{R}$ = _____

2. $\bar{R}_x$ = _____

   $\bar{R}_y$ = _____

3. By proportion $\frac{|\bar{R}|}{5} = \frac{|\bar{R}_x|}{3} = \frac{\quad}{4}$

Correct response to preceding frame

$$\bar{R} = 15\left(\frac{3\bar{i} + 4\bar{j}}{\sqrt{3^2 + 4^2}}\right) = 15\left(\frac{3\bar{i} + 4\bar{j}}{5}\right)$$

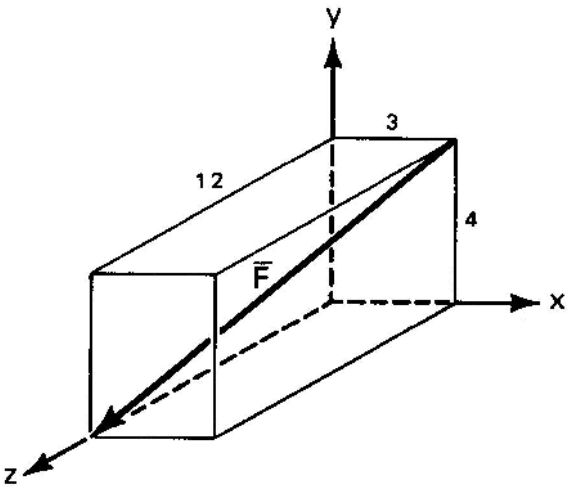
$$\bar{R}_x = 9\bar{i} \text{ lb}$$

$$\bar{R}_y = 12\bar{j} \text{ lb}$$

$$\frac{|\bar{R}|}{5} = \frac{|\bar{R}_x|}{3} = \frac{|\bar{R}_y|}{4}$$

Frame 5-14

Rectangular Components



Write $\bar{F}$ as a product of a magnitude and a unit vector. Call its magnitude F.

$\bar{F}$ = _____

Write the components of the vector along the x, y, and z axes.

$\bar{F}_x$ = _____

$\bar{F}_y$ = _____

$\bar{F}_z$ = _____

Correct response to preceding frame

$$\overline{F} = F\left(\frac{-3\overline{i} - 4\overline{j} + 12\overline{k}}{\sqrt{9 + 16 + 144}}\right) = \frac{F(-3\overline{i} - 4\overline{j} + 12\overline{k})}{13}$$

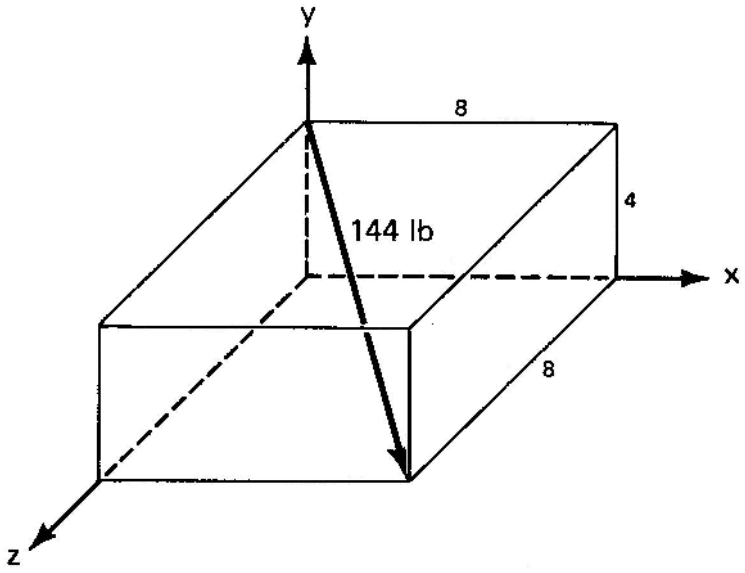
$$\overline{F}_x = -\frac{3}{13} F\overline{i}$$

$$\overline{F}_y = -\frac{4}{13} F\overline{j}$$

$$\overline{F}_z = \frac{12}{13} F\overline{k}$$

Frame 5-15

Rectangular Components



Find the components of the force shown above which are parallel to the coordinate axes.

Correct response to preceding frame

$\overline{R}_x = 96\overline{i}$
$\overline{R}_y = -48\overline{j}$
$\overline{R}_z = 96\overline{k}$

Frame 5-16

Rectangular Components

Do problems 5-1 and 5-2 in your notebook.

Correct response to preceding frame

Example 5-1

$\overline{F}_x = 120\overline{i}$

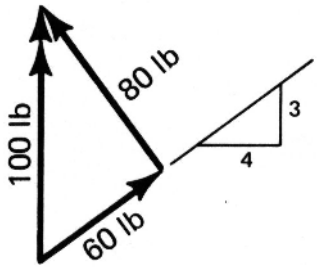
$\overline{F}_y = -60\overline{j}$

$\overline{F}_z = 40\overline{k}$

$$F = 140\left(\frac{6\overline{i} - 3\overline{j} + 2\overline{k}}{\sqrt{6^2 + 3^2 + 2^2}}\right)$$

$$= 140\left(\frac{6i - 3j + 2k}{7}\right)$$

Example 5-2



Frame 5-17

Transition

You have mastered the concept of rectangular components -- very probably without difficulty. Now we will look at components which are not at right angles.

There are, of course, an infinite number of pairs of components for any vector, but we will limit our discussion to the cases most commonly encountered in engineering problems. The cases we will consider are as follows:

1. The magnitude and direction of one component are given, but nothing is known about the other.
2. The directions of both components are given, but neither magnitude is known.

The next transition is 8 frames ahead.

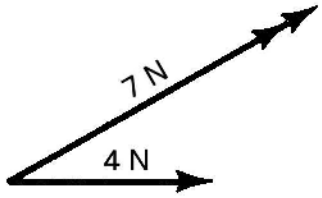
When you are ready, go to the next frame.

Correct response to preceding frame

No response

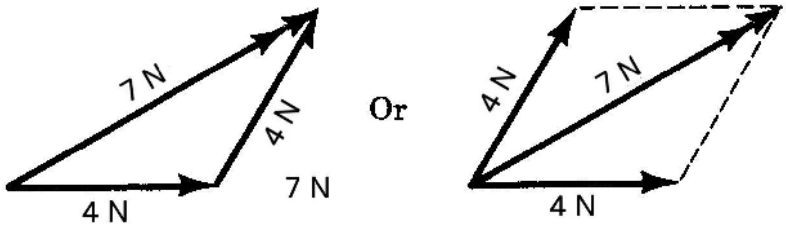
Frame 5-18

Oblique Components by Graphical Method



The 4 N force is one component of the 7 N force. Draw the other component and measure to find its magnitude.

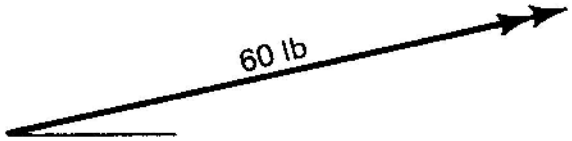
Correct response to preceding frame



The force may be broken into components by either the method of triangles or the method of parallelograms, but since most people seem to prefer the triangle, future answers will be given in that form.

Frame 5-19

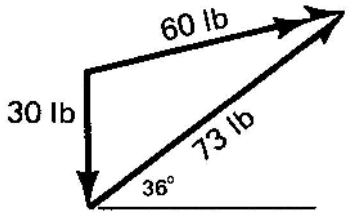
Oblique Components by Graphical Method



The force shown is to be broken into components, one of which is 30 lb downward.

Determine the other component by graphical means. Find its magnitude and the angle it makes with the horizontal.

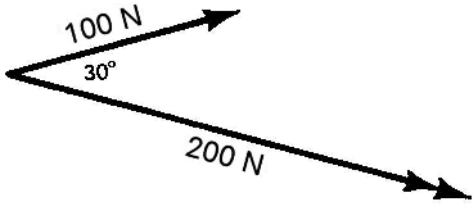
Correct response to preceding frame



Note that the 73 lb component is larger than the 60 lb resultant. This is not unusual with non-rectangular components.

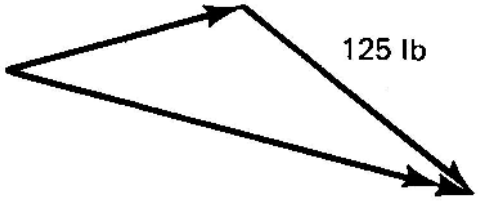
Frame 5-20

Oblique Components--Graphical



The 100 N force is one component of the 200 N force shown. Determine the other component.

Correct response to preceding frame



Frame 5-21

Oblique Components by Graphical Method

When one is given a scale drawing of any problem in vectors, a graphical solution is apt to be as good as any, provided one is carrying his drafting equipment. If, by some ill fortune, one isn't, a trigonometric solution can be used as follows:

1. Sketch the resultant and the known component.

2. Sketch the unknown component by either the triangle or parallelogram method.

3. Solve by whatever trigonometric strategy appeals to you.

Go to the next frame.

Correct response to preceding frame

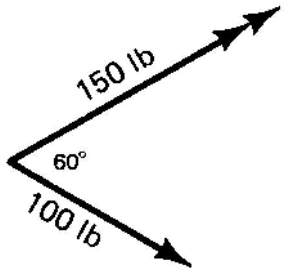
No response

Frame 5-22

Oblique Components by Trigonometric Method

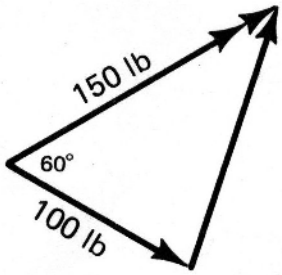
The steps in the solution of the following problem are:

1. Sketch the resultant and its known component
2. Sketch the unknown component by the method of triangles or parallelograms
3. Solve by trigonometry



Find the component which, when added to the 100 lb shown, will result in the 150 lb force.

Correct response to preceding frame



C = 133 lb

by the law of cosines

$C^2 = (100)^2 + (150)^2 - 2(100)(150)\cos 60°$

$C = \sqrt{17500}$

Frame 5-23

Review

Complete the next section of your notebook.

Correct response to preceding frame

No response

Frame 5-24

Transition

You have been concerned with oblique components of a vector when one component is completely specified. You have learned to solve such problems by means of graphical and trigonometric methods.

Two-dimensional problems in which the directions of both components are specified can be solved by methods similar to those you have just learned.

The next section will deal with such problems and will take about 20 minutes. When you are ready, begin.

Correct response to preceding frame

No response

Frame 5-25

Review

Which of the following did you use more often?

- ☐ construction of a triangle
- ☐ construction of a parallelogram

In your trigonometric solutions which of the following did you use more often?

- ☐ Law of Sines
- ☐ Law of Cosines

In the preceding section you solved problems dealing with oblique components. What was given you?

- ☐ magnitude and direction of one component
- ☐ direction of both components

Correct response to preceding frame

Though there is no right or wrong to it, most people solve problems where one component is completely known by means of a triangle.

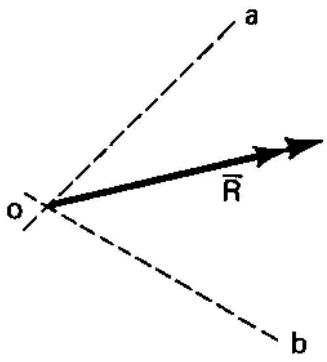
The givens in such a problem make the law of cosines the practically unanimous choice.

One component was always given completely.

Frame 5-26

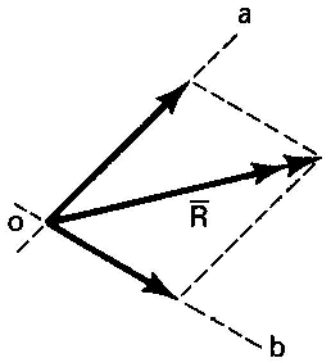
Oblique Components--Graphical

When the directions of two components are specified it is easy to use a parallelogram construction to solve for their magnitudes.



On the sketch show two components of $\overline{R}$ such that one has a line of action along oa , the other along ob .

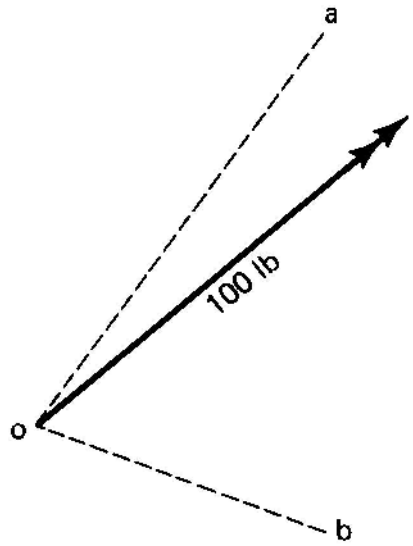
Correct response to preceding frame



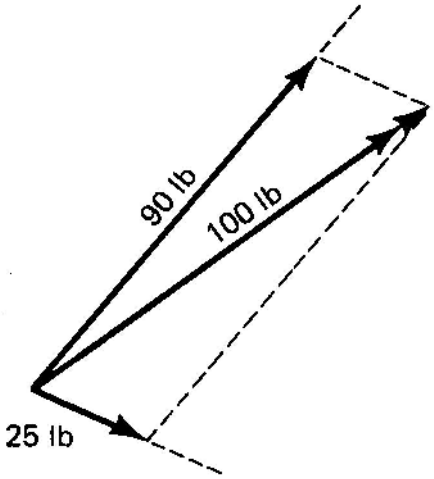
Frame 5-27

Oblique Components

The 100 lb force shown has components along oa and ob . Determine their magnitudes by drawing and measuring.



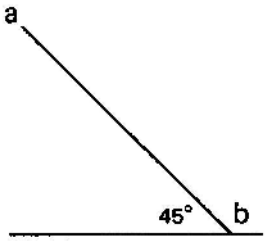
Correct response to preceding frame

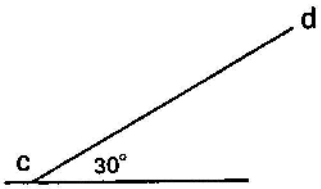


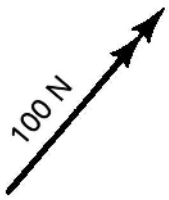
Frame 5-28

Oblique Components -- Graphical

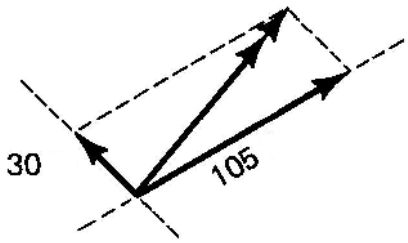
Divide the force shown into components parallel to lines ab and cd and find their magnitudes.







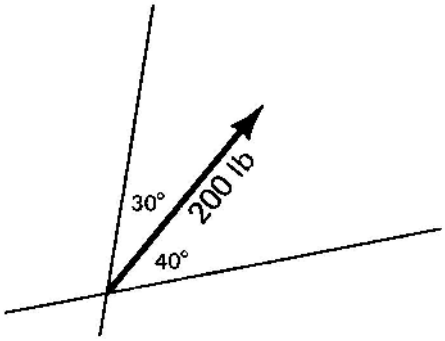
Correct response to preceding frame



Frame 5-29

Oblique Components -- Trigonometric

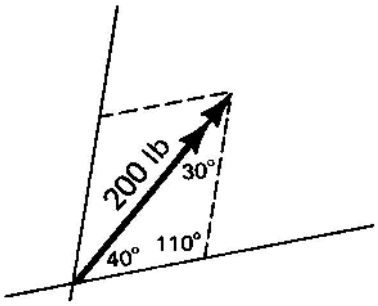
It is often more convenient to solve a problem trigonometrically than to work from a scale drawing. In such a case the figure should be sketched, showing the resultant and the known directions of the components on a parallelogram. The magnitudes may then be determined by means of trigonometry.



Complete the parallelogram to find the components along the lines shown and find the magnitude of all angles. Decide which of the following would yield the easier solution.

- ☐ law of sines
- ☐ law of cosines

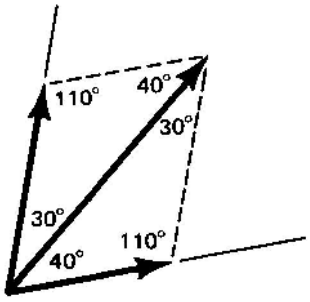
Correct response to preceding frame



The law of sines will give the more direct solution.

Frame 5-30

Oblique Components



Find the components of the 200 lb force. Remember that parallel sides of a parallelogram are equal.

Correct response to preceding frame

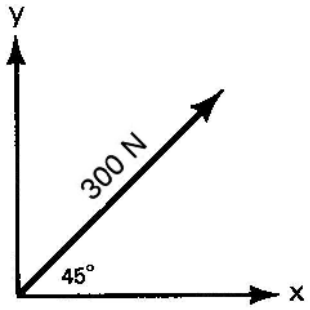
$R_1 = 107$ lb Solution:

$$\frac{R_1}{\sin 30°} = \frac{R_2}{\sin 40°} = \frac{200}{\sin 110°}$$

$R_2 = 137$ lb

Frame 5-31

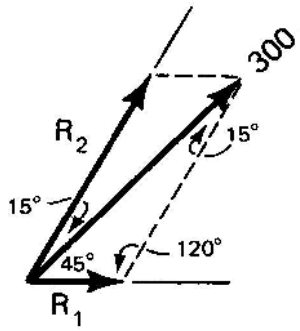
Oblique Components--Trigonometric



The 300 N force shown is to be divided into two components. One component is to be horizontal, the other is to make an angle of 60° with the positive x-axis. Find the components by means of a trigonometric solution.

Correct response to preceding frame

$R_1 = 90.0$
$R_2 = 245$
Solution:



$$\frac{R_1}{\sin 15°} = \frac{R_2}{\sin 45°} = \frac{300}{\sin 120°}$$

Frame 5-32

Oblique Components

Beginning with problem 5-4, complete the next section of your notebook.

Correct response to preceding frame

No response

Frame 5-33

Transition

The earlier parts of this unit have been devoted to teaching you graphical and trigonometric methods for breaking forces into oblique components. However, in most three dimensional problems and in many two dimensional ones the simplest method for solving such problems involves vector algebra.

The remainder of this unit will be devoted to algebraic solutions of problems involving oblique components. It will probably take you about 30 minutes to complete. When you are ready to settle down to it, go to the next frame.

Correct response to preceding frame

No response

Frame 5-34

Oblique Components -- Algebraic

The simplest algebraic solution results from the case where the force and one component are completely known. In that case simple subtraction will supply the missing component. Consider a force, $\overline{R}$.

$$\overline{R} = 7\overline{i} + 14\overline{j} + 3\overline{k}$$ Newtons

We wish to break it into two components, $\overline{C}_1$ and $\overline{C}_2$,

$$\overline{C}_1 = 5\overline{i} + 5\overline{j}$$

What is the other component?

$$\overline{C}_2 = \overline{R} - \overline{C}_1$$

$\overline{C}_2 =$ _____

Correct response to preceding frame

$\overline{C}_2 = 2\overline{i} + 9\overline{j} + 3\overline{k}$

Frame 5-35

Oblique Components -- Algebraic

$\overline{F}$ represents a force.

$\overline{F} = 8\overline{i} + 12\overline{j} - 5\overline{k}$ pounds

Break it into components so that one component, $\overline{C}_1$, is given by the following expression.

$\overline{C}_1 = 12\overline{i} - 6\overline{j} - 3\overline{k}$

$\overline{C}_2 =$ _____

Correct response to preceding frame

$\overline{C}_2 = -4\overline{i} + 18\overline{j} - 2\overline{k}$

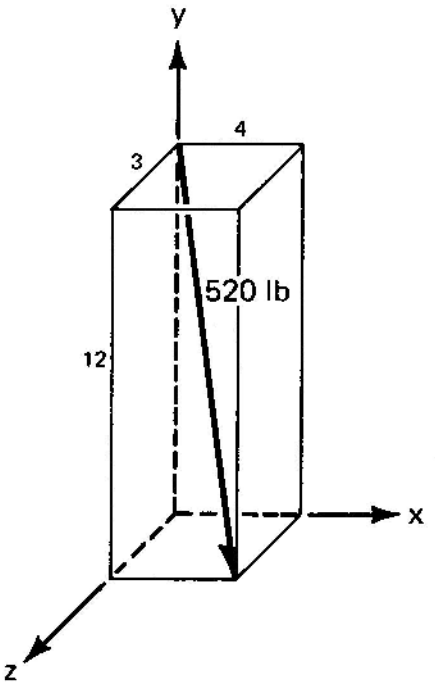
Solution:

$\overline{C}_2 = \overline{F} - \overline{C}_1 = (8\overline{i} + 12\overline{j} - 5\overline{k}) - (12\overline{i} - 6\overline{j} - 3\overline{k})$

Frame 5-36

Oblique Components -- Algebraic

The method of algebraic subtraction is so easy that it is often well worth the trouble of writing a force in vector form just to be able to use it.



The 520 lb force acts as shown. It is to be broken into components such that one component is $100\overline{i} + 100\overline{j}$. Write the force in vector form and find the second component by subtraction.

$\overline{F} =$ _____

$\overline{C}_2 =$ _____

Correct response to preceding frame

$$\overline{F} = 520\left(\frac{4\overline{i} - 12\overline{j} + 3\overline{k}}{13}\right)$$

$$\overline{C}_2 = 60\overline{i} - 580\overline{j} + 120\overline{k}$$

Solution:

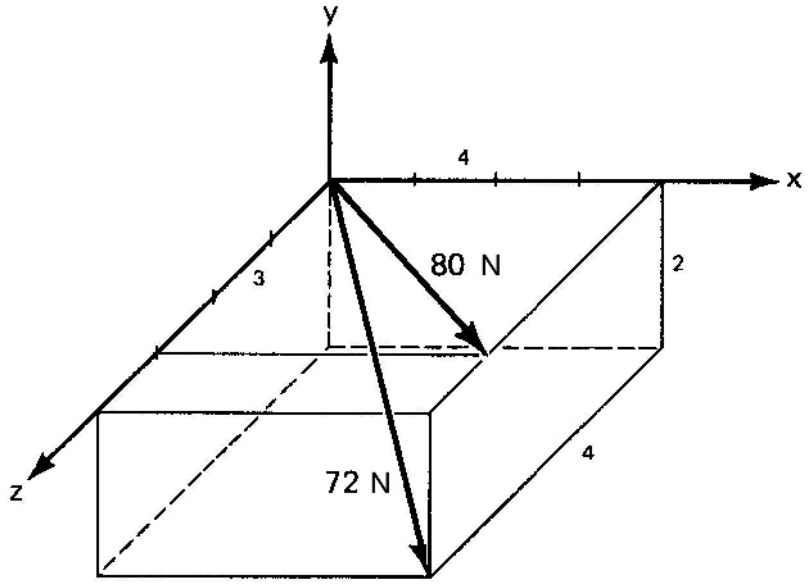
$$\overline{F} = 160\overline{i} - 480\overline{j} + 120\overline{k}$$

$$\overline{C}_2 = \overline{F} - \overline{C}_1$$

$$= 160\overline{i} - 480\overline{j} + 120\overline{k} - 100\overline{i} - 100\overline{j}$$

Frame 5-37

Oblique Components -- Algebraic



The 80 N force shown has one component equal to 72 N with a line of action as shown.

Find the other component by subtraction.

$\overline{F}$ = _____

$\overline{C}_1$ = _____

$\overline{C}_2$ = _____

Correct response to preceding frame

$$\overline{F} = 80\left(\frac{4\overline{i} + 3\overline{k}}{5}\right)$$

$$= 64\overline{i} + 48\overline{k}$$

$$\overline{C}_1 = 72\left(\frac{4\overline{i} - 2\overline{j} + 4\overline{k}}{6}\right)$$

$$= 48\overline{i} - 24\overline{j} + 48\overline{k}$$

$$\overline{C}_2 = 16\overline{i} + 24\overline{j}$$

Frame 5-38

Oblique Components -- Algebraic

Do example 5-5 in your notebook.

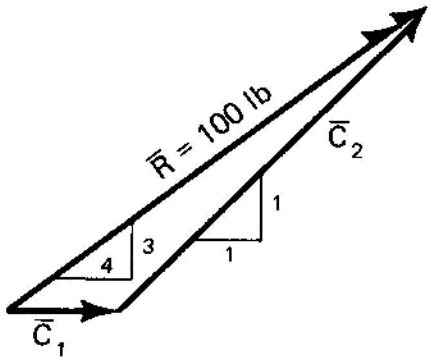
Correct response to preceding frame

$\bar{C}_2 = -9\bar{j} + 30\bar{k}$

Frame 5-39

Oblique Components -- Algebraic

When the directions of both components are known the problem becomes a little more complex.



The force $\bar{R}$ is divided into components whose directions are shown.

The first step is to write all forces as vectors thus

$\bar{R} = 100\ (.8\bar{i} + .6\bar{j})$

$\bar{C}_1 = A\ (\bar{i})$

$\bar{C}_2 = B\ (.707\bar{i} + .707\bar{j})$

Then set $\bar{R} = \bar{C}_1 + \bar{C}_2$, for magnitudes **A** and **B**, and then write the component vectors $\bar{C}_1$ and $\bar{C}_2$.

A = _____

B = _____

$\bar{C}_1$ = _____

$\bar{C}_1$ = _____

Correct response to preceding frame

$A = 20$

$\overline{C}_1 = 20\overline{i}$

$B = \frac{60}{.707}$ or $B = 84.8$

$\overline{C}_2 = 60\overline{i} + 60\overline{j}$

Solution:

$80\overline{i} + 60\overline{j} - A\overline{i} + .707\ B\overline{i} + .707\ B\overline{j}$

$\overline{i}$ coefficients

$80 = A + .707B$

$\overline{j}$ coefficients

$60 = .707B$

$A = 20$

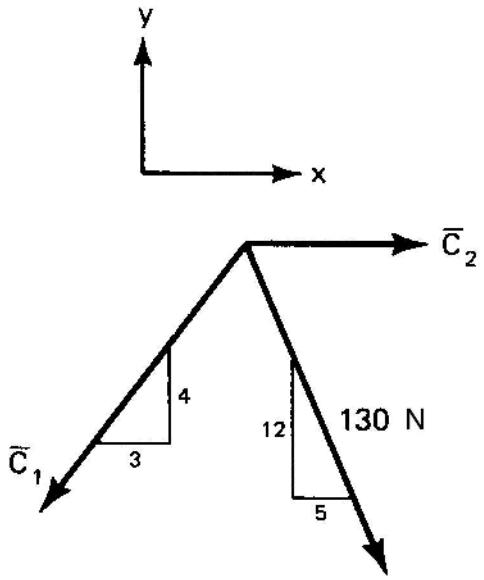
$\overline{C}_1 = 20\overline{i}$

$B = \frac{60}{.707}$

$\overline{C}_2 = 60\overline{i} + 60\overline{j}$

Frame 5-40

Oblique Components--Algebraic



The 130 N force is to be divided into components in the indicated directions. Find the components $\overline{C}_1$ and $\overline{C}_2$.

Correct response to preceding frame

$\overline{C}_2 = 140\overline{i}$

$\overline{C}_1 = -90\overline{i} - 120\overline{j}$ or

$\overline{C}_1 = 150\ (-.6\overline{i} - .8\overline{j})$

Solution:

$\overline{F} = 50\overline{i} - 120\overline{j}$

$\overline{C}_1 = A(-.6\overline{i} - .8\overline{j})$

$\overline{C}_2 = B(\overline{i})$

$\overline{F} = \overline{C}_1 + \overline{C}_2$

$50\overline{i} - 120\overline{j} = -.6A\overline{i} - .8A\overline{j} + B\overline{i}$

$\overline{j}$ coefficients

$-120 = -.8\ A\overline{j}$

$A = 150$

$\overline{i}$ coefficients

$50 = -.6A + B$

$50 = -90 + B$

$B = 140$

Frame 5-41

Oblique Components--Algebraic

In an algebraic solution of a problem involving components of known directions the key step is to (check one)

- ☐ write each component as a vector sum
- ☐ write each component as the product of a magnitude and a unit vector

Correct response to preceding frame

write each component as the product of a magnitude and a unit vector

---

Frame 5-42

Notebook

Do problem 5-6 in your notebook.

Correct response to preceding frame

$\overline{C}_2 = 280\overline{i} + 210\overline{j}$

$\overline{C}_1 = -250\overline{i} - 250\overline{j}$

Solution:

$\overline{F} = 30\overline{i} - 40\overline{j}$

$\overline{C}_1 = A(-.707\overline{i} - .707\overline{j})$

$\overline{C}_2 = B(.8\overline{i} + .6\overline{j})$

coefficient equations

$30 = -.7A + .8B$

$-40 = -.7A + .6B$

$70 = .2B$

$B = 350$

$30 = -.7A + 280$

$A = \frac{250}{.7}$

Frame 5-43

Conclusion

That does it. You are now able to find components, both rectangular and oblique, by three different methods:

1. Graphical
2. Trigonometric
3. Algebraic

One method, at least, should suit your taste for any component problem -- and it is often merely a matter of personal preference which to use.

Many other configurations and statements of problems are possible, of course, but you have now worked the commonest kinds. Furthermore you can apply the same methods to the other weird problems in components that fate may occasionally hand you.