Introduction to Statics

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Unit 3
Vectors: Algebraic Representation

Helen Margaret Lester Plants
Late Professor Emerita
Wallace Starr Venable
Emeritus Associate Professor
West Virginia University, Morgantown, West Virginia

Unit 3
Vectors: Algebraic Representation

Frame 3-1

Introduction

In the preceding sections you learned to identify vector quantities as having magnitude and direction and to represent them as directed line segments.

It is not always particularly convenient to work with directed line segments. (In fact, in three dimensions it becomes downright nasty even to draw them to look right.) Consequently the next section will teach you how to write mathematical expressions representing vectors.

The first section of this unit will be devoted to teaching you the rules for the algebraic notation of vectors. Next you will learn to write vectors as algebraic sums. The last part of the unit will be devoted to teaching you to write vectors as algebraic products.

Go to the next frame.

Correct response to preceeding frame

No response

Frame 3-2

Notation

There are several ways in which vectors are denoted in mathematical expressions. One way which has been used in books is to print vectors in bold type.

Using the bold-faced notation, in the expression $\mathbf{V} = V\mathbf{e}$, $\mathbf{V}$ and $\mathbf{e}$ would be vectors and V would be a scaler.

This is a cheap and easy solution in print, but it is difficult to do bold-facing in handwritten equations.

A simple solution in hand written expressions is to draw a bar over (or under) vector quantities.

In this book we will use the top-bar notation.

Any quantity written with a bar above is a vector. For example, $\overline{V}$ and $\overline{e}$ indicate vectors. Any quantity written without the bar is a scalar.

Circle the vectors in the following equations:

$$\overline{R} = 17\overline{i} + 14\overline{j} + 12\overline{k}$$

$$\overline{A} = \overline{Q} + \overline{R}$$

$$\overline{N} = N\overline{n}$$

Correct response to preceeding frame

$$\bar{R} = 17\bar{i} + 14\bar{j} + 12\bar{k}$$

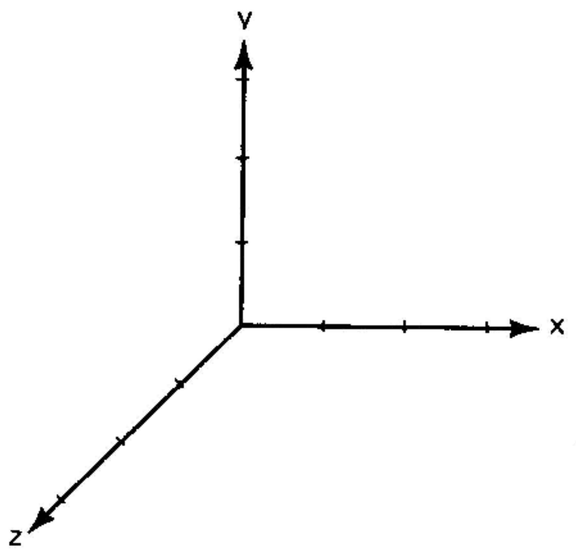
$$\bar{A} = \bar{Q} + \bar{R}$$

$$\bar{N} = N\bar{n}$$

Frame 3-3

Unit Vectors

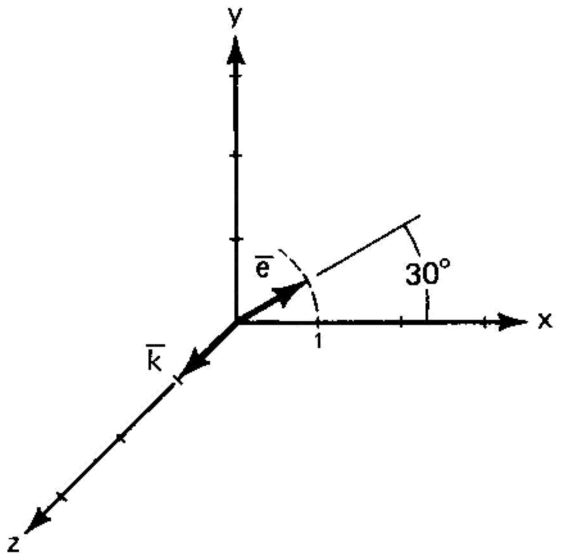
Before we can do much with vector expressions you must learn about unit vectors. A unit vector is simply a vector that is one unit long. It may be in any direction.



1. Draw a unit vector along the z-axis. Call it $\bar{k}$.

2. Draw a unit vector in the xy plane that makes an angle of 30° with the positive x-axis. Call it $\bar{e}$.

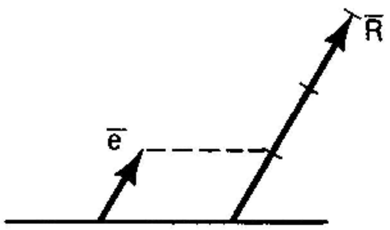
Correct response to preceeding frame



Frame 3-4

Unit Vectors

Any vector may be written as a scalar magnitude times a unit vector in the appropriate direction.



$\bar{e}$ is a unit vector in the direction shown.

$\bar{R}$ is a vector parallel to $\bar{e}$ and is three units in length.

Write $\bar{R}$ in terms of $\bar{e}$.

$\bar{R}$ = ________ $\bar{e}$

Correct response to preceeding frame

$\overline{R} = 3\overline{e}$

Frame 3-5

Unit Vectors

1. A certain vector, $\overline{Q}$, is 30 units in magnitude and is parallel to unit vector $\overline{q}$. Write an expression for the vector.

$\overline{Q} =$ _____________

Sometimes the scalar magnitude of a vector is written as the vector without the bar, thus:

$\overline{V} = V\overline{e}_v$ or $\overline{V} = V\overline{v}$

2. What is the magnitude of vector $\overline{V}$ in the above notation? __________________

3. What are $\overline{e}_v$ and $\overline{v}$? ____

Correct response to preceeding frame

1. $\overline{Q} = 30\overline{q}$

2. Magnitude of $\overline{V}$ is V

3. $\overline{e}_v$ and $\overline{v}$ are unit vectors, parallel to $\overline{V}$

Frame 3-6

Unit Vectors

Magnitudes are not always known. Consequently a vector of unknown magnitude, but known to be in the direction of unit vector, $\overline{a}$, could be written

$$\overline{V} = x\overline{a}$$

In this case x stands for ____

and $\overline{a}$ is ____ .

Correct response to preceeding frame

x is the unknown magnitude of $\bar{V}$

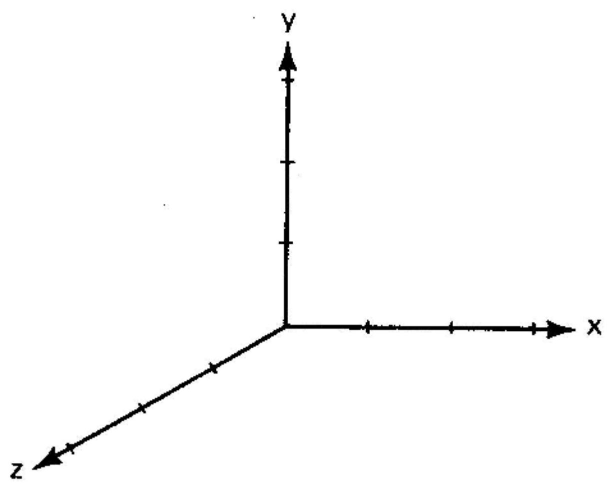
$\bar{a}$ is a known unit vector

Frame 3-7

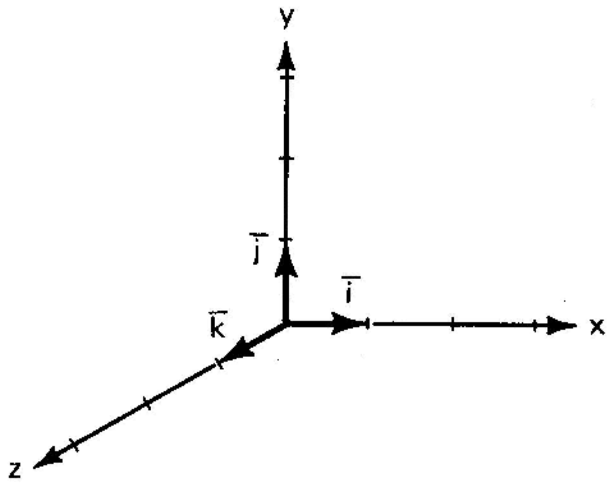
Unit Vectors

In this text a unit vector in the x-direction will always be denoted by $\bar{i}$, a unit vector in the y-direction will always be $\bar{j}$ and a unit vector in the z-direction will be $\bar{k}$.

Draw $\bar{i}$, $\bar{j}$ and $\bar{k}$ on the coordinate system shown.



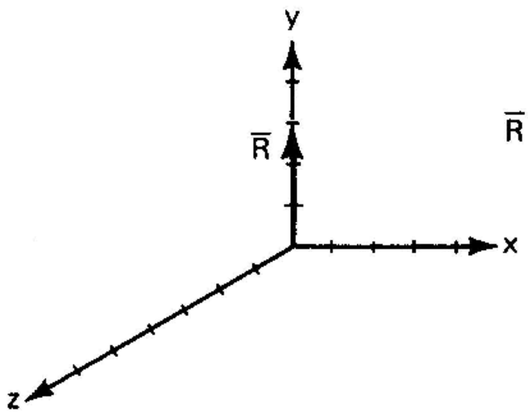
Correct response to preceeding frame



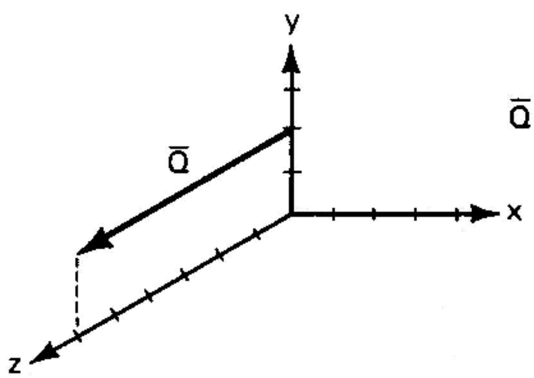
Frame 3-8

Unit Vectors

Any vector parallel to a coordinate axis can be represented as the product of its scalar magnitude and the appropriate unit vector. Do so for the vectors shown.



$\bar{R}$ = __________



$\bar{Q}$ = __________

Correct response to preceeding frame

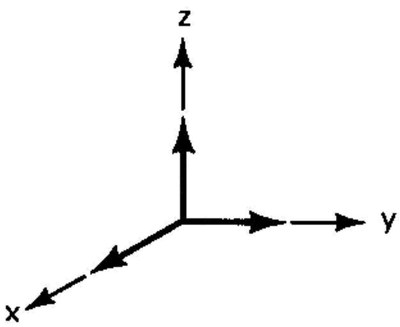
$\bar{R} = 3\bar{j}$

$\bar{Q} = 6\bar{k}$

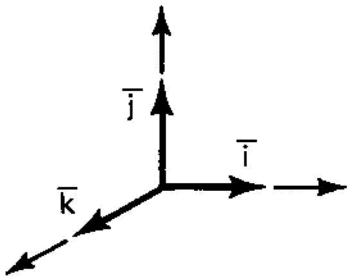
Frame 3-9

Notation

$\bar{i}$, $\bar{j}$, and $\bar{k}$ will denote unit vectors directed along the x, y, and z axes respectively in a Cartesian coordinate system.

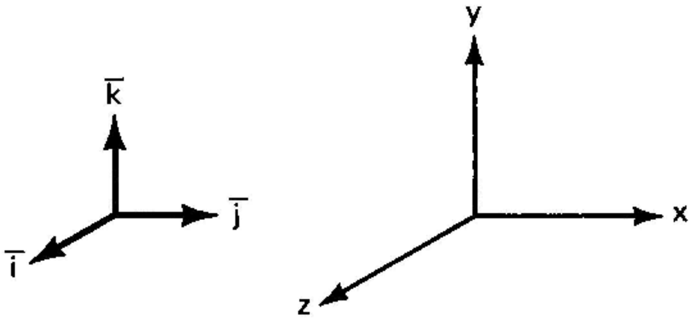


Label the unit vectors.



Label the coordinate axes.

Correct response to preceeding frame



Frame 3-10

Transition

In the preceding frames you have learned to write vectors as the product of a magnitude and a unit vector. You have also seen that we tend to associate unit vectors with specific coordinate systems.

The next short section will show you the sort of coordinate system you can expect to see throughout this course. You will find that in addition to being orthogonal we will restrict our thinking to "right-hand systems." For an explanation of that term, go to the next frame.

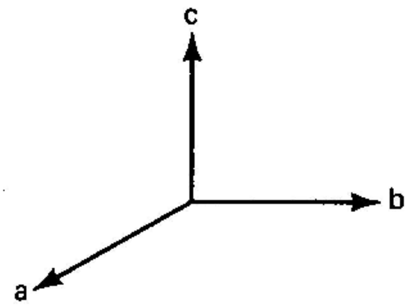
Correct response to preceeding frame

No response

Frame 3-11

Right Hand Systems

In a right hand coordinate system, rotation of a right hand screw from one positive axis toward a second positive axis would cause the screw to advance in the direction of the third positive axis.



1. In the (abc) system shown, rotation of a toward b would cause a right hand screw to advance along the

positive ___________ axis.

2. Rotation of b toward c would cause a right hand screw to advance along the ______ axis, and rotation of c toward a would cause it to advance along the _______ axis.

3. The right hand system shown is described as (abc), (bca), or (________).

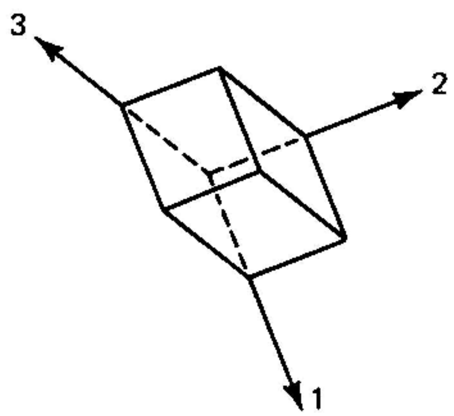
Systems (bac), (__________), and (________) are not right hand systems.

Correct response to preceeding frame

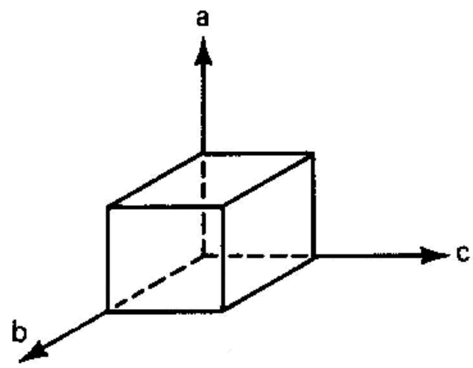
1. c
2. Positive a
   Positive b
3. (cab)
   (cba)
   (acb)

Frame 3-12

Right Hand Systems

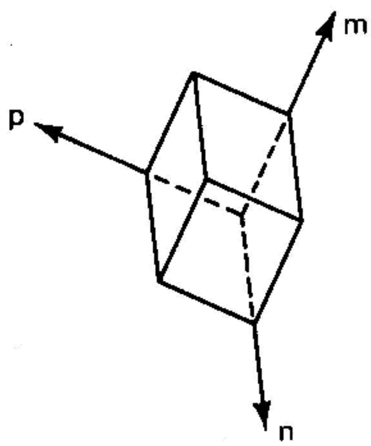


1. (1,2,3) is a right hand system. Rotation of 1 toward 2 would cause a right hand screw to advance in the direction of __________ .



2. Is (a,b,c) a right hand system?

☐ Yes ☐ No



3. Is (m,n,p) a right hand system?

☐ Yes ☐ No

4. Is (n,m,p) a right hand system?

☐ Yes ☐ No

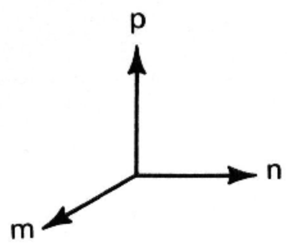
Correct response to preceeding frame

1. 3
2. Yes
3. No
4. Yes

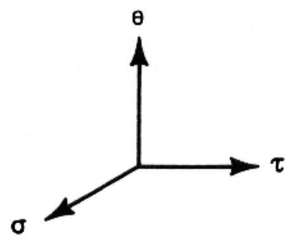
Frame 3-13

Right Hand Systems

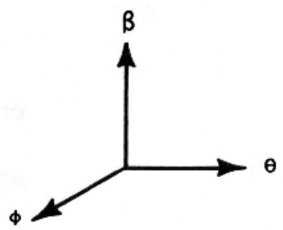
Are these systems right hand systems? (Yes or No)



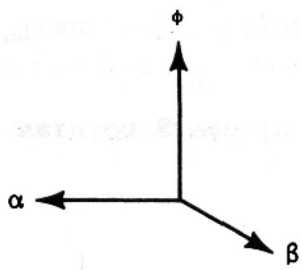
1. (m,n,p) ☐ Yes ☐ No



2. (τ , θ , σ) ☐ Yes ☐ No



3. (β , θ , ϕ) ☐ Yes ☐ No



4. (α , β , ϕ) ☐ Yes ☐ No

Correct response to preceeding frame

1. Yes
2. Yes
3. No
4. Yes

Frame 3-14

Right Hand Systems

1. A right hand system is one in which the rotation of the first axis given in the description toward the _________________ axis will cause a right hand screw to advance in a (positive, negative) direction along the _______________ axis.

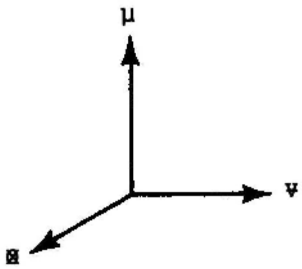
2. Must you be given a description of the system to determine its "handed-ness"?

☐ Yes ☐ No

Correct response to preceeding frame

1. second
   positive
   third

2. Yes*



*Note: The description of the reference system shown would certainly have to be given since you have no idea how the symbols ¥ , μ , and ⊠ are usually ordered.

However, many authors assume that numerals, the Roman (English) alphabet, and certain parts of the Greek alphabet are self-ordering, so you may assume the orders (123), (αβγ), (pqr), etc., unless a different arrangement is stated.

Frame 3-15

Coordinate Systems

All coordinate systems used in these units will be right hand orthogonal systems.

1. "Orthogonal" means that all coordinate axes are ______________________________

2. Will we consider left hand systems? ☐ Yes ☐ No

Correct response to preceeding frame

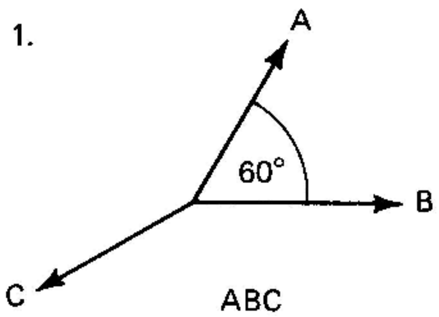
1. Orthogonal axes are *perpendicular to each other.*
2. No

Frame 3-16

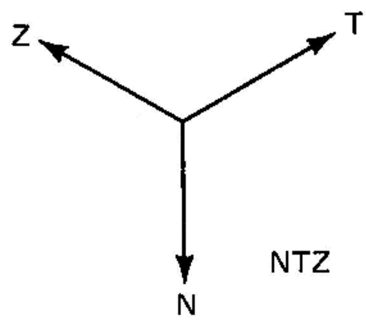
Coordinate Systems

Circle those systems which we will not consider. (Angles which are not dimensioned are 90°.)

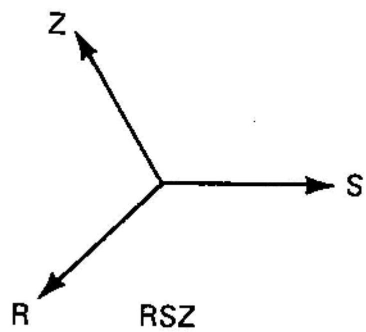
1.



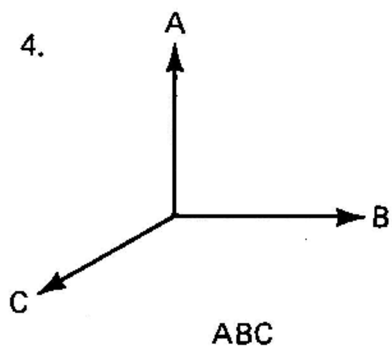
2.



3.



4.



Correct response to preceeding frame

1 is not orthogonal and 4 is not right hand, therefore neither will be considered.

Frame 3-17

Review

Complete the first section of Page 3-1 in your notebook.

Correct response to preceeding frame

No response

Frame 3-18

Transition

You have completed the section on the ground rules of notation and coordinate systems, and have just summarized them in your notebook.

The next section will be devoted to writing algebraic expressions for vectors as sums of orthogonal components. You will find it relatively quick and easy.

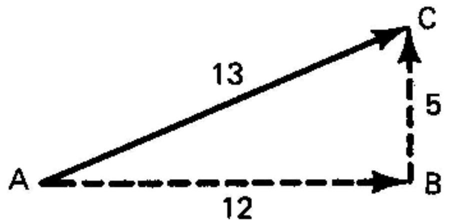
Go to the next frame.

Correct response to preceeding frame

No response

Frame 3-19

Vectors as Sums



Consider the vector $\overline{AC}$ as a displacement in the x-y plane. It is possible to go directly from A to C. It is equally possible to go from A to B along a horizontal line and from B to C along a vertical line. Written as a vector, the foregoing statements would read

$\overline{AC} = 12\overline{i} +$ ______________

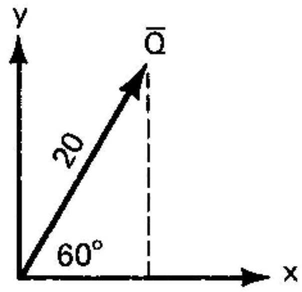
Correct response to preceeding frame

$\overline{AC} = 12\overline{i} + 5\overline{j}$

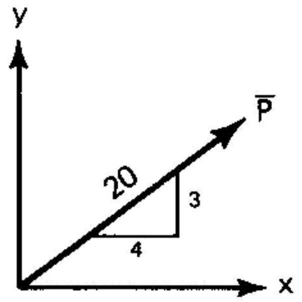
Frame 3-20

Vectors as Sums

Write the following vectors as sums



$\overline{Q}$ = _____ $\overline{i}$ + _____ $\overline{j}$



$\overline{P}$ = ______________________

Correct response to preceeding frame

$\overline{Q} = 20 \cos 60° \, \overline{i} + 20 \sin 60° \overline{j} = 10\overline{i} + 17.32\overline{j}$

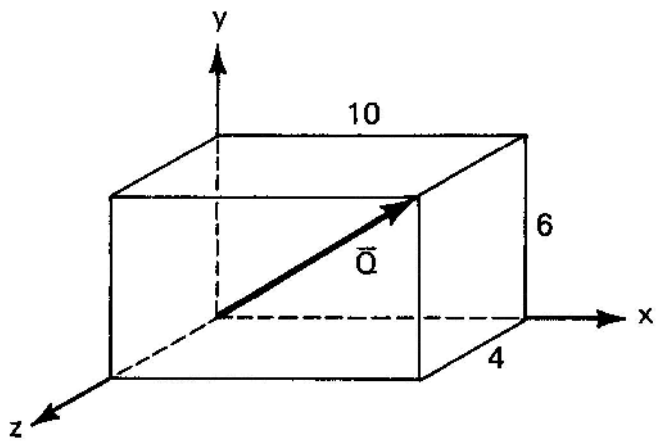
$\overline{P} = 4/5 \, (20) \, \overline{i} + 3/5 \, (20) \, \overline{j} = 16\overline{i} + 12\overline{j}$

Frame 3-21

Vectors as Sums

Two (or more) vectors which, when added together, equal a vector, $\overline{R}$, are called the components of $\overline{R}$. If the components are at right angles, they arecalled rectangular components.

Write the vector shown as the sum of three rectangular components.



$\overline{Q}$ = ____

Correct response to preceeding frame

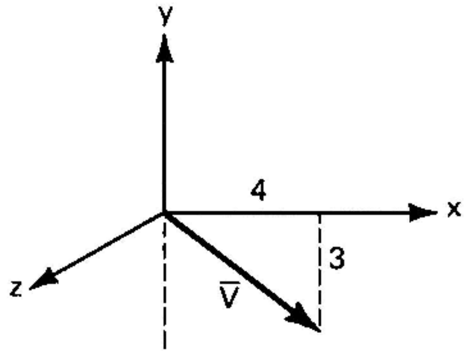
$\overline{Q} = 10\overline{i} + 6\overline{j} + 4\overline{k}$

Frame 3-22

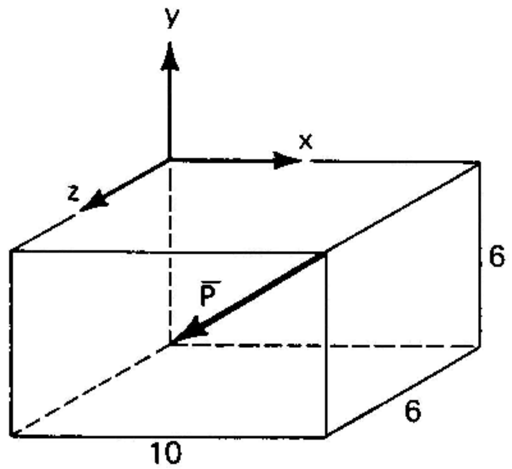
Vectors as Sums

It is quite possible for vectors to have negative components as well as positive ones.

Write expressions for the vectors shown.



$\overline{V}$ = ______________________________



$\overline{P}$ = ______________________________

Correct response to preceeding frame

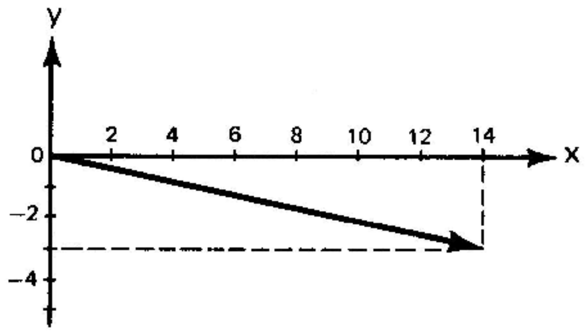
$\overline{V} = 4\overline{i} - 3\overline{j}$

$\overline{P} = -10\overline{i} - 6\overline{j} - 6\overline{k}$

Frame 3-23

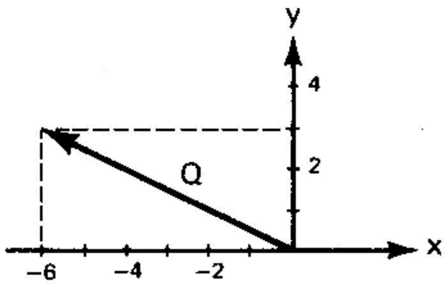
Graphical Representation

A vector may be drawn from its mathematical statement. Thus, $\overline{R} = 14\overline{i} - 3\overline{j}$ would be drawn as shown:



Draw the vector $\overline{Q} = -6\overline{i} + 3\overline{j}$

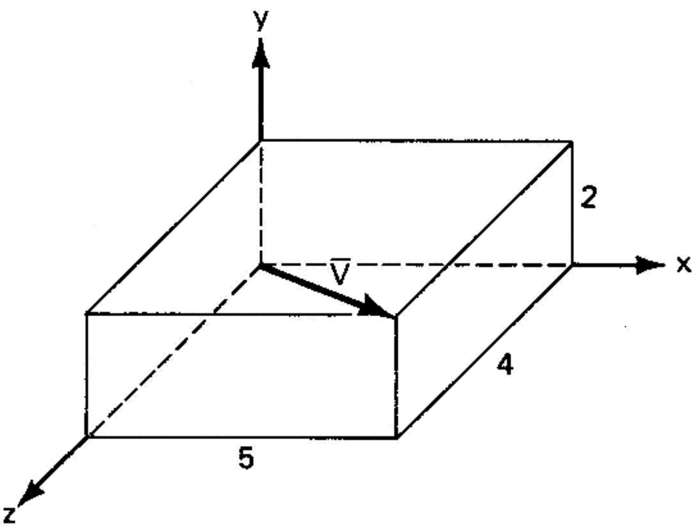
Correct response to preceeding frame



Frame 3-24

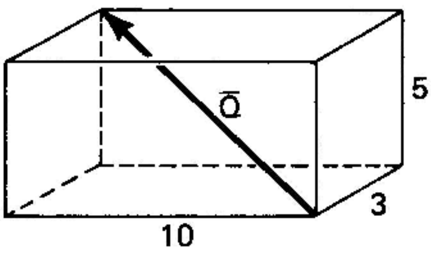
Graphical Representation

The three dimensional vector $\overline{V} = 5\overline{i} + 2\overline{j} + 4\overline{k}$ may be represented by the drawing below.



Draw the vector $\overline{Q} = -10\overline{i} + 5\overline{j} - 3\overline{k}$

Correct response to preceeding frame



Frame 3-25

Review

Complete the next section in your notebook.

Correct response to preceeding frame

If you have doubts as to the accuracy of your answers in the notebook you should review the last 10 frames of this unit.

Frame 3-26

Transition

The first section of this unit taught you to represent vectors graphically; the second section taught you to write them as sums. The next section will show you how to write vector quantities as products.

This is about the halfway point of this unit. If you need a break take it now.

Otherwise, turn to the next frame.

Correct response to preceeding frame

No response

Frame 3-27

Review

When a vector is represented by a line segment the magnitude of the vector is

____

Correct response to preceeding frame

the length of the line

Frame 3-28

Review

Draw the following vectors and compute their magnitudes.

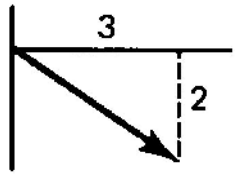
1. $\overline{F} = 3\overline{i} - 2\overline{j}$

magnitude = ____________

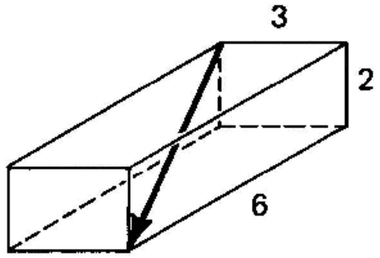
2. $\overline{F} = 3\overline{i} - 2\overline{j} + 6\overline{k}$

magnitude = _____________

Correct response to preceeding frame



magnitude = $\sqrt{13}$ = 3.61



$\ell = \sqrt{6^2 + 2^2 + 3^2}$

magnitude = 7

Frame 3-29

Magnitude of a Vector

It is possible to find the magnitude of a vector from its equation without first drawing the vector. Tell how.

Correct response to preceeding frame

By taking the square root of the sum of the squares of the coefficients of the unit vectors (Or equivalent response)

Frame 3-30

Magnitude of a Vector

Write an expression for the magnitude of a vector whose equation is

$$\overline{V} = x\overline{i} + y\overline{j} + z\overline{k}$$

V = Magnitude = ________________________

Correct response to preceeding frame

$$\text{Magnitude} = V = \sqrt{x^2 + y^2 + z^2}$$

Frame 3-31

Magnitude of a Vector

Use the expression you just found to obtain the magnitude of the following vectors. (Use the unbarred letter to indicate magnitude.)

1. $\overline{P} = 15\overline{j} - 8\overline{k}$

   P = ______________

2. $\overline{Q} = 8\overline{i} + 9\overline{j} + 12\overline{k}$

   Q = ______________

Correct response to preceeding frame

1. $P = 17 = \sqrt{15^2 + 8^2} = \sqrt{225 + 64} = \sqrt{289}$

2. $Q = 17 = \sqrt{8^2 + 9^2 + 12^2} = \sqrt{64 + 81 + 144} = \sqrt{289}$

Frame 3-32

Vector Magnitude

You have learned to find the magnitude of any vector. Does giving its magnitude alone sufficiently describe a vector?

☐ Yes ☐ No

If your answer is "Yes", draw a vector showing magnitude only.

If your answer is "No", tell what else is needed.

Correct response to preceeding frame

No. Direction is also needed.

Frame 3-33

Direction of a Vector

A vector can be completely specified by multiplying its magnitude by a unit vector in the proper direction. In the following simple cases underline the part giving the magnitude of the vector once. Underline the part giving the direction twice.

$$\bar{R} = 12\ \bar{i}$$

$$\bar{Q} = 17\ \bar{j}$$

Correct response to preceeding frame

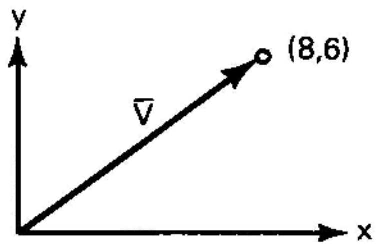
$\bar{R}$ = <u>12</u> <u>$\bar{i}$</u>

$\bar{Q}$ = <u>17</u> <u>$\bar{j}$</u>

Frame 3-34

Direction of a Vector

Unfortunately, not all vectors are in the coordinate directions. To write them as products of magnitudes and unit vectors, it is first necessary to construct a unit vector in the desired direction. For example, suppose you need a unit vector parallel to the vector shown.



1. Write an expression for the vector $\bar{V}$ as the sum of components.

   $\bar{V}$ = ______________________________

2. What is the magnitude of $\bar{V}$?

   V = ___________________________

3. Divide $\bar{V}$ by its magnitude and call the resulting vector $\bar{e}$.

   $\bar{e} = \frac{\bar{V}}{V} =$ _____________

4. What is the magnitude of $\bar{e}$? ______________

Correct response to preceeding frame

1. $\overline{V} = 8\overline{i} + 6\overline{j}$
2. $V = 10$
3. $$\overline{e} = \frac{8\overline{i} + 6\overline{j}}{10} = .8\overline{i} + .6\overline{j}$$
4. $e = 1$

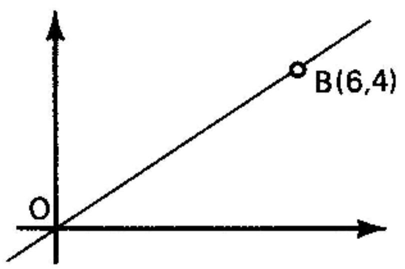
Frame 3-35

Direction of a Vector

A unit vector in any desired direction may be written by

1. writing an expression for any convenient vector in the desired direction
2. dividing the vector thus obtained by its magnitude.

Using the above procedure, write an expression for a unit vector parallel to line OB.



$\overline{V}$ = ______________________

$\overline{e}$ = ______________________

Correct response to preceeding frame

$\overline{V} = 6\overline{i} + 4\overline{j}$

$$\overline{e} = \frac{6\overline{i} + 4\overline{j}}{\sqrt{52}}$$

Frame 3-36

Direction of a Vector

Write a unit vector parallel to each of the given vectors.

$\overline{R} = -12\overline{i} + 5\overline{j}$

$\overline{r} =$ ______________________________

$\overline{Q} = 6\overline{i} + 8\overline{j} + 24\overline{k}$

$\overline{q} =$ ______________________________

Correct response to preceeding frame

$$\bar{r} = \frac{-12\bar{i} + 5\bar{j}}{13}$$

$$\bar{q} = \frac{6\bar{i} + 8\bar{j} + 24\bar{k}}{26}$$

Frame 3-37

Transition

The preceding group of frames was devoted to showing you how to determine the magnitude and direction of a vector when given the vector written in the form of a sum.

The next section will teach you to use this information in writing a vector as the product of a scalar magnitude and a properly selected unit vector.

There are four more frames and then -- oh joyous thought -- you'll be done with this unit.

Go to the next frame.

Correct response to preceeding frame

No response

Frame 3-38

Vectors as Products

Write each of the following vectors as some magnitude times the given unit vector.

$-12\bar{i} + 5\bar{j} =$ ____________ $\left(\frac{-12\bar{i} + 5\bar{j}}{13}\right)$

$6\bar{i} + 8\bar{j} + 24\bar{k} =$ ____________ $\left(\frac{\overset{3}{\cancel{6}}\bar{i} + \overset{4}{\cancel{8}}\bar{j} + \overset{12}{\cancel{24}}\bar{k}}{\underset{13}{\cancel{26}}}\right)$

Correct response to preceeding frame

$$-12\bar{i} + 5\bar{j} = 13\left(\frac{-12\bar{i} + 5\bar{j}}{13}\right)$$

$$6\bar{i} + 8\bar{j} + 24\bar{k} = 26\left(\frac{3\bar{i} + 4\bar{j} + 12\bar{k}}{13}\right)$$

Frame 3-39

Vectors as Products

$\bar{R}$ is a vector 15 units in length which passes through the origin and the point (-2,1).

1. What is its magnitude?

$R =$ ________________

2. What is the unit vector parallel to a line passing through (0,0) and (-2,1)?

$\bar{e} =$ ________________________________

3. Write the vector as the product of the magnitude and the unit vector.

$\bar{R} =$ ____

Correct response to preceeding frame

1. $R = 15$

2. $\bar{e} = \dfrac{-2\bar{i} + \bar{j}}{\sqrt{5}}$

3. $\bar{R} = 15 \left(\dfrac{-2\bar{i} + \bar{j}}{\sqrt{5}} \right)$

Frame 3-40

Vectors as Products

Write the following vectors as the product of a magnitude and an appropriate unit vector.

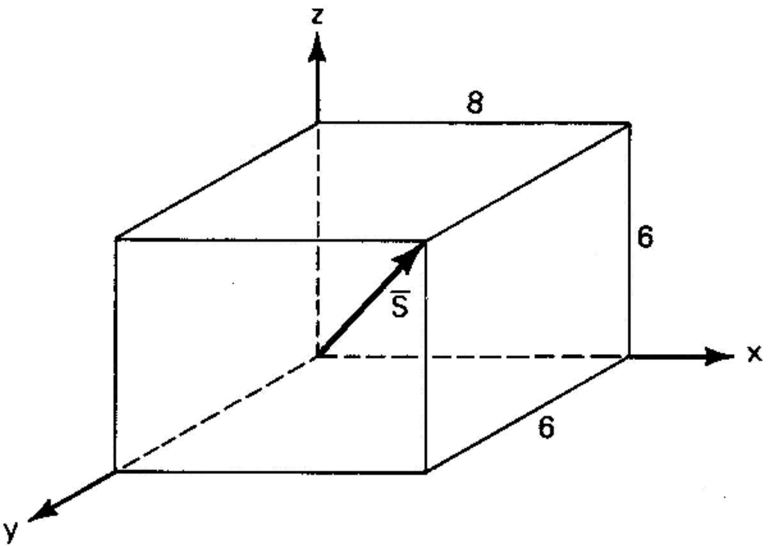
1. $\bar{Q} = 60\bar{j} + 80\bar{k}$

$\bar{Q}$ = ______________________________

2. $\bar{R} = 6\bar{i} - 4\bar{j} + 12\bar{k}$

$\bar{R}$ = ______________________________

3.



$\bar{S}$ = ______________________________

Correct response to preceeding frame

1. $\bar{Q} = 100 \left(\frac{6\bar{j} + 8\bar{k}}{10}\right)$
2. $\bar{R} = 14 \left(\frac{3\bar{i} - 2\bar{j} + 6\bar{k}}{7}\right)$
3. $\bar{S} = 11.7 \left(\frac{4\bar{i} + 3\bar{j} + 3\bar{k}}{\sqrt{34}}\right)$

Frame 3-41

Vectors as Products

Complete the next section of your notebook.

Correct response to preceeding frame

Similar problems are found in the last five frames. Refer to them to check your method.

Frame 3-42

Closure

Now you have learned to write a vector as a sum, to write it as a product, and to represent it graphically. You're developing quite a bag of tricks and you'll find that each way has its own advantages for certain problems.

However, you'll probably be happy to know that these are the only ways you need to learn to express vectors.

Briefly the things you should be able to do are as follows:

1. Identify vector quantities

2. Represent vectors as directed line segments

3. Identify right hand coordinate systems

4. Use unit vectors corresponding to Cartesian coordinates to write expressions for vectors as sums

5. Determine the magnitude of a vector

6. Write a unit vector in any specified direction

7. Write expressions for vectors as products of magnitude and a unit vector.