Introduction to Statics

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Unit 2

Vector Notation & Graphic Vectors

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Unit 2
Vector Notation & Graphic Vectors

Frame 2-1

Introduction

In the preceding unit you were introduced to the concept of force as a vector. Perhaps your reaction to this little gem of information is "So what?".

The answer to that good question is that vector quantities operate in a very different manner than do the scalars to which you are accustomed. Consequently, if you are going to deal with forces, and if forces are vectors, you are going to need to know it so that you can learn how to handle them.

This unit will present you with some basic ideas regarding vectors. These notions will apply not only to forces but to other vector quantities as well. First we will talk about identifying vectors as such and will see a few of the physical quantities that are vectors. Next, we will learn to represent vectors graphically. Last, we will learn to see vectors as the sum of components at right angles to one another.

When you are ready to begin, go to the next frame.

Correct response to preceding frame

No response

Frame 2-2

Definition of a Vector

A vector quantity has two characteristics:
1. magnitude
2. direction.

James jogs 3 kilometers north from his home.

1. What is the magnitude of his displacement from home? ____________
2. What is the direction of his displacement from home? ___________
3. On the evidence so far, can displacement be a vector? __________

Correct response to preceding frame

1. 3 kilometers
2. north
3. Yes, since it has magnitude and direction.

Frame 2-3

Characteristics

What are the characteristics of a vector?

1. ____

2. ____

Correct response to preceding frame

1. magnitude

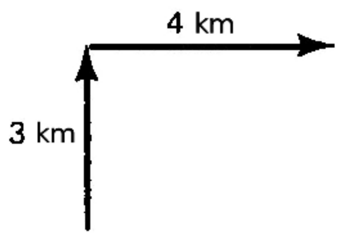
2. direction

Frame 2-4

Definition of a Vector

In addition to having magnitude and direction all vectors add according to the parallelogram law.

After jogging 3 kilometers north, James turns and joggs 4 kilometers east, as shown.

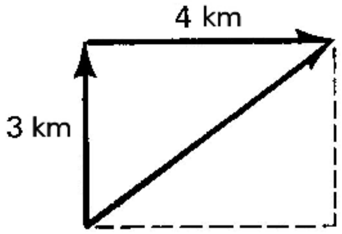


Draw the line that represents his total displacement from home.

Does displacement add by the parallelogram law? ☐ Yes ☐ No

Is displacement a vector quantity? ☐ Yes ☐ No

Correct response to preceding frame



Yes
Yes

Frame 2-5

Definition

1. In order to be a vector a quantity must not only have magnitude and direction, it must

also ____

2. Can vector quantities have additional characteristics beside magnitude and direction.

☐ Yes

☐ No

☐ Don’t know

Correct response to preceding frame

1. add according to the parallelogram law

2. Yes. Specific sorts of vectors may have additional characteristics. For example, you have already learned that the characteristics of force are:

magnitude
direction
point of application

Frame 2-6

Vector or Scalar?

Vector notation is suitable to the expression of most quantities having both magnitude and direction. Quantities having only magnitude are called scalars.

Write vector beside those phrases which suggest both magnitude and direction and scalar beside those statements which deal with magnitude alone.

1. 2 dozen eggs _________

2. Two tens _________

3. A thousand feet down _________

4. 100^0C __________

Correct response to preceding frame

1. Scalar
2. Scalar
3. Vector
4. Scalar

Frame 2-7

Scalar or Vector?

Write ***S*** beside the scalar quantities. Write ***V*** beside the vector quantities

1. 150 kilometers per hour straight up _________
2. A mile a minute ________
3. 2000 miles from my home _______
4. Ten blocks from the Eiffel Tower _______
5. Ten blocks northeast of the Eiffel Tower ______
6. 12 meters long _______

Correct response to preceding frame

1. **V**
2. **S**
3. **S**
4. **S**
5. **V**
6. **S**

Frame 2-8

Scalar or Vector?

1. Scalar quantities have ______________________________ .

2. Vector quantities have ______________________________ .

3. Write ***V*** by the vector quantities, ***S*** by the scalar quantities.

   Length ______

   Mass ______

   Force ______

   Velocity ______

Correct response to preceding frame

1. Scalar quantities have ***magnitude***.
2. Vector quantities have ***magnitude and direction***.
3. ***S*** Length
   S Mass
   V Force (A force always acts in some direction)
   V Velocity (If an object moves it must move in some direction.

Frame 2-9

Definition of Vector

Vector quantities are those which have two characteristics, ________________________

and ____________________________ , and which __________________________________

____ .

Correct response to preceding frame

Vector quantities are those which have two characteristics, ***magnitude*** and ***direction*** and which ***add by the parallelogram law***.

Frame 2-10

Footnote

While most quantities which have only magnitude are scalars and most quantities which have magnitude and direction are vectors, there are a couple of noteable exceptions.

Angle, which can be said to have magnitude and a sort of direction, and finite rotation, which certainly has both magnitude and direction, are neither vectors nor scalars. The reason for this is that neither quantity obeys the commutative law of vector addition:

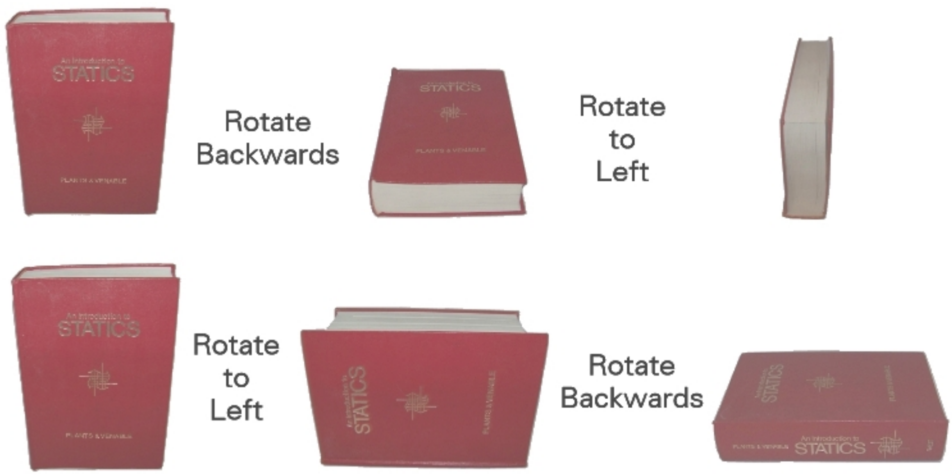
$$\bar{A} + \bar{B} = \bar{B} + \bar{A}$$

or the commutative law of scalar addition:

$$A + B = B + A$$

Consequently they cannot be classed with quantities that do obey these laws.

The two sequences below demonstrates this fact for rotations.



You will learn more about these quantities when you take dynamics. At the present moment it is sufficient for you to know that the class of non-vector, non-scalar quantities does exist.

Go to the next frame.

Correct response to preceding frame

No response

Frame 2-11

Vector or Scalar?

Fill in the first section of Page 2-1 in your notebook.

Correct response to preceding frame

No response

Frame 2-12

Transition

In the preceding section we have been talking about what a vector is. Since it is a quantity with magnitude and direction it may easily be represented by a directed line segment.

The next group of frames will concern themselves with such representation.

They should take you around 10 minutes to do, so if you have that much time and energy proceed to the next frame.

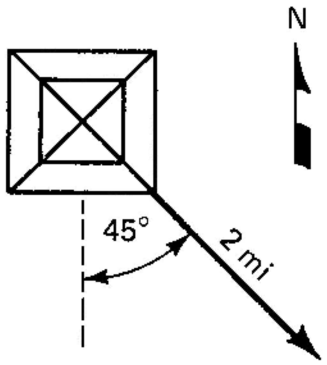
Correct response to preceding frame

No response

Frame 2-13

Graphical Representation

Since vector quantities have both magnitude and direction it is easy to represent them as directed line segments. For instance, the vector described as 2 miles south-east of the Washington Monument could be represented thus:

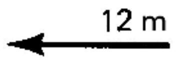


1. Draw a vector to represent a downward force of 10 lb.

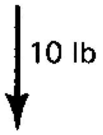
2. Draw a vector to represent a displacement of 12 meters to the left.

Correct response to preceding frame

1.



2.



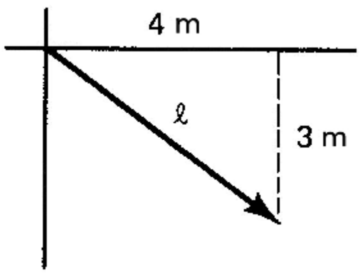
Frame 2-14

Graphical Representation

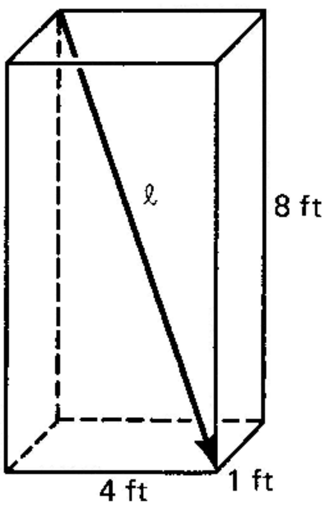
The following directed line segments represent vector quantities. Their directions are as shown.

Determine their magnitudes by using the Pythagorean Theorem.

1.



2.



3. The magnitudes are indicated by the ______________________ of the line segments.

4. The directions are indicated by the ______________________ of the line segments.

Correct response to preceding frame

1. 5 m $\left(\ell = \sqrt{3^2 + 4^2}\right)$
2. 9 ft $\left(\ell = \sqrt{8^2 + 4^2 + 1^2}\right)$
3. lengths
4. slopes

Frame 2-15

Graphical Representation

1. When a vector is represented by a directed line segment, the magnitude is

   indicated by ______________________________ .

2. Check the correct word.
   The magnitude of a vector is a ☐ scalar ☐ vector.

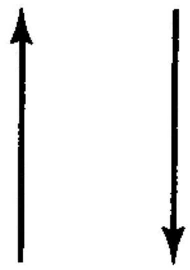
Correct response to preceding frame

1. the length of the line segment
2. scalar

Frame 2-16

Sense of a Vector

The direction of a vector includes both its inclination and its sense. You might say the "sense" of a vector is its sign. The two vectors shown have the same inclination (vertical) but opposite senses. (One is up, the other, down.)



In a graphical representation the sense of a vector is indicated by ________________

____ .

Correct response to preceding frame

an arrow head

Frame 2-17

Graphical Representation

Complete the next section of Notebook Page 2-1.

Correct response to preceding frame

No response

Frame 2-18

Transition

For the last several frames you have been talking about representing vectors graphically. The next few frames will give you a chance to practice doing so.

In particular you will be drawing a kind of vector called a position vector. It is an important one to know about in statics and it is also an easy one to start with because it can be visualized as a line on a map.

The next transition is about ten minutes away. When you are ready, go to the next frame.

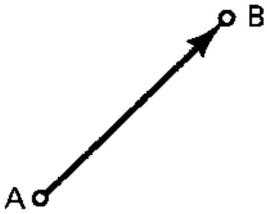
Correct response to preceding frame

No response

Frame 2-19

Position Vector

A position vector shows the position of one point with respect to another.



The vector AB shows the position of B with respect to A, the "reference point".

A position vector always starts at a "reference point" and goes to the point whose position is being defined.

For the position vector shown on this page the reference point is _______ and the vector shows the position of ______ .

Correct response to preceding frame

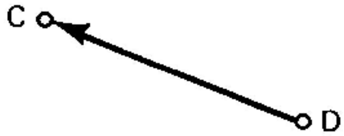
A is the reference point.
The position of B is shown by the vector.

Frame 2-20

Position Vector

Draw the vector showing the position of C with respect to D. ("With respect to D" means that D is the reference point.)

Correct response to preceding frame



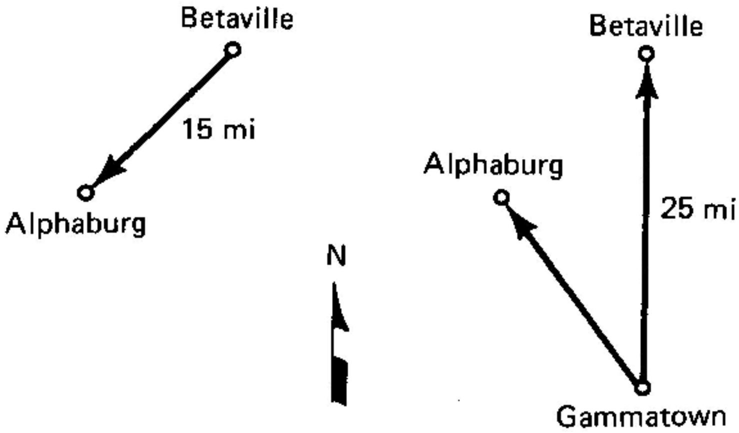
Frame 2-21

Position Vector

Alphaburg is 15 miles southwest of Betaville. Draw a vector showing the position of Alphaburg with respect to Betaville.

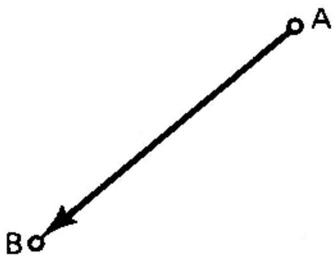
Gammatown is 25 miles due south of Betaville. Draw vectors showing the positions of both Alphaburg and Betaville with respect to Gammatown.

Correct response to preceding frame



Frame 2-22

Position Vector



The position vector shown is the position of ________ with respect to ________ .

The reference point is ________ .

Correct response to preceding frame

B with respect to A
The reference point is A.

Frame 2-23

Position Vector

Choose the words that make the sentence correct.

A position vector always runs (***from, to***) the reference point (***from, to***) the point in question.

Correct response to preceding frame

from the reference point

to the point in question

Frame 2-24

Transition

So far in working with vectors we have been somewhat handicapped by being unable to describe most vector directions. The next section will call upon the parallelogram law of vector addition to correct this.

You will see that considering a vector as the sum of rectangular components makes it much easier to describe and draw.

When you are ready to verify this wondrous truth go to the next frame.

(The upcoming section is the last in the unit and should take about 15 minutes.)

Correct response to preceding frame

No response

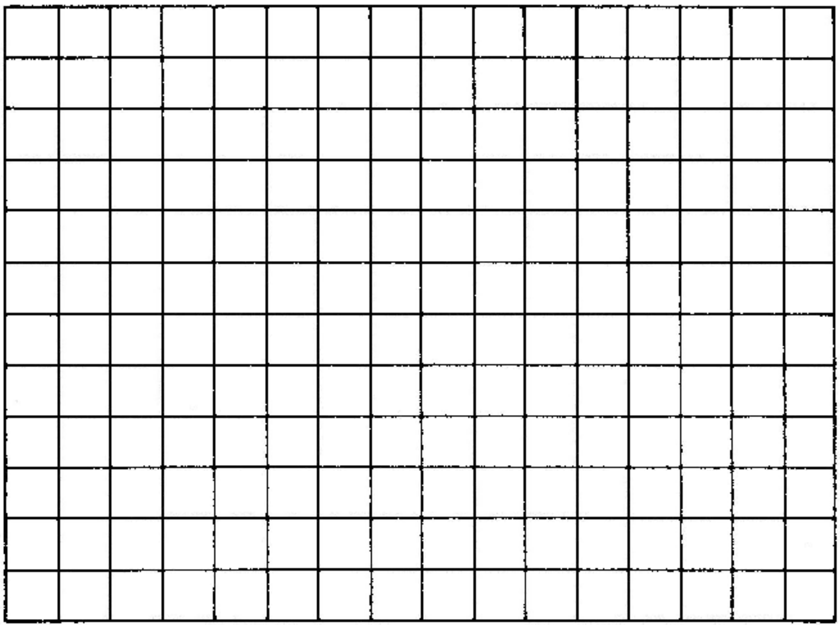
Frame 2-25

Vectors as Sums

(Do this frame to scale.)

A is 4 ft to the left of B.

Draw the position vector for A with respect to B.

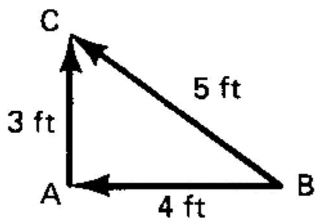


C is 3 ft straight up from A.

On the same sketch draw the position vector for C with respect to A.   Now draw the position vector for C with respect to B.

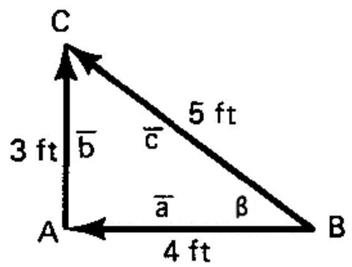
Now draw the position vector for C with respect to B.

Correct response to preceding frame



Frame 2-26

Vectors as Sums

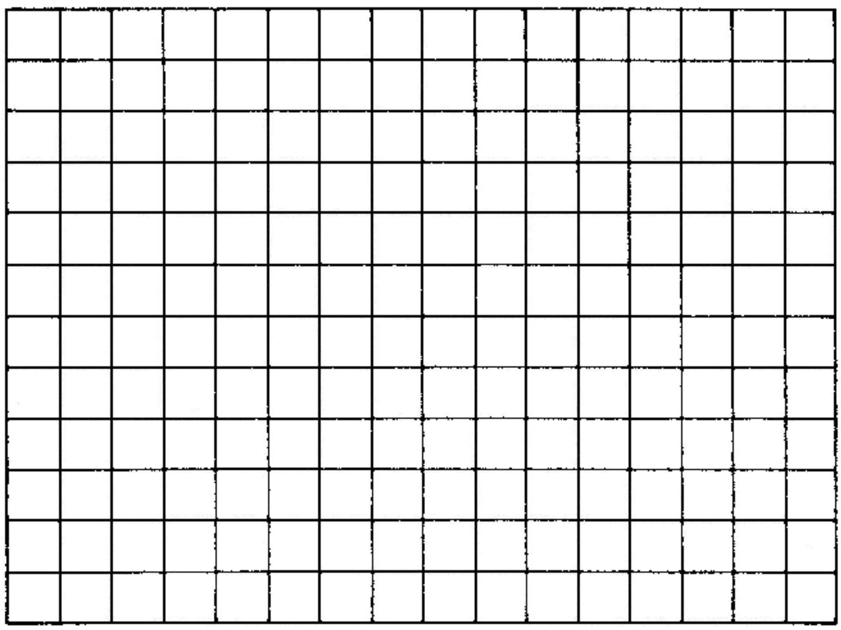


1. The position vector from B to C has a magnitude of _______ and is directed so that

   β = arc tan _________

   β= ______________

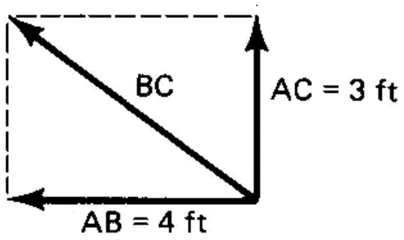
2. Add vectors $\bar{a}$ and $\bar{b}$ according to the parallelogram law



3. Does $\bar{c} = \bar{a} + \bar{b}$ by vector addition? ☐ Yes ☐ No

Correct response to preceding frame

1. BC = 5 ft
   ß = arc tan 3/ 4
   ß = 36.9^0
2.



3. Yes

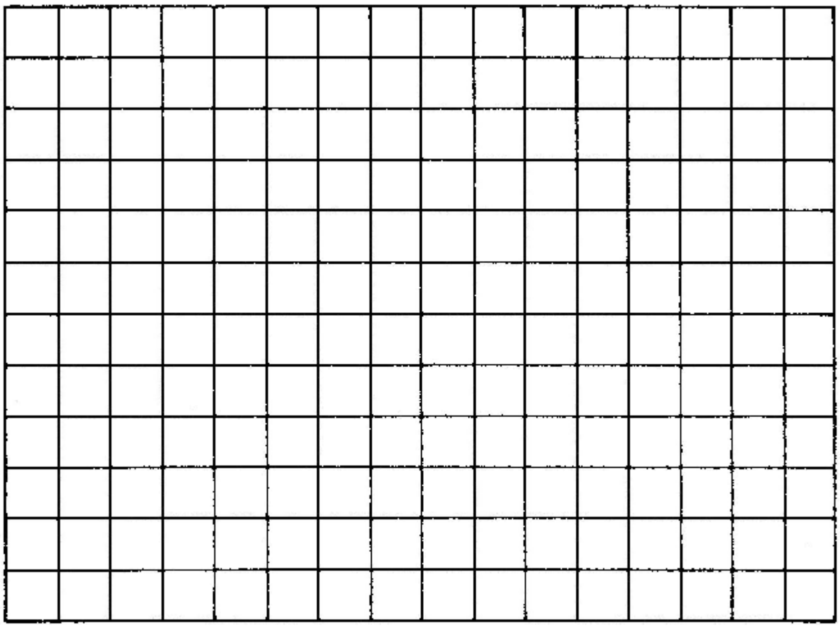
Frame 2-27

Vectors as Sums

The stringing of vectors nose to tail is a variation of addition by the parallelogram law and is called the triangle method of addition.

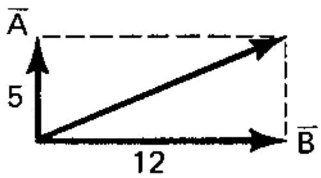
Solve the following with scale drawings.

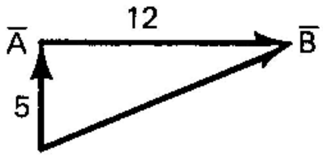
Vector A is 5 units in magnitude and is directed north.
Vector B is 12 units in magnitude and is directed east.



Add vectors A and B by the parallelogram law.
Add vectors A and B by the triangle law.
Did you get the same resultant? ☐ Yes ☐ No

Correct response to preceding frame





The resultants are the same.

Frame 2-28

Vectors as Sums

A ship leaves its harbor and sails 20 nautical miles east and 10 nautical miles south. Then it runs aground. Draw a vector showing its final position from harbor.

1. How far from harbor is it? ___________________

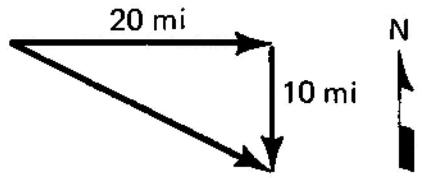
2. What is its bearing from the harbor? _________________

3. In drawing the position you made use of one of the properties of vectors.

   Which one was it? ____

____

Correct response to preceding frame



1. 22.4 nautical miles

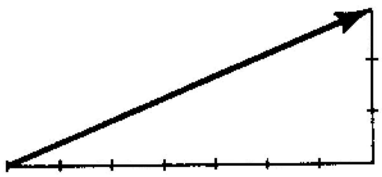
2. on a bearing of 117^0 or 26.6^0 (26^0 34') S of E.

3. Addition by parallelogram law gave you the picture. You got the characteristics of the vector from the picture.

Frame 2-29

Vectors as Sums

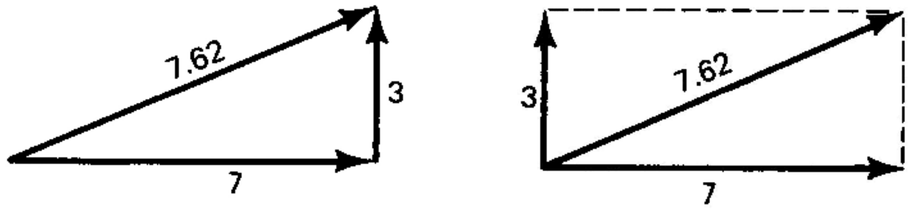
The vector in the figure can be shown as the sum of two perpendicular vectors which we call its components.



Make a drawing of the components of the vector showing the magnitude and direction of each.

Show the magnitude of the resultant on your drawing.

Correct response to preceding frame



(Or equivalent response)

Frame 2-30

Review

To review what you have learned in this unit, complete your notebook for Unit 2.

Correct response to preceding frame

The treasure is 10 miles from your home on a line approximately 54^0 S of W.

Frame 2-31

Summary

In this unit you have learned to identify vectors as those quantities which have magnitude and direction and add by the parallelogram law.

You have learned to represent vectors graphically and to determine their characteristics from graphical representations. You have applied this knowledge to drawing position vectors.

Last you have learned to consider a vector as the sum of rectangular components and to make a graphical representation from the description of the components.

That seems like a pretty good record of accomplishments when we list it, and indeed it is.