

# Introduction to Statics

.PDF Edition – Version 0.95

## Unit 4

# Vector Addition: Resultant Forces

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# Unit 4

## Vector Addition: Resultant Forces

Frame 4-1

### **Introduction**

The preceding unit taught you to represent vectors graphically and in two different algebraic forms. The first part of this unit will be devoted to the beginning of vector algebra and will teach you to:

1. Add and subtract vectors graphically
2. Add and subtract vectors algebraically

The latter part will let you make use of your new knowledge by finding the resultant of some simple force systems.

While it is not absolutely necessary, you may find it useful to have a small ruler or straight-edge at hand while working this unit.

Go to the next frame.

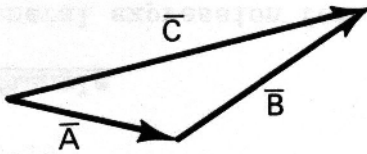
Correct response to preceding frame

No response

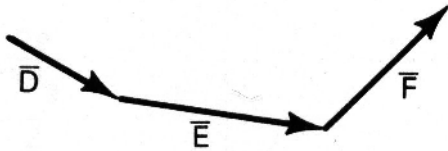
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Frame 4-2

### Graphical Addition of Vectors

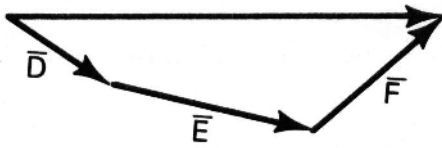


Vector  $\vec{C}$  is the sum of vectors  $\vec{A}$  and  $\vec{B}$ .



Draw the vector that represents  $\vec{D} + \vec{E} + \vec{F}$

Correct response to preceding frame



Frame 4-3

### Graphical Addition of Vectors

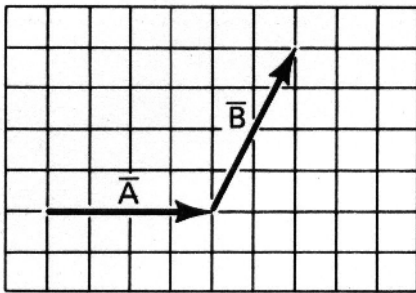


Figure (a)

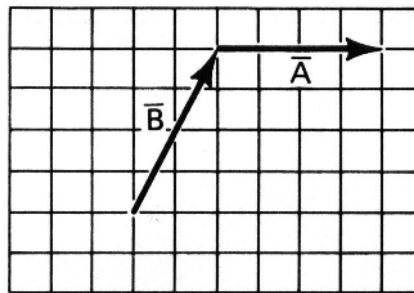


Figure (b)

1. Figure (a) shows vector  $\vec{B}$  added to vector  $\vec{A}$ . Draw the vector that represents the sum. Call it  $\vec{A} + \vec{B}$ .

2. Figure (b) shows vector  $\vec{A}$  added to vector  $\vec{B}$ . Draw the vector that represents the sum. Call it  $\vec{B} + \vec{A}$ .

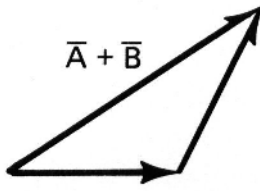
3. Does  $\vec{A} + \vec{B}$  have the same magnitude as  $\vec{B} + \vec{A}$  ?  Yes  No

Do they have the same direction?  Yes  No

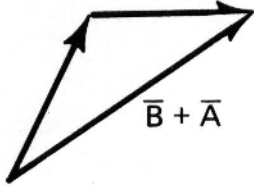
4. Does  $\vec{A} + \vec{B}$  equal  $\vec{B} + \vec{A}$  ?  Yes  No

Correct response to preceding frame

1.



2.



3. Yes, the magnitudes are the same.  
Yes, the directions are the same.

4. Yes.  $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

---

Frame 4-4

### Vector Addition

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$  is the commutative law of vector addition.

In words the commutative law of vector addition states that in vector addition the order in which vectors are added \_\_\_\_\_

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Correct response to preceding frame

The order in which vectors are added does not affect the result. (Or equivalent response)

---

Frame 4-5

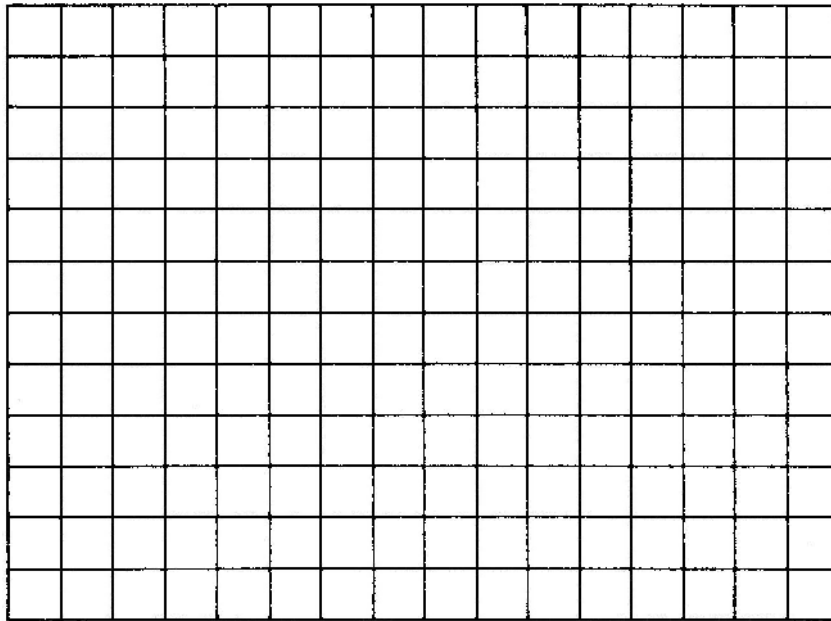
### Graphical Addition of Vectors

Vectors may be added graphically by drawing the vectors to be added in the correct directions, and to some common scale, laying them out successively so that the "tail" of the second begins at the "nose" of the first, and the "tail" of the third begins at the "nose" of the second and so forth. The vector sum will be the vector connecting the tail of the first to the nose of the last.

Add the following vectors graphically.

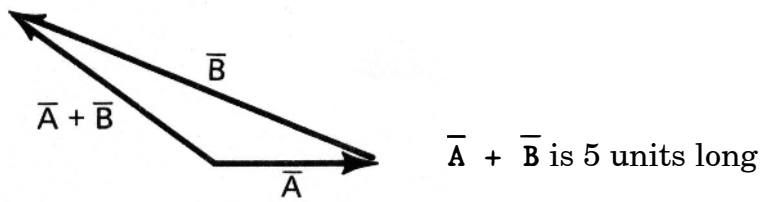
$$\vec{A} = 3\vec{i}$$

$$\vec{B} = -7\vec{i} + 3\vec{j}$$



What is the magnitude of  $\vec{A} + \vec{B}$ ?

Correct response to preceding frame

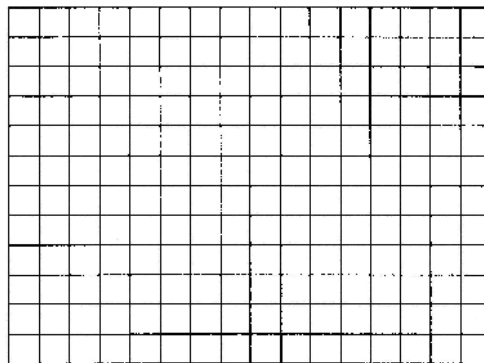


Frame 4-6

### Graphical Addition of Vectors

1. Add the following vectors graphically

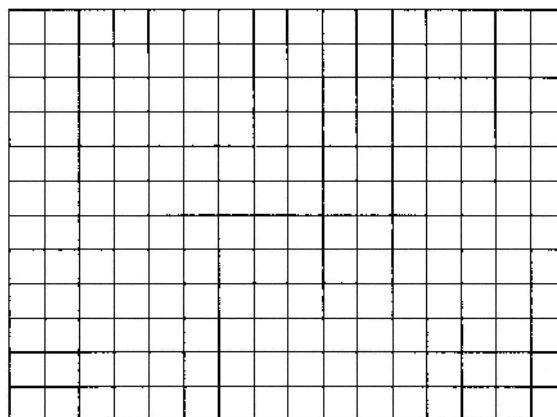
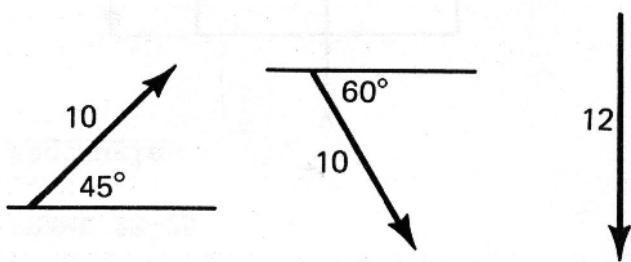
$$\begin{aligned}\vec{A} &= 3\vec{i} + 12\vec{j} \\ \vec{B} &= -6\vec{i} - 8\vec{j} \\ \vec{C} &= 3\vec{i} - 7\vec{j}\end{aligned}$$



From your solution write the vector that represents the sum.

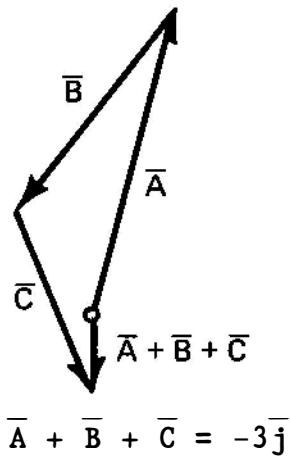
$$\vec{A} + \vec{B} + \vec{C} = \underline{\hspace{2cm}}$$

2. Add the vectors shown graphically and by measuring your figure determine the magnitude of the sum.

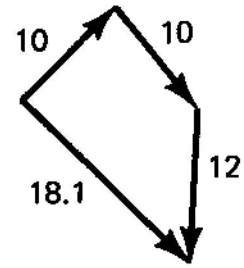


Correct response to preceding frame

1.



2.



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Frame 4-7

### Graphical Vector Addition

Complete the section of your notebook beginning on Page 4-1 and work out the example problems.



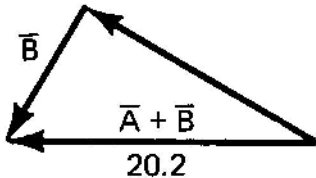
Correct response to preceding frame

My statement of method is contained in Frame 4-5. You may have found a more elegant way of putting it.

Problem 4-1

The magnitude of your vector sum should be 13.

Problem 4-2



---

Frame 4-8

### Transition

All the time you have been humoring the author by working graphical solutions, I hope you have been saying,

"There must be an easier way!"

and

"Yeah, but how about three dimensional vectors?"

There is, indeed, an easier way and it handles three dimensional problems as easily as two dimensional ones. (I don't even like to think about working a three dimensional problem graphically.)

Graphical methods may be ideal for certain problems and will furnish you a good way of checking others but for the easy way through vector addition, go to the next frame.

Correct response to preceding frame

No response

---

Frame 4-9

### Algebraic Vector Addition

To add vectors algebraically one merely adds up all the terms containing each unit vector separately, thus:

$$\begin{aligned}\bar{A} &= 10\bar{i} + 12\bar{j} - 6\bar{k} \\ \bar{B} &= -8\bar{i} + 5\bar{j} \\ \bar{C} &= -3\bar{i} - 16\bar{j} + 12\bar{k}\end{aligned}$$

---

$$\bar{A} + \bar{B} + \bar{C} = -\bar{i} + 2\bar{j} + 6\bar{k}$$

Add the following vectors:

$$\begin{aligned}\bar{P} &= 10\bar{i} + 12\bar{j} \\ \bar{Q} &= -8\bar{i} + 6\bar{j} \\ \bar{R} &= 3\bar{i} - 6\bar{j}\end{aligned}$$

Correct response to preceding frame

$$\bar{P} + \bar{Q} + \bar{R} = 5\bar{i} + 12\bar{j}$$

(These are the same vectors you added in Problem 4-1. Look back at your graphical solution.)

---

Frame 4-10

### Subtraction of Vectors

To subtract vectors, one simply changes the sign of the vector to be subtracted and adds either graphically or algebraically.

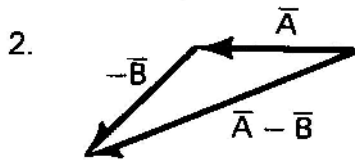
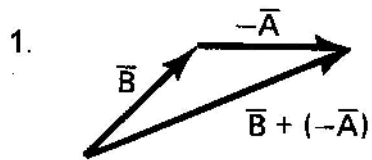


1. Draw the vector representing  $\bar{B} + (-\bar{A})$

2. Draw the vector representing  $\bar{A} - \bar{B}$

3. Subtract  $\bar{A} = 4\bar{i} + 3\bar{j} - 12\bar{k}$  from  $\bar{B} = -6\bar{i} + 2\bar{j} + 10\bar{k}$

Correct response to preceding frame



3.  $\vec{B} - \vec{A} = -10\vec{i} - \vec{j} + 22\vec{k}$

---

Frame 4-11

### Algebraic Addition of Vectors

Complete problems 4-3 and 4-4 in your notebook.

Correct response to preceding frame

Problem 4-3

$$\begin{aligned}\vec{A} &= -8\vec{i} - 6\vec{j} \\ \vec{B} &= -12\vec{i} + 9\vec{j} \\ \vec{A} + \vec{B} &= -20\vec{i} + 3\vec{j}\end{aligned}$$

Problem 4-4

$$\begin{aligned}\vec{A} + \vec{B} &= 2\vec{i} + 11\vec{j} + 13\vec{k} \\ \vec{A} - \vec{B} &= 22\vec{i} - 5\vec{j} - \vec{k} \\ \vec{B} - \vec{A} &= -22\vec{i} + 5\vec{j} + \vec{k}\end{aligned}$$

---

Frame 4-12

### Transition

In the preceding frames you have been learning to find the sum of vectors both algebraically and graphically.

We will now apply what you have learned to finding resultant forces -- since forces are the most important kind of vectors to the student of statics.

This is about the halfway point in this unit. Estimated time to the end -- 30 minutes.

When you are ready to invest that time, go to the next frame.

Correct response to preceding frame

No response

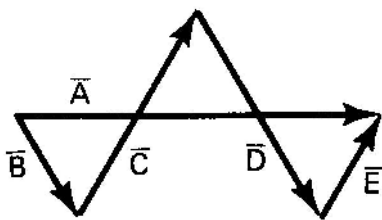
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Frame 4-13

### Resultants

When two or more forces are added, the vector representing the sum is called the "resultant".

In the figure below, which vector is the resultant of the other forces?



\_\_\_\_\_ is the resultant

Correct response to preceding frame

$\bar{A}$

---

Frame 4-14

**Vocabulary**

The sum of vectors is called their \_\_\_\_\_ .

Correct response to preceding frame

resultant

---

Frame 4-15

**Review**

The characteristics of a force are

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_



Correct response to preceding frame

1. magnitude
  2. direction
  3. point of application
- 

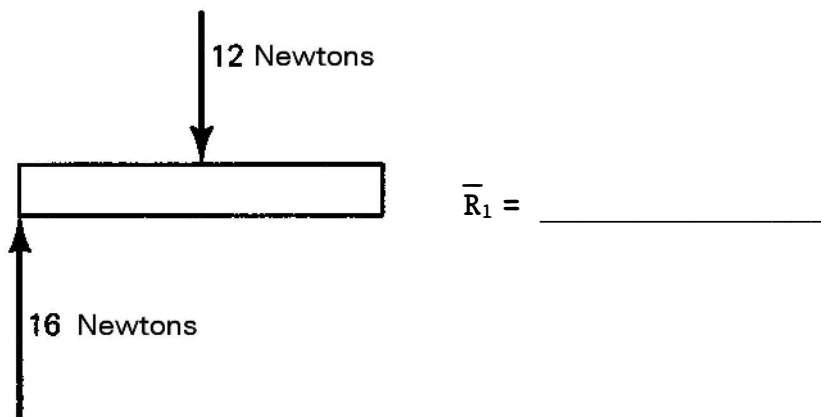
Frame 4-16

### Resultants

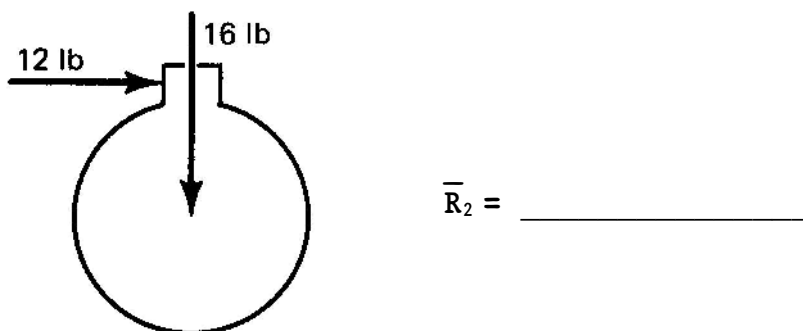
Read Page 4-3 of your notebook.

In the force systems below determine the magnitude and direction of the resultant force. If you can determine the point of application of the resultant, draw it on the figure.

1.



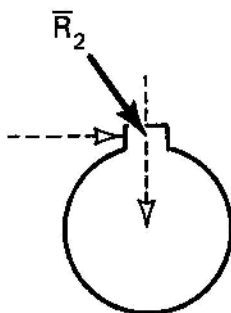
2.



Correct response to preceding frame

1.  $\bar{R}_1 = 16\bar{j} - 12\bar{j} = 4\bar{j}$  point of application unknown

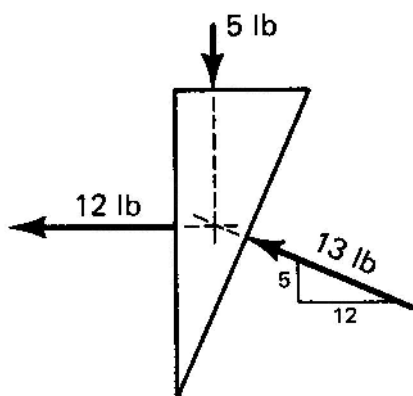
2.  $\bar{R}_2 = 12\bar{i} - 16\bar{j} = 20 [(3\bar{i} - 4\bar{j})/5]$



Frame 4-17

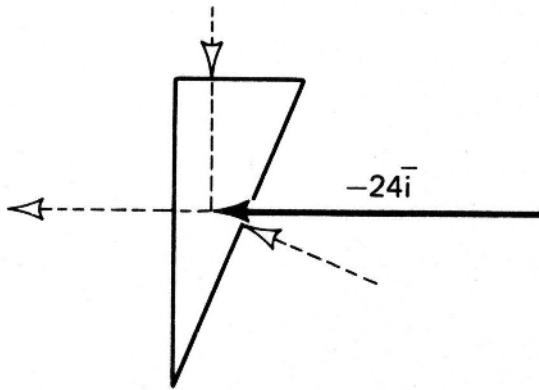
### Resultants

Find the resultant of the force system shown and show it on the sketch.



$\bar{R} =$  \_\_\_\_\_

Correct response to preceding frame



$$\bar{R} = -24\bar{i}$$

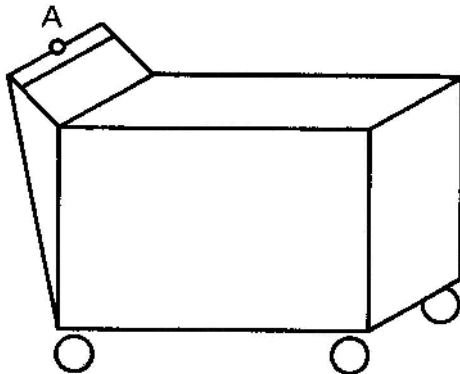
Solution:

$$\begin{aligned} \bar{R} &= 13 \left( \frac{5\bar{j} - 12\bar{i}}{13} \right) - 5\bar{j} - 12\bar{i} \\ &= -24\bar{i} \end{aligned}$$

Frame 4-18

**Resultants**

Two forces  $\bar{A} = 6\bar{i} + 12\bar{j} - 3\bar{k}$  N, and  $\bar{B} = 14\bar{i} + 6\bar{j} - 12\bar{k}$  N act through point A in the figure below. Find the resultant force and its point of application.



$$\bar{R} = \underline{\hspace{10em}}$$

Correct response to preceding frame

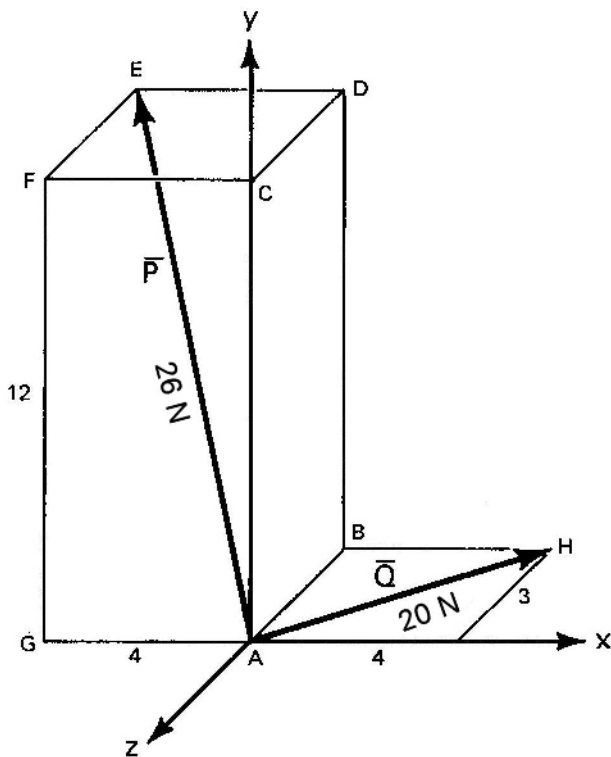
$$\bar{R} = 20\bar{i} + 18\bar{j} - 15\bar{k} \text{ N acting through A}$$

---

Frame 4-19

### Resultants

Determine the resultant of the force system shown and give its point of application. (Start by calculating the components of  $\bar{P}$  and  $\bar{Q}$ .)



$$\bar{P} = \underline{\hspace{10cm}}$$

$$\bar{Q} = \underline{\hspace{10cm}}$$

$$\bar{R} = \bar{P} + \bar{Q} = \underline{\hspace{10cm}}$$

Correct response to preceding frame

$$\bar{P} = 26 \left( \frac{-3\bar{k} - 4\bar{i} + 12\bar{j}}{13} \right) = -6\bar{k} - 8\bar{i} + 24\bar{j}$$

$$\bar{Q} = 16\bar{i} - 12\bar{k}$$

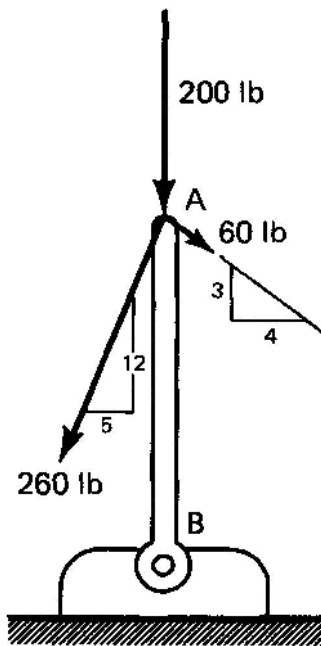
$$\bar{R} = 8\bar{i} + 24\bar{j} - 18\bar{k} \quad \text{acting through A}$$

Frame 4-20

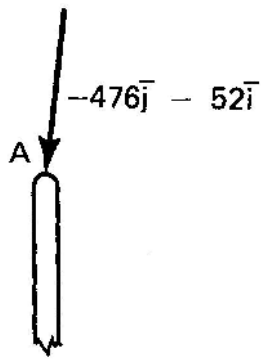
Resultants

The resultant of any number of concurrent forces may be found by simply adding vectorially and placing the resultant through the point of concurrency.

Find the resultant of the forces shown and show it on the sketch.



Correct response to preceding frame



---

Frame 4-21

**Resultants**

Work problem 4-5 in your notebook

Correct response to preceding frame

$$300 \left( \frac{-14\bar{i} + 2\bar{j} - 5\bar{k}}{15} \right) + 240 \left( \frac{4\bar{i} + 2\bar{j} - 4\bar{k}}{6} \right) - 200 \bar{j} = -120\bar{i} - 80\bar{j} - 260\bar{k}$$

---

Frame 4-22

### Closure

This concludes this unit. You should now be able to:

1. Add two dimensional vectors graphically
2. Add vectors analytically
3. Apply vector addition to find the resultant of concurrent forces