Unit 12
Centroids

The centroid of an area is defined as the point at which ______________________________

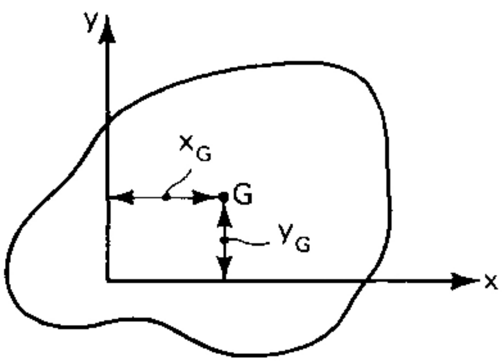
____

____ (12-2)

The distance from the centroid of a given area to a specified axis may be found by

____

____ (12-5)



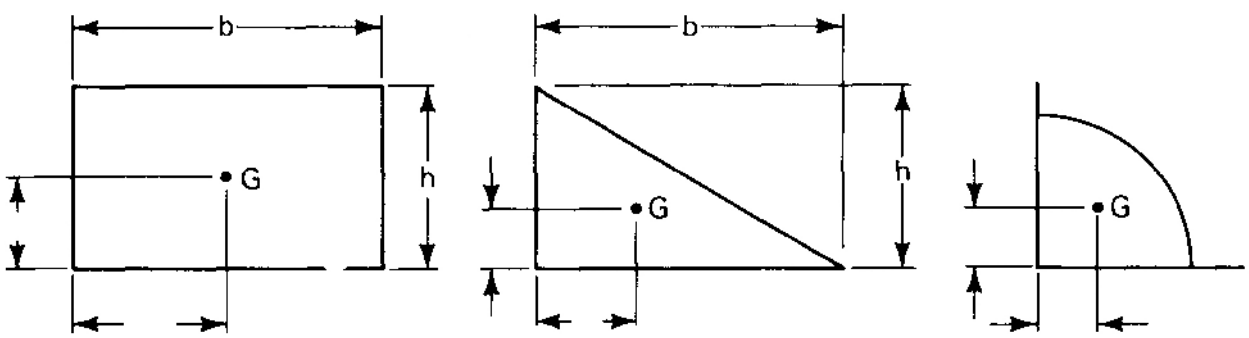
For the area shown A = 6 in.^{2}, Q_y = 18 in.^{3} and Q_x = 22 in.^{3}

G denotes ______________________________

y_G = ______________________________

x_G = ______________________________ (12-5)

Fill in the locations of the centroids of the figures below. You will find it convenient to know these.

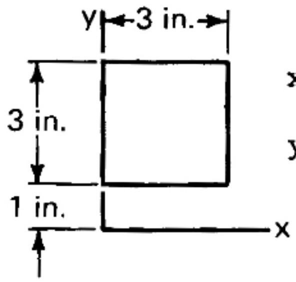


STOP Return to Frame 12-15

Centroids by Symmetry

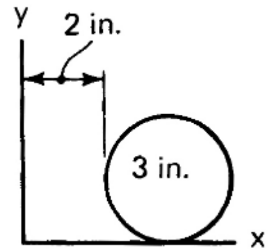
The centroid of any symmetrical area will fall ______________________________ (12-16)

By symmetry, find the centroids of the areas below.



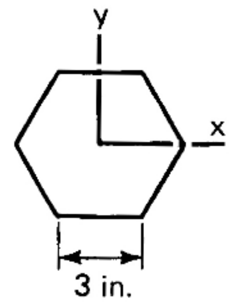
x_G = _____

y_G = _____



x_G = _____

y_G = _____



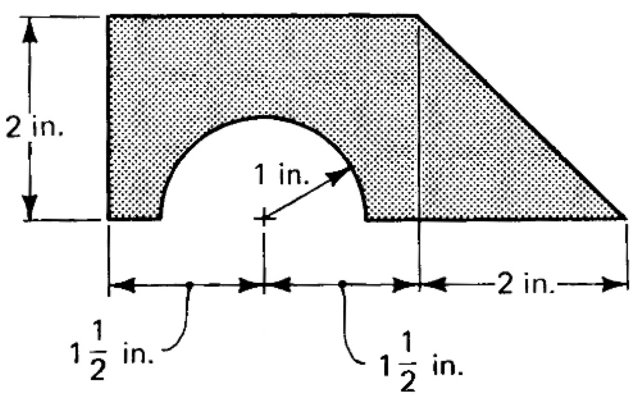
x_G = _____

y_G = _____

(12-18)

Problem 12-1

Find the centroid of the area shown and locate it on the sketch.



Part	A_P	x_{GP}	$A_P x_{GP}$	y_{GP}	$A_P y_{GP}$
Total		X		X	

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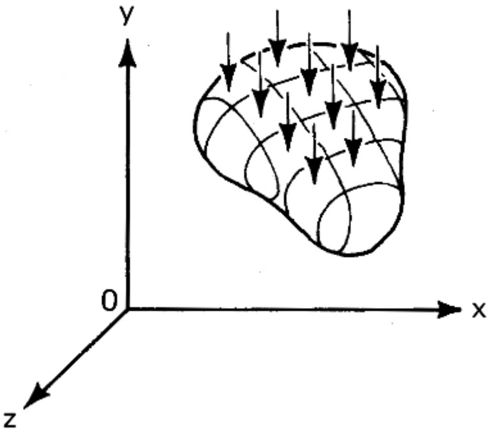
Unit 13
Center of Mass and Center of Pressure

Center of Gravity



Every body can be considered to be made up of a large number of small particles, each having a mass, m. The earth exerts an attractive force on each particle which we call the weight of the particle. If we call each of the individual forces W_i , we might draw a picture of a body acted upon by its own weight that would look like this:

The resultant of all the weights would be

(1.) $\bar{R} = \sum \bar{W}_i = W\bar{j}$

where W is the total weight of the body. The resultant would have a line of action such that its moment about 0 would equal the sum of the moments of all the little weights.

(2.) $\bar{q} \times \bar{W} = \sum \left[\bar{r}_i \times W_i \bar{j} \right]$

where q is a vector from the origin to some point on the line of action of W. The simplest spot would be where it pierces the x-z plane, where

(3.) $\bar{q} = X_G \bar{i} + Z_G \bar{k}$

X_G and Z_G are two of the three coordinates of the center of gravity of the body. If you expand equation (2.) you can write

(4.) $\left[X_G \bar{i} + Z_G \bar{k} \right] \times W\bar{j} = \sum \left[X_i \bar{i} + Y_i \bar{j} + Z_i \bar{k} \right] \times W_i \bar{j}$

If you work out the cross products and break down the results into coefficient equations you will get

(5.) $X_G = \frac{\sum \bar{W}_i X_i}{W} \qquad Z_G = \frac{\sum \bar{W}_i Z_i}{W}$

The third coordinate of the center of gravity can be found by rotating the body and the coordinate system repeating the process we just completed giving

(5a.) $Y_G = \frac{\sum \bar{W}_i Y_i}{W}$

The center of gravity is that point in a body at which its entire weight may be concentrated as a resultant force without changing the external effects on the body. If we shrink our small elements of mass to the "infinitesimals" of calculus, these equations may be re-written as

$$X_G = \frac{\int x dW}{W} \qquad Y_G = \frac{\int y dW}{W} \qquad Z_G = \frac{\int z dW}{W}$$

STOP Return to Frame 13-3

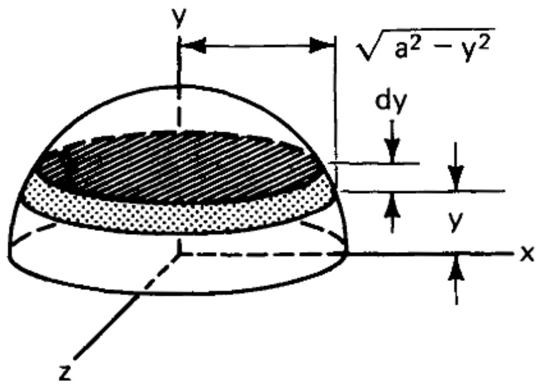
Center of Mass

The coordinates of the *center of mass*, or *mass-center*, are given by the expressions

$$x_G = \frac{\int x\,dM}{\int dM} \qquad y_G = \frac{\int y\,dM}{\int dM} \qquad z_G = \frac{\int z\,dM}{\int dM}$$

Problem 13-1

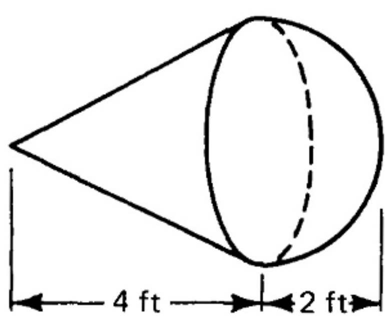
Locate the mass center of a homogeneous hemisphere of density ρ.



STOP Return to Frame 13-13

Problem 13-2

The "ice-cream cone" shown is to be cast in a material weighing 25 lb/cu ft for an advertising display. Find its mass center.



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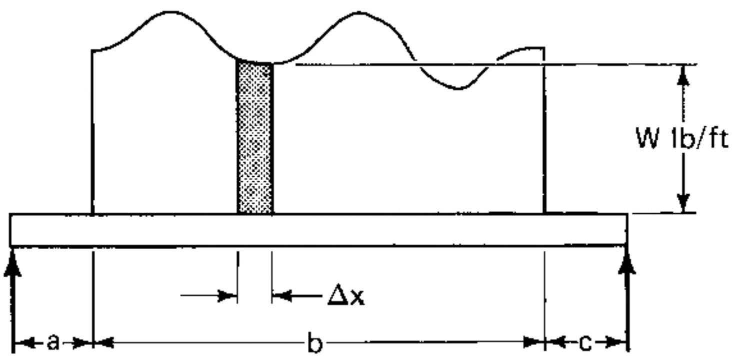
Center of Pressure

If a distributed force system is considered as a system of parallel forces acting on a plane, the total force exerted is the resultant of the system and the *center of pressure* is the point at which the resultant acts on the plane.

Total Force



Thus the weight acting on the element Δx is $W\Delta x\overline{j}$ and the total downward force acting in the body is $\sum W\Delta x\overline{j}$

STOP Return to Frame 13-26

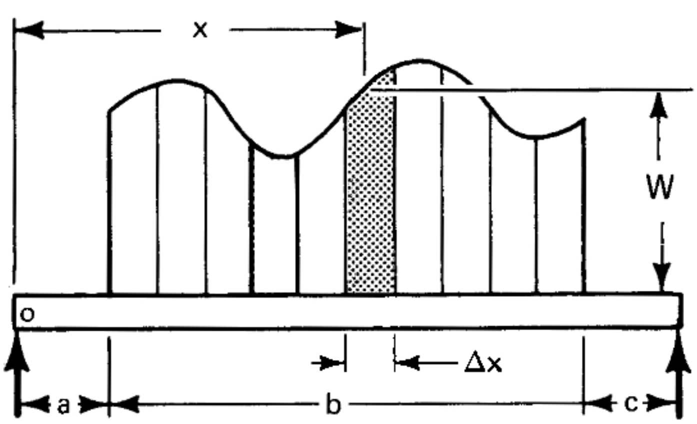
Another way of saying the same thing is

$$\overline{F} = \int_a^{a+b} d\overline{F} \qquad (13\text{-}27)$$

This shows us that the total force is proportional to the ______________________________

____ (13-29)

Location of Center of Pressure



$$d\overline{F} = -wdx\ \overline{j}$$

$$\overline{F} = \int_a^{a+b} -wdx\ \overline{j}$$

$$d\overline{M}_o = -xwdx\,\overline{k}$$

$$\overline{M}_o = \int_a^{a+b} -x(wdx)\overline{k}$$

STOP Return to Frame 13-33

The resultant force $\overline{F}$ must be applied so as to produce a moment about 0 equal to M_0.

$$x_R\overline{i} \times \int_a^{a+b} -wdx\overline{j} = \int_a^{a+b} -x(wdx)\overline{k}$$

and
$$x_R \int_a^{a+b} -wdx = \int_a^{a+b} -x(wdx)$$

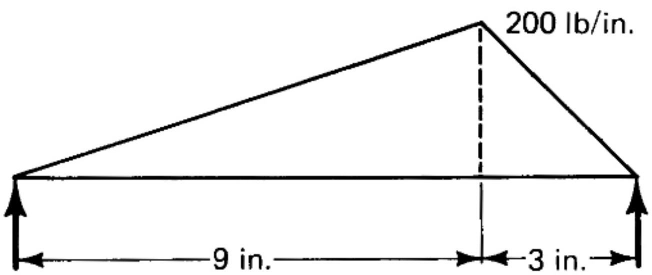
so
$$x_R = \frac{\int_a^{a+b} x\ wdx}{\int_a^{a+b} wdx} \quad \text{or} \quad x_R = \frac{\int_a^{a+b} x\ dF}{F}$$

This means that the resultant force must pass through the ______________________

____ (13-34)

Problem 13-3

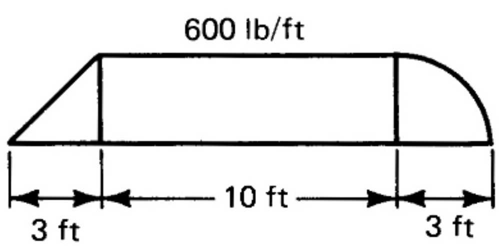
Find and locate the resultant force for the distributed load shown.



STOP Return to Frame 13-38

Problem 13-4

Find and locate the distributed load shown.



STOP Return to Frame 13-43

Unit 14
Free Body Diagrams of Single-Body Systems

Free Body Diagrams

The process of drawing a *free body diagram* (FBD) consists of two steps:

(1.) ____

(2.) ____ (14-3)

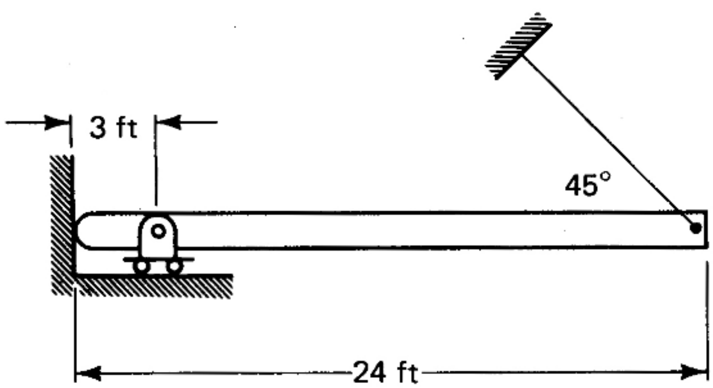
The weight of the body is one of the forces which must generally be included.

It acts through ______________________________ (14-9) which often

coincides with ______________________________ (14-10)

Problem 14-1

Use what you have learned so far and draw the free body diagram for this beam.



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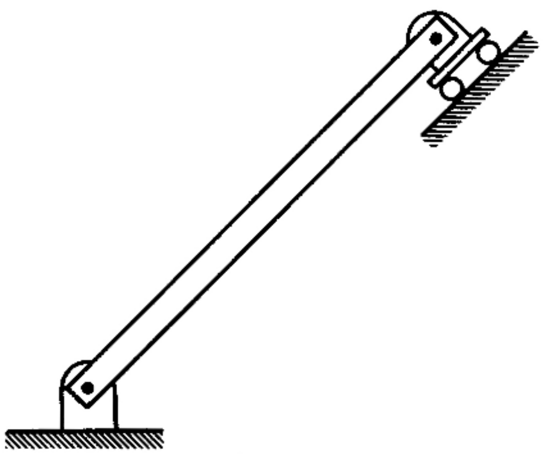
Problem 14-2

Draw a FBD of this system.



STOP Return to Frame 14-36

Unit 15
Equilibrium of Bodies

Equilibrium of Rigid Bodies

A rigid body is in equilibrium when the external forces acting on it form a system of forces equivalent to zero. This means that the necessary and sufficient conditions for the equilibrium of a rigid body are:

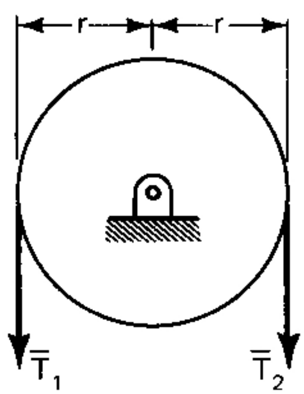
$$\sum \overline{F} = 0 \text{ and } \sum \overline{M}_0 = 0$$

The moments may be taken about any point whatsoever and must always equal zero.

STOP Return to Frame 15-3

Problem 15-1

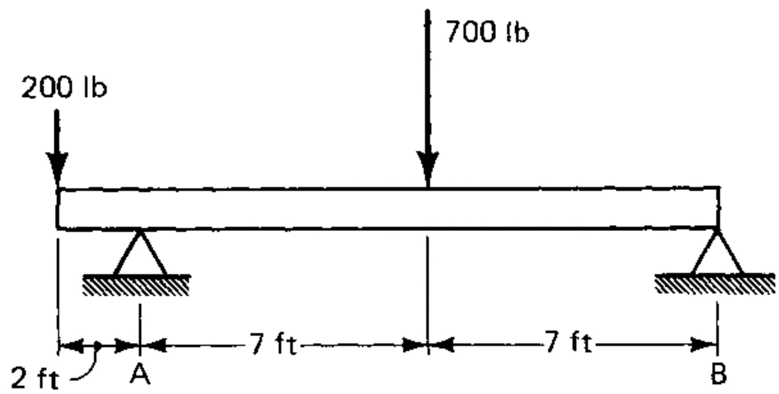
Use the information above to solve for T_2 in terms of T_1 assuming the pulley shown to be perfectly frictionless. Also find the reaction in terms of T_1.



STOP Return to Frame 15-5

Problem 15-2

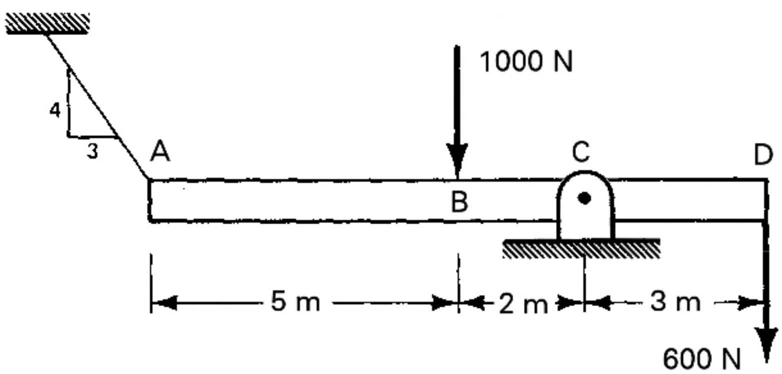
Find the reactions at A and **B** for the beam shown, and check your work.



STOP Return to Frame 15-12

Problem 15-3

Find the tension in the cable at A and the reactions at C.



(Continue on to the next page when you have done the problem.)

The steps in solving a problem involving the equilibrium of a rigid body are as follows:

(1.) ____________________

(2.) ____________________

(3.) ____________________

(4.) ____________________

(5.) ____________________ (15-19)

STOP Return to Frame 15-21

The maximum number of unknowns that can be found by the laws of statics for a given force system is as follows:

(1.) Concurrent coplanar ________

(2.) Concurrent parallel coplanar ________

(3.) Parallel coplanar ________

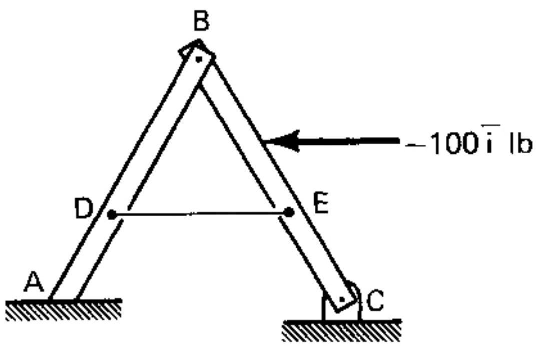
(4.) General coplanar ________ (15-25)

If the number of unknowns exceeds the number that can be found by statics, the system is statically indeterminate. This does not mean it cannot be solved. It often can be solved by including additional information obtained from the deformation of bodies. It is, however, insoluble by analysis as a completely rigid body.

STOP Return to Frame 15-28

Unit 16
Free Body Diagrams of Multi-Body Systems

Problem 16-1



Bars AB and BC are of equal length and weight. A rests on a frictionless surface while joints B and C are pinned. DE is a weightless cord.

Draw free body diagrams of each of the two bars.

STOP Return to Frame 16-14

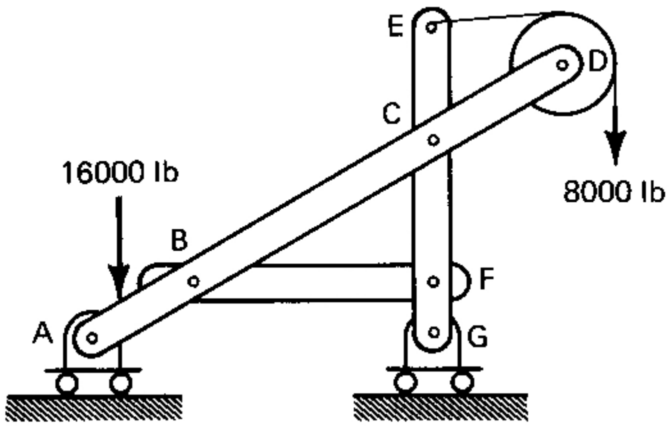
Problem 16-2

Draw a FBD of the system consisting of both bars and the cord in the problem above.

STOP Return to Frame 16-21

Problem 16-3

Draw a free body diagram of the system below.



Draw free body diagrams of each of the members of the system above.

STOP Return to Frame 16-23

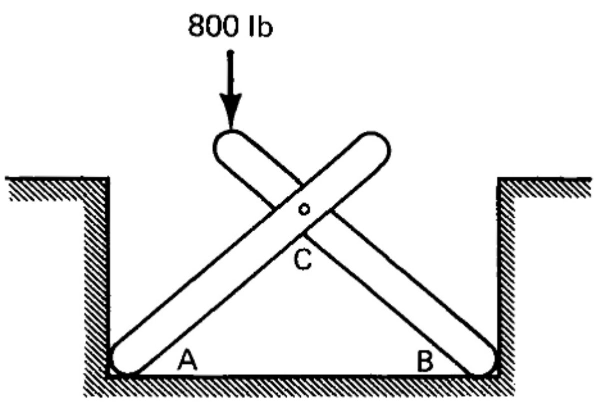
Free Body Diagrams of Systems

Reactions

In statics, the word "reactions" is used to refer to ______________________________

____ (16-4)

Draw a FBD of the system below which shows the reactions.



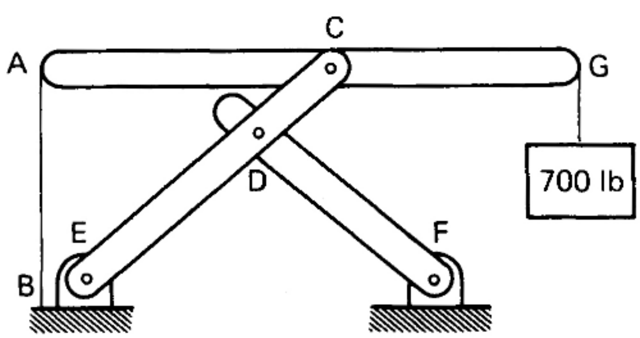
Necessary and Sufficient FBDs

If it is necessary to find all the unknown forces acting on a system it is necessary to draw

as many free body diagrams as ______________________________________ (16-30)

When only selected forces are to be found it is possible to find them from a few of the possible FBDs for the system.

Draw the FBD which will be needed to find the tension in cable AB and the forces at C.

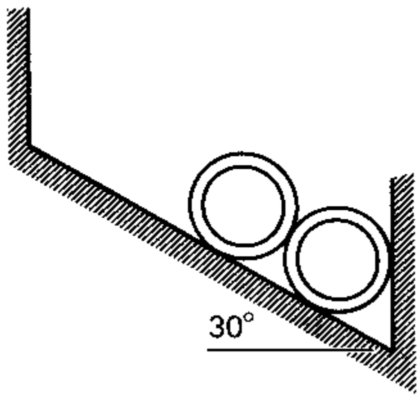


STOP Return to Frame 16-33

Unit 17
Equilibrium of Frames

Problem 17-1

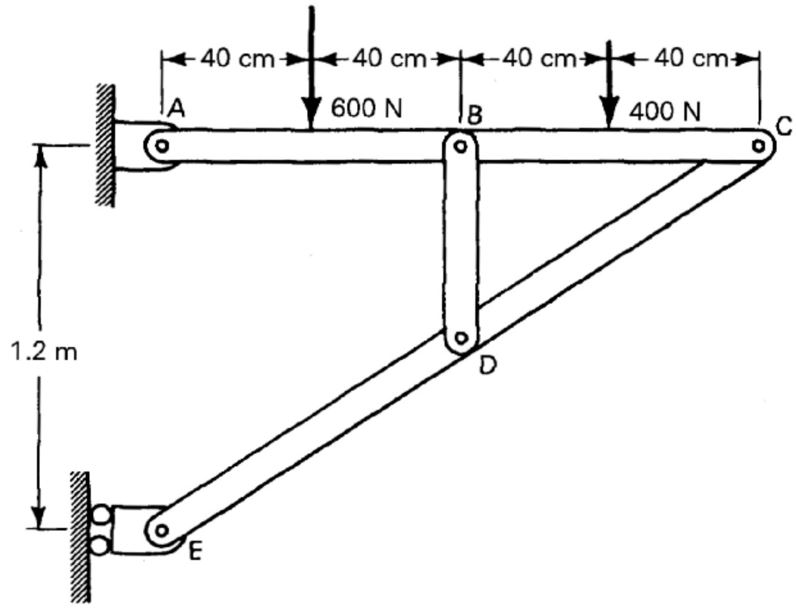
A piece of metal is bent to make a pipe rack as shown. Two similar lengths of pipe, each weighing 20 lb, are in the rack. Find the force exerted by the wall on the lower pipe.



STOP Return to Frame 17-13

Problem 17-2

Find all the forces acting on the horizontal member.



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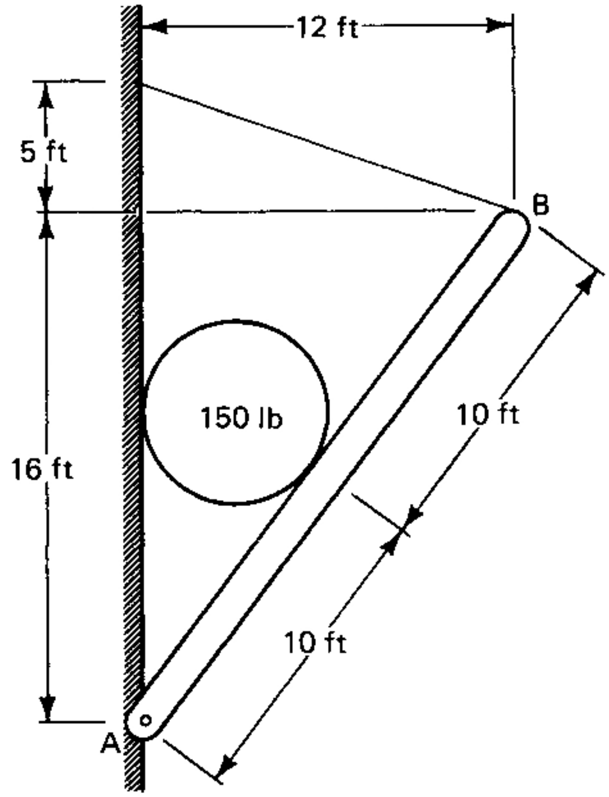
Problem 17-3

Find the forces on member AB at A and B.



Return to Frame 17-28

Unit 18
Trusses: Method of Joints

Trusses are rigid bodies made up of a number of members fastened at their ends. In their analysis three simplifying assumptions are made.

1. ____

2. ____

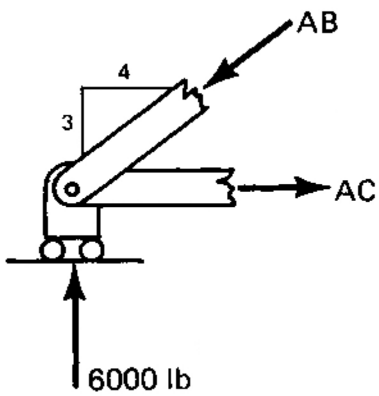
3. ____ (18-3)

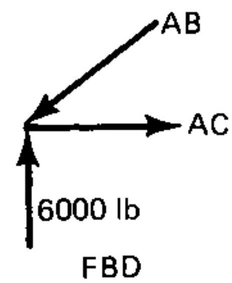
This results in a series of two force members, so that the line of action of the force on any member is ____ __ (18-7)

Thus, the line of action of such a force is apparent by inspection.

Method of Joints

In the method of joints the forces on members are determined by drawing free body diagrams of joints and solving $\sum \bar{F} = 0$ for each joint.



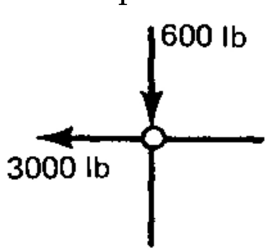


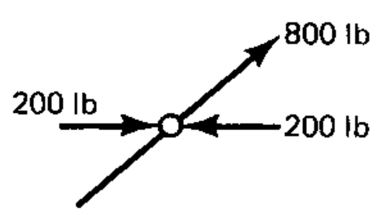
$$\sum \bar{F} = 0$$

$$6000\bar{j} + AC\bar{i} + AB\,[-.8\bar{i} - .6\bar{j}] = 0$$

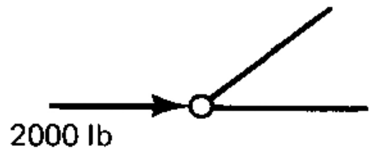
Special Joints

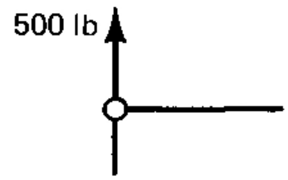
Case I. When a joint is acted upon by four forces arranged in two collinear pairs opposite forces must be equal. Complete the free body diagrams below.



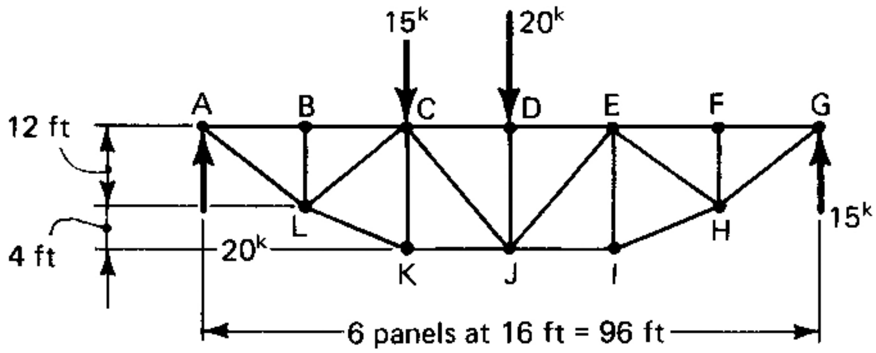


Case II. When a joint connects three members, two of which form a straight line, the forces in the collinear members are equal and the force in the third member is zero.

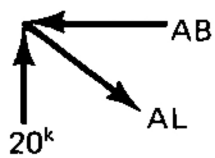




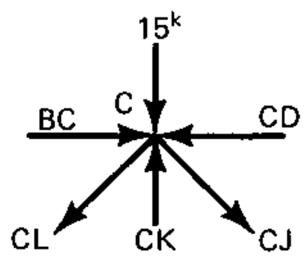
Example



Typical Joints



$$-AB\bar{i} + 20^{k}\bar{j} + AL(.8\bar{i} - .6\bar{j}) = 0$$



$$BC\bar{i} - CD\bar{i} - 15^{k}\bar{j} + CK\bar{j} + CL(-.8\bar{i} - .6\bar{j})$$
$$+ CJ(.707\bar{i} - .707\bar{j}) = 0$$

Draw the FBD and write the equation for joint E.

Special Joints

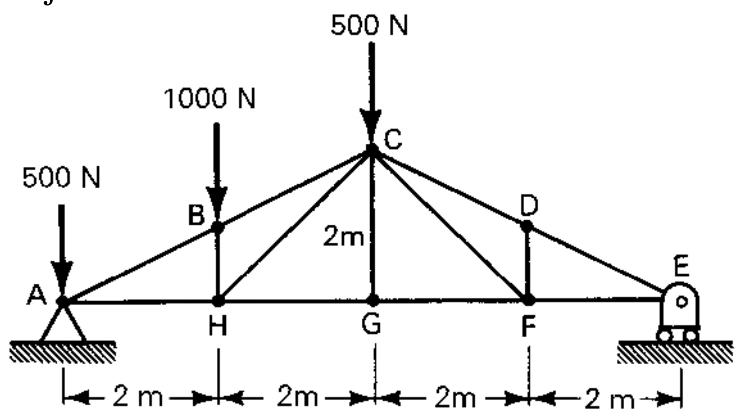
Draw the FBDs and write the equations for joints D and B.

Joint K is not a special joint because members KL and KJ are ________________________ .

STOP Return to Frame 18-30

Problem 18-1

A Pratt roof truss is loaded as shown. Determine the force on each of the members of the truss by the method of joints.



STOP Return to Frame 18-34

Unit 19
Trusses: Method of Sections

Method of Sections

In the method of sections, the first step is to draw a line on the truss which _________

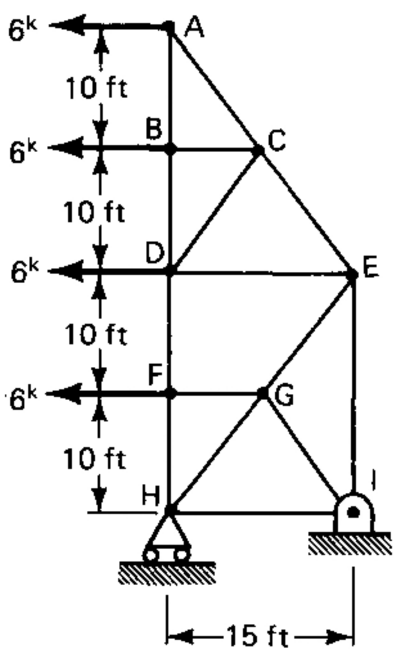
____

Next, a FBD is made of ____

The solution for the forces on the members is then made by using the equations $\sum \overline{F} = 0$ and $\sum \overline{M} = 0$. (19-6)

Problem 19-1

Find the forces in AB, BD, DE, and CE by using a combination of the method of joints and the method of sections.

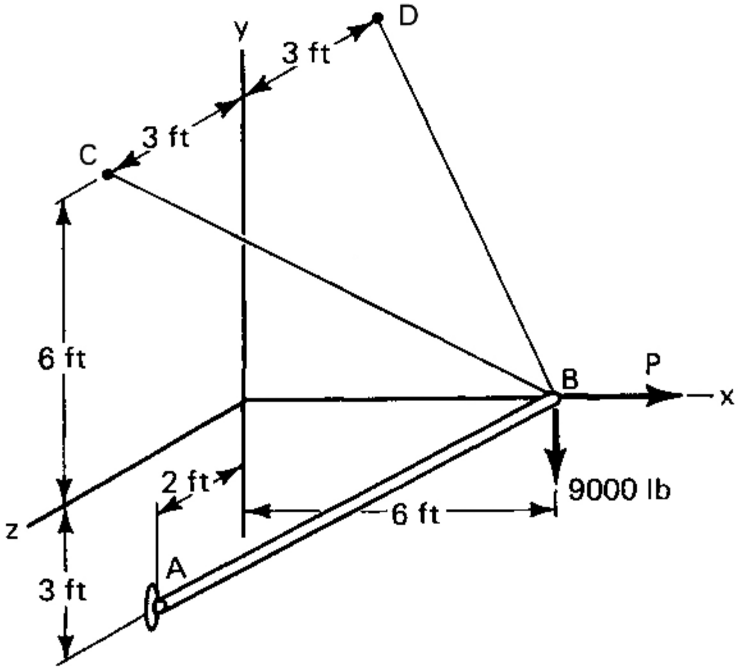


STOP Return to Frame 19-19

Unit 20
Equilibrium of Non-Coplanar Force Systems

Problem 20-1

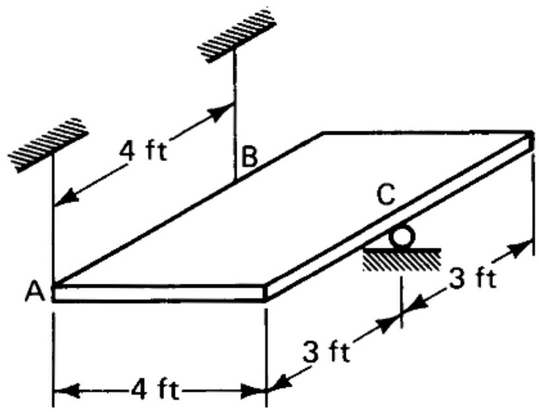
A 7 ft boom is held in place by a ball and socket at A and two cables, BC and BD. Find the magnitude of P if the tensions in the cables are known to be equal. Also find the force acting along AB.



STOP Return to Frame 20-41

Problem 20-2

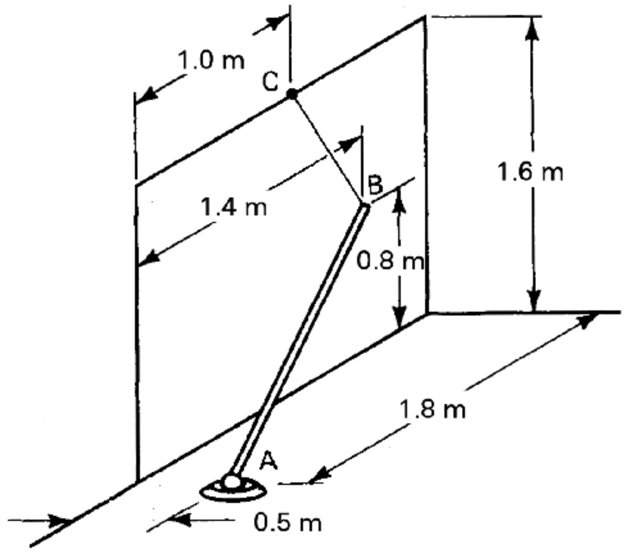
The 600 lb homogeneous plate is supported in a horizontal position by two wires at A and B and a roller at C. Find all reactions.



Return to Frame 20-18

Problem 20-3

A slender homogeneous rod weighing 50 kilograms is supported by a ball and socket at A, and a smooth wall at B, and a cable BC. Find all unknown forces acting on the bar.



STOP Return to Frame 20-29