Introduction to Statics

.PDF Edition – Version 1.0

Notebook

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Contents: Notebook

Unit 0
Instructions to the Student

You should start your study with Unit 0 in the text. Read the first pages of the text and return here when instructed.

Using the Notebook

The notebook has been designed to provide the student with concise recapitulation of the major points in the units at the same time that it provides an annotated index for ready reference and review.

It is an integral part of the programs so that you are directed to work first in the programs, then to summarize your work in the notebook and to return to the programs for the next topic.

Every item called for in the notebook is referred to a frame in the program where the item is discussed. For example, a question followed by the notation (8-23) means that the answer will be found in Frame 8-23. This enables you to check your reply and to build up a completely correct notebook for future reference, at the same time that it provides you with immediate access to the programmed treatment of the topic should you wish more detail.

Since the entire notebook forms an annotated index to the material, it seems reasonable that the alphabetical index to topics should be included in the notebook. Thus, you may look up a given topic in the alphabetical index. There, you will be referred to the appropriate page of the notebook for a concise treatment of the topic. Should that not be sufficient the parenthetical numbers will direct you to the portion of the unit which gives a detailed treatment of the topic.

The authors hope that the notebook will help you in your night-before-the-examination reviews of the programs and that the reference system will make it possible for you to locate quickly material which you may recall and wish to find at a later date.

STOP Return to Frame 1-3

Unit 1
Forces as Vectors

Force may be defined as ____

____ (1-5)

Mass, Weight, and Force of Gravity

Sir Isaac Newton postulated a ***law of gravitational attraction***, primarily to use for explaining planetary motion.

For our earthly work, his equation is

$$F = G\,\frac{M\,m}{r^2}$$

where F is the force of gravity on a body, m is the mass of the body, M is the mass of the earth, r is the radius of the earth, and G is the universal constant of gravitation. According to experimental evidence, $G = 66.73 \times 10^{-12}\ m^3/(kg \bullet s^2)$

We define the local gravitational constant as g, and we get the simple equation

$$F = m\,g$$

Note that the earth is not a perfect sphere, so the precise value of r, and therefor g, is different at different locations.

According to some measurements made by the U.S. Coast & Geodetic Survey, at sea level g varies from 32.088 feet per second squared (fps^2) [9.780 meters per second squared (mps^2)] at the equator to 32.258 [9.832] at the North Pole. It also decreases by 0.003 fps^2 for each increase of 1000 feet in altitude [0.003 mps^2 per 1000 m].

In this book we will use values of $g = 32.2\ fps^2$ (a good value for Washington, DC or New York) and $g = 9.81\ mps^2$ (a good value for Paris or London) for our calculations.

STOP Return to Frame 1-9

The characteristics of a force are

1. ______________________________

2. ______________________________

3. ______________________________ (1-14)

The direction of a force is defined by

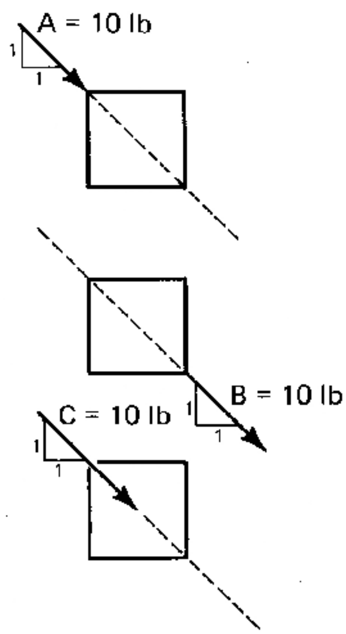
1. ______________________________

2. ______________________________ (1-18)

STOP Return to Frame 1-22

Principle of Transmissibility

The principle of transmissibility states that a force may be applied at any point along its line of action without changing its external effect on a rigid body. (The internal effect will usually be changed.)

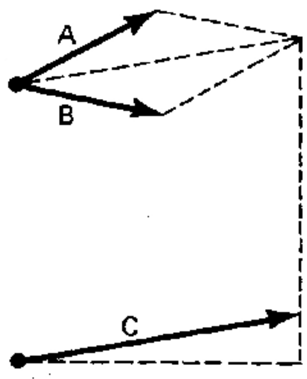


For example, force A has the same external effect on the body as force B or force C, since all three forces have the same line of action. Thus we could say that A, B, and C are "equivalent" forces.

STOP Return to Frame 1-26

Resultant of Two Forces

Experimental evidence shows us that forces do not obey the laws of arithmetic addition. Instead they add according to the "parallelogram law."



This means that forces A and B acting together on a particle have exactly the same effect on the particle as force C which is proportional to the diagonal of the parallelogram constructed on forces A and B. Force C is called the resultant of A and B. This is an observed law. It cannot be proven mathematically.

STOP Return to Frame 1-31

Unit 2
Vector Notation: Graphic Vectors

Vectors are those quantities which have ____________________ and ___________________

and add by the ____ (2-9)

Scalars are quantities which have only ______________________ and add by

____ (2-8)

(There are a few quantities which are neither vector nor scalar but they are rare.)

STOP Return to Frame 2-12

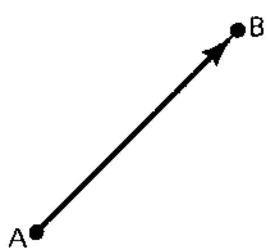
Vector quantities may be represented by directed line segments. In this case the length of the line indicates _________________ , the slope of the line indicates _______________ , and the position of the arrowhead indicates ________________________ (2-14, 2-15, 2-16)

STOP Return to Frame 2-18

A position vector shows the position of one point with respect to another.

The sketch below shows the position of _____ with respect to ______ .

In this case _______ is called the reference point. (2-22)



Problem 2-1

A buried treasure is located 6 miles west and 8 miles south of your home. Sketch its position vector with respect to your home. Determine its distance and bearing from your home.

In the above example a variation of the parallelogram law was used to add two components. Draw the components and add them by the parallelogram law.

STOP Return to Frame 2-31

Unit 3
Vectors: Algebraic Representation

Vector Algebra

Notation $\overline{R} = 7\overline{e}$ indicates a vector whose magnitude is ______ and whose direction is parallel to unit vector ____________ . (3-4)

$\overline{M} = M\overline{m}$ is a vector of magnitude _____ directed parallel to ________ (3-6)

Cartesian Unit Vectors

The symbol $\overline{i}$ denotes a vector of ____________ magnitude parallel to the __________________ , $\overline{j}$ is a similar vector parallel to the ___________ , and $\overline{k}$ is a similar vector parallel to the ____________ (3-9)

Coordinate Systems

The discussion will be limited to coordinate systems which are _____________________ and __________________________ . Sketch such a system below.

(3-16)

A right hand system is one in which ____

____

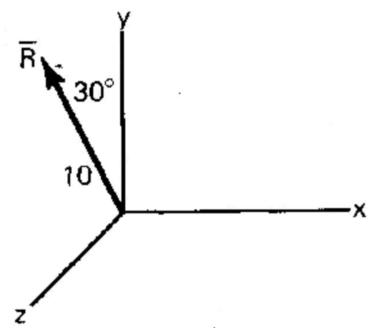
____

____ . (3-14)

STOP Return to Frame 3-18

Vectors as Sums

The vector, $\overline{R}$, lying in the yz plane may be represented by the sum of its rectangular components thus:



$\overline{R}$ = ______________________

The vector $\overline{Q} = 7\overline{i} + 4\overline{j} - 5\overline{k}$ may be shown as follows:

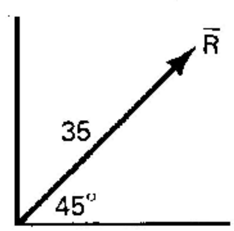
STOP Return to Frame 3-26

Vectors as Products

The vector $\overline{Q} = 7\overline{i} + 4\overline{j} - 5\overline{k}$ may also be written as the product of a magnitude and a unit vector as follows:

$\overline{Q}$ = ______________________________________

The vector $\overline{R}$ shown in the figure may be written as the product of its magnitude (35) and a unit vector as follows:



$\overline{R}$ = ______________________________________

STOP Return to Frame 3-42

Unit 4
Vector Addition and Resultant Forces

One graphical method of vector addition may be stated as follows:

____

____

____

______________________________________ . (4-5)

Problem 4-1

Find the vector sum of the following vectors by using graphical addition.

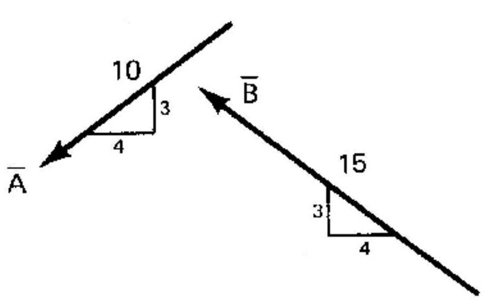
$\bar{A} = 10\bar{i} + 12\bar{j}$

$\bar{B} = -8\bar{i} + 6\bar{j}$

$\bar{C} = 3\bar{i} - 6\bar{j}$

Problem 4-2

Use graphical addition to add the vectors shown.



Return to Frame 4-8

Problem 4-3

Write the vectors in Problem 4-2 as sums of components and find $\overline{A} + \overline{B}$ algebraically.

$\overline{A}$ = _______ $\overline{i}$ + _______ $\overline{j}$

$\overline{B}$ = ________________________________

$\overline{A} + \overline{B}$ = ________________________________

Problem 4-4

$\overline{A} = 12\overline{i} + 3\overline{j} + 6\overline{k}$

$\overline{B} = -10\overline{i} + 8\overline{j} + 7\overline{k}$

Find: $\overline{A} + \overline{B}$ = ________________________________

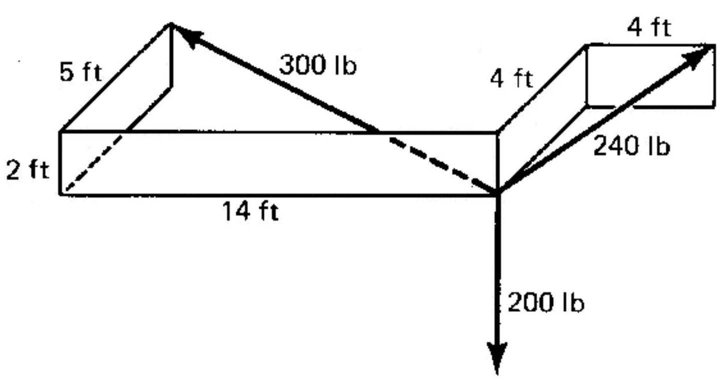
$\overline{A} - \overline{B}$ = ________________________________

$\overline{B} - \overline{A}$ = ________________________________

STOP Return to Frame 4-12

Problem 4-5

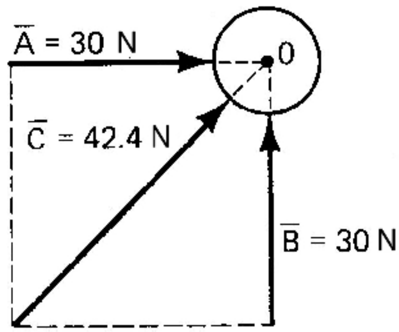
Find the resultant of the forces shown.



STOP Return to Frame 4-22

Resultants of Force Systems

When forces are concurrent (have a common point of application) their resultant will also have the same point of application. The resultant of such a force system can be obtained by adding the forces vectorially and showing the resultant acting through the intersection of the lines of action of the original forces. Thus C shown below is the resultant of forces A and B. Stated algebraically



$$\bar{C} = 30\bar{i} + 30\bar{j}\text{, or}$$

$$\bar{C} = 42.4\left(\frac{\bar{i} + \bar{j}}{\sqrt{2}}\right)$$

acting through 0.

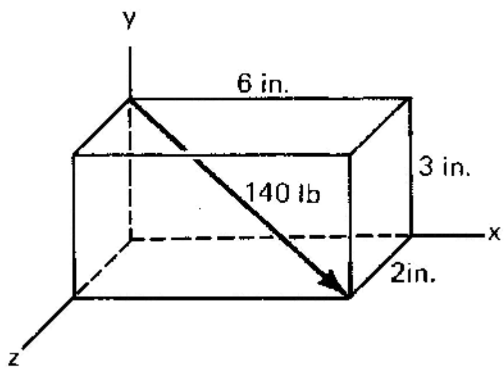
When forces are not concurrent the magnitude and direction of the resultant can be found by simple addition but its point of application must be found by the use of moments. (You will learn about problems of this nature in Unit 10.)

Unit 5
Components of Forces

If a vector is replaced by two or more vectors, which, when taken together, have the same effect as the original vector, the replacing vectors are called ***components*** of the original vector. A resultant vector is the sum of its components.

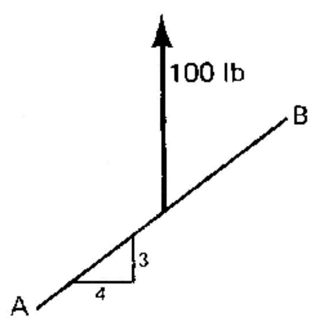
STOP Return to Frame 5-2

Problem 5-1



Break the force shown into three components, each of which is parallel to a coordinate axis.

Problem 5-2



Divide the 100 lb force into rectangular components, one of which lies along AB. Evaluate the components and show them on the sketch.

STOP Return to Frame 5-17

The steps in solving a component problem by means of a figure are as follows:

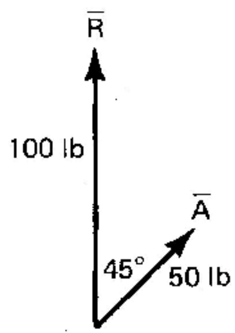
1. ______________________________

2. ______________________________

3. ______________________________ (5-12)

Can a component be larger than the resultant? ______________ (5-19)

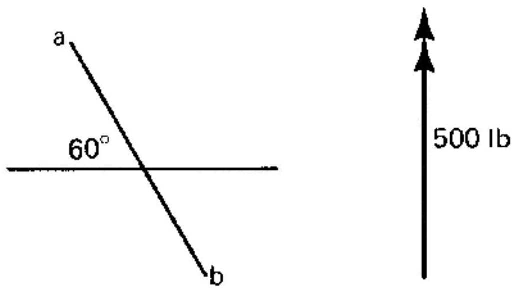
Problem 5-3



$\bar{A}$ is one component of $\bar{R}$. Find the component which, when added to $\bar{A}$, will produce $\bar{R}$ by a trigonometric solution. Then check it by drawing the figure to scale and measuring.

STOP Return to Frame 5-24

Problem 5-4



Break the 500 lb force shown into two components, one horizontal and one parallel to ab. Solve the problem trigonometrically and check your answer graphically.

When directions of both components are given the most direct solution involves the law of

______________________ .

When the magnitude and direction of one component are given, the most direct solution

involves the law of _________________________ . (5-29)

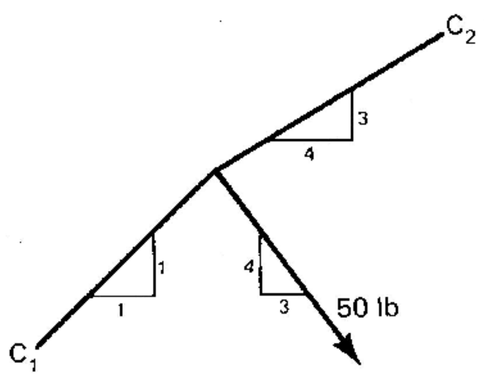
Return to Frame 5-33

Problem 5-5

The force $\overline{F} = 25\overline{i} + 16\overline{j} + 30\overline{k}$ has one component, $\overline{C}_1 = 25\overline{i} + 25\overline{j}$. Find the other component which, added to $\overline{C}_1$, will be equal to $\overline{F}$.

STOP Return to Frame 5-39

Problem 5-6



Find the components in the indicated directions.

STOP Return to Frame 5-43

Unit 6
Equilibrium of a Particle: Concurrent Force Systems

In the last part of the seventeenth century Sir Isaac Newton, by the observation of natural phenomena, formulated three basic laws upon which all Newtonian mechanics is based. It should be emphasized that these laws cannot be proven mathematically but may be verified by physical measurement.

Statics is concerned primarily with the first of these laws which may be stated as follows:

If the resultant of all forces acting on a particle is zero, the particle will remain at rest (if originally at rest) or will travel at a constant speed along a straight path (if originally in motion).

When the resultant of all forces on a particle is zero, the particle is said to be in equilibrium.

STOP Return to Frame 6-2

A particle is a body whose equilibrium is ____

____ .

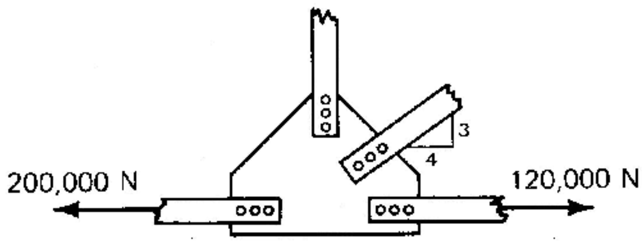
A body may be considered a particle if the forces acting on it form a ________________ force system.

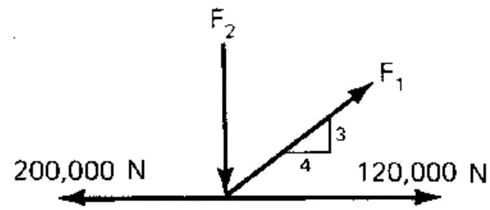
Draw an example of a particle acted upon by an appropriate force system.

STOP Return to Frame 6-19

Problem 6-1

The figures show a detail of a joint in a truss and its free body diagram. Find the two unknown forces.

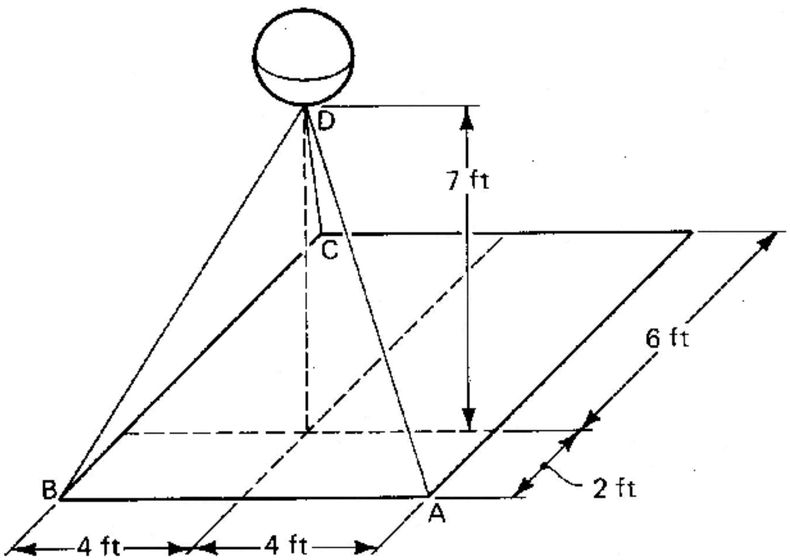


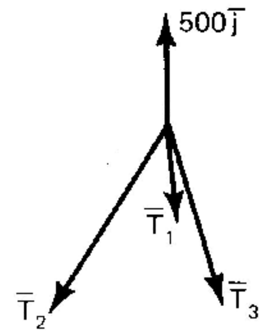


STOP Return to Frame 6-32

Problem 6-2

A balloon is moored by three cables as shown. The upward lift on the balloon is 500 lb. Find the tension in each cable.





In solving problems involving equilibrium of a particle, the steps in the solution are as follows:

1. ____

2. ____

3. ____

4. ____ (6-33)

STOP Return to Frame 6-37

Unit 7
Vector Products

Product of a Scalar and a Vector

The product of a scalar and a vector is a ______________ directed _______________ to the original vector. (7-3)

Example: $\bar{R} = 4\bar{i} + 3\bar{j}$ and $\bar{Q} = 3\bar{R}$

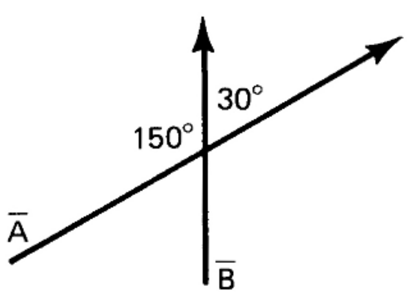
$\bar{Q}$ = ____________________________________ (7-4)

Dot Product

The dot product of two vectors is a scalar and is given by the following expression:

$\bar{A} \cdot \bar{B}$ = _____________________________ (7-7)

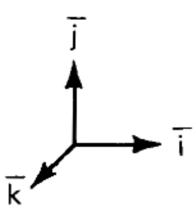
Example:



Vector $\bar{A}$ has a magnitude of 20 units.
Vector $\bar{B}$ has a magnitude of 5 units.

$\bar{A} \cdot \bar{B}$ = _________________________ (7-8)

Dot Products of Unit Vectors



$\bar{i} \cdot \bar{i}$ = _____	$\bar{j} \cdot \bar{i}$ = _____	$\bar{k} \cdot \bar{i}$ = _____	
$\bar{i} \cdot \bar{j}$ = _____	$\bar{j} \cdot \bar{j}$ = _____	$\bar{k} \cdot \bar{j}$ = _____	
$\bar{i} \cdot \bar{k}$ = _____	$\bar{j} \cdot \bar{k}$ = _____	$\bar{k} \cdot \bar{k}$ = _____	(7-10)

$\bar{R} = 3\bar{i} - 4\bar{j} - 2\bar{k}$
$\bar{S} = 9\bar{i} + 6\bar{j} - 3\bar{k}$

$\bar{R} \cdot \bar{S}$ = _____________________________________

STOP Return to Frame 7-14

Cross Product

The cross product of any two vectors may be obtained from the following expression

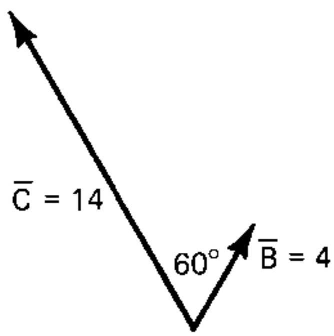
$\bar{A} \times \bar{B} =$ ______________________________ (7-17)

The magnitude of $\bar{A} \times \bar{B}$ is ______________________________ (7-16)

The direction of $\bar{A} \times \bar{B}$ is ______________________________ (7-17)

The sense of $\bar{A} \times \bar{B}$ is ______________________________ (7-18)

Evaluate and sketch $\bar{B} \times \bar{C}$

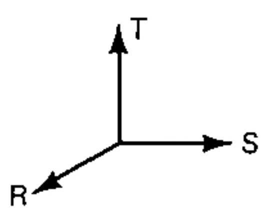


STOP Return to Frame 7-23

The magnitude of the cross product of parallel unit vectors is ____________________

The magnitude of the cross product of perpendicular unit vectors is ______________

____ (7-30)



$\bar{e}_R \times \bar{e}_T =$ _____ $\bar{e}_T \times \bar{e}_R =$ _____

$\bar{e}_S \times \bar{e}_T =$ _____ $\bar{e}_T \times \bar{e}_S =$ _____

$\bar{e}_T \times \bar{e}_T =$ _____

STOP Return to Frame 7-32

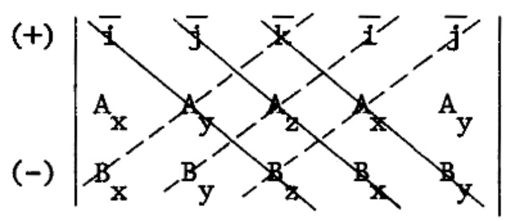
While it is quite possible to find the cross product of the two vectors

$$\bar{A} \times \bar{B} = \left[A_x\bar{i} + A_y\bar{j} + A_z\bar{k}\right] \times \left[B_x\bar{i} + B_y\bar{j} + B_z\bar{k}\right]$$

by multiplying it out term by term, most people find it easier to write it as a determinant.

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

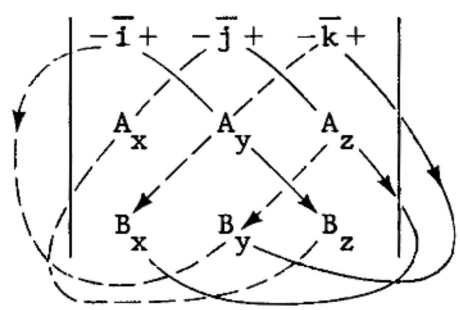
We can evaluate this determinant in several ways. One way that is used by many is to rewrite the first two columns and take ordinary products along the diagonals subtracting the south-western diagonals from the south-eastern diagonals thus

$\bar{A} \times \bar{B} =$ 

$$= \bar{i}A_yB_z + \bar{j}A_zB_x + \bar{k}A_xB_y - B_xA_y\bar{k} - B_yA_z\bar{i} - B_zA_x\bar{k}$$

$$= \left[A_yB_z - B_yA_z\right]\bar{i} + \left[A_zB_x - B_zA_x\right]\bar{j} + \left[A_xB_y - B_xA_y\right]\bar{k}$$

Another way that amounts to the same thing, but avoids rewriting is to follow first the plus, then the minus, path from each unit vector, taking products as you go.

$\bar{A} \times \bar{B} =$ 

$$= \bar{i}A_yB_z - \bar{i}B_yA_z + \bar{j}A_zB_x - \bar{j}A_xB_z + \bar{k}B_yA_x - \bar{k}A_yB_x$$

Collecting terms yields the same form as before. There are several other ways of achieving the same result.

STOP Return to Frame 7-36

Example:

$\overline{A} = 7\overline{i} + 4\overline{j} + 6\overline{k}$
$\overline{B} = 2\overline{i} + 3\overline{j} + 4\overline{k}$

$\overline{A} \times \overline{B} =$ ____________________

$\overline{A} \cdot \overline{B} =$ ____________________

$\overline{B} \times \overline{A} =$ ____________________

$\overline{B} \cdot \overline{A} =$ ____________________

Properties of Products

In this unit we have observed (not proved) the following laws governing multiplication involving vectors.

Multiplication of a vector by a scalar is distributive.

Example: $a(\overline{x} + \overline{y}) = a\overline{x} + a\overline{y}$

The dot product of two vectors is distributive.

Example: $\overline{a} \cdot (\overline{b} + \overline{c}) = \overline{a} \cdot \overline{b} + \overline{a} \cdot \overline{c}$

The dot product of two vectors is commutative.

Example: $\overline{u} \cdot \overline{v} = \overline{v} \cdot \overline{u}$

The cross product of two vectors is distributive.

Example: $\overline{A} \times (\overline{B} + \overline{C}) = \overline{A} \times \overline{B} + \overline{A} \times \overline{C}$

The cross product of two vectors is not commutative.

Example: $\overline{A} \times \overline{B} \neq \overline{B} \times \overline{A}$

The cross product is not associative.

Example: $(\overline{P} \times \overline{Q}) \times \overline{R} \neq \overline{P} \times (\overline{Q} \times \overline{R})$

Proof of all these statements may be found in any text on vector analysis.

STOP Return to Frame 7-40

Unit 8
Moment About a Point

Moment of a Vector with Respect to a Point

Definition (in your own words)

Moment of a vector with respect to a point is ______________________________

____ (8-10)

Problem 8-1

Find the moment of the vector

$$\bar{V} = 20\left(\frac{\sqrt{3}}{2}\,\bar{i} - \frac{1}{2}\,\bar{j}\right)$$

which acts through the point (0,4,0) with respect to the origin.

STOP Return to Frame 8-14

Moments by Vector Products

It is frequently advantageous to use vector algebra to compute moments. This is done

with the ___________________ or _________________ product.

$\overline{M}$ = ______________________________

The vector $\overline{a}$ denotes a vector which runs from __________________________ to

____ (8-18)

Problem 8-2

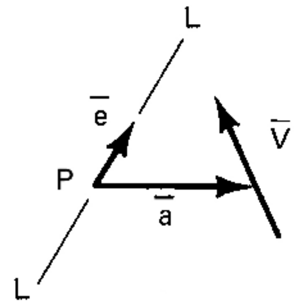
The vector $\overline{V} = -3\overline{i} + 4\overline{j} + 5\overline{k}$ passes through the point B which has the coordinates (0,0,4). Find its moment about the point (0,2,2).

STOP Return to Frame 8-24

Unit 9
Moment About a Line

Moment of a Vector About a Line

The moment of a vector about a line is a rectangular component of the moment of the vector about a point on the line. The component is parallel to the direction of the line.



$$\overline{M}_L = \left[(\overline{a} \times \overline{V}) \cdot \overline{e} \right] \overline{e}$$

In the figure above, $\overline{e}$ is a unit vector along line L-L, P is a point on the line, and $\overline{a}$ is an arm from P to the vector $\overline{V}$.

The moment of the vector $\overline{V}$ about point P is given by the equation $\overline{M} = \overline{a} \times \overline{V}$

Since $\overline{M} \cdot \overline{e} = M \cos \theta$, the expression $(\overline{a} \times \overline{V}) \cdot \overline{e}$ represents the magnitude of the rectangular component of the moment along L-L. This magnitude is then multiplied by the unit vector along the line in order to form a vector component.

STOP Return to Frame 9-2

Problem 9-1

A force $\overline{F} = 5\overline{i} - 3\overline{j} + 2\overline{k}$ acts through the point P with coordinates (-3,1,1). Find the moments (1.) about the x axis, (2.) about the y axis, (3.) about a line through the origin and the point Q with coordinates (-8,-4,1).

$\overline{M}_x$ = ______________________________

$\overline{M}_y$ = ______________________________

$\overline{M}_{0-Q}$ = ______________________________

STOP Return to Frame 9-21

Unit 10

Moments of a Force System: Resultant of a Coplanar Force System

Moment of a Force System

State in your own words the definition of the resultant moment of a force system.

____

____

____ (10-7)

Resultant of a Non-concurrent Force System

The resultant of a two dimensional force system is a force which:

(a) has a magnitude and direction determined by (in words) ____________________

__________ or the equation $\overline{R}$ = ______________ (10-16)

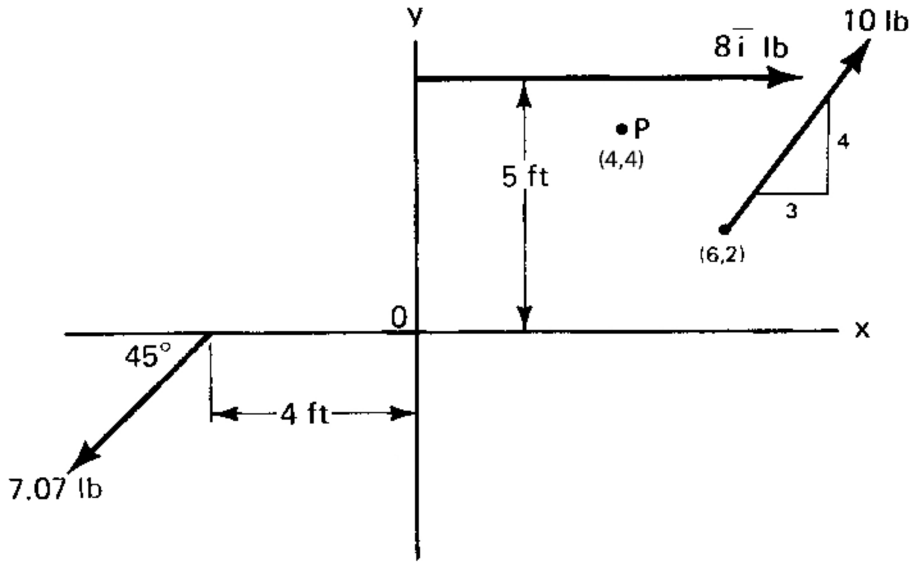
(b) has a line of action which must be located such that (in words) ________________

____

____ (10-17)

Problem 10-1

Given the system of forces:



(a) Find its moment about the origin 0.

$\bar{M}_0$ = ______________________________

(b) Find its moment about P, the point (4,4).

$\bar{M}_P$ = ______________________________

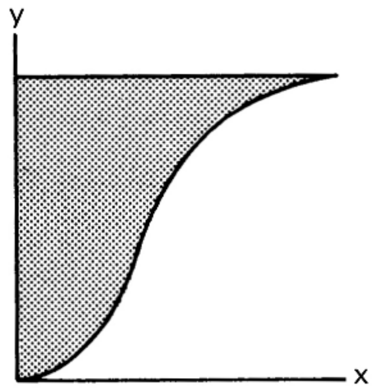
(c) Find the resultant of the force system and locate its line of action on the drawing above.

$\bar{R}$ = ______________________________

STOP Return to Frame 10-30

Unit 11
First Moments

First Moment of Area



The rule for successful selection of an element for determination of first moment

by integration is: ________________________________

____

____ (11-8)

Draw an appropriate element on the figure for finding the first moment of the shaded area with respect to the x-axis.

The general expression for the first moment of an area with respect to an axis is:

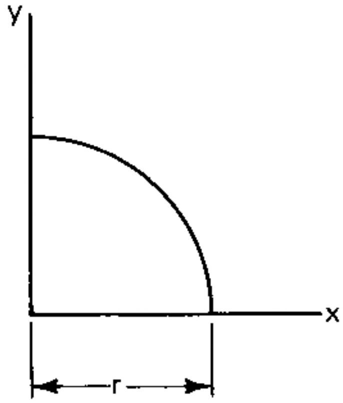
Q_x = ____________________

The steps in the determination of a first moment are as follows:

1. Sketch the problem.
2. Select an element. Dimension it in terms of x and y and known constants.
3. Write the integral $Q_y = \int x dA$ or $Q_x = \int y dA$ as required.
4. From your dimensioned sketch write dA in terms of x, y, and known constants.
5. From the sketch determine your limits.
6. Reduce your expression for xdA or I ydA to an expression in one variable by substituting x in terms of **y** or vice versa.
7. Integrate and evaluate your expression.

STOP Return to Frame 11-11

Problem 11-1



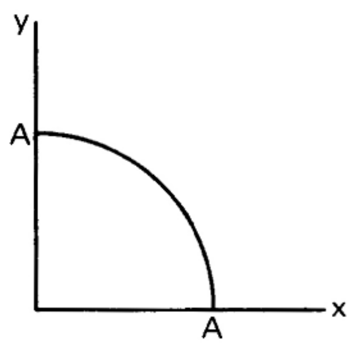
Find the first moment of the quarter-circle shown with respect to the x-axis.

Return to Frame 11-18

First Moment of a Line

The method for finding the first moment of a line about an axis is very similar to that used for first moments of areas. The difference is that a differential line element is used, rather than an area.

Problem 11-2



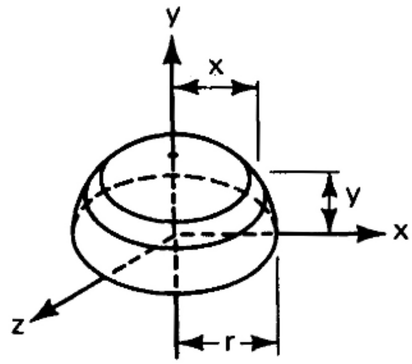
Find the first moment of a quarter of a circular arc of radius a about the x- axis.

Return to Frame 11-23

First Moments of Volume about a Plane

The first moment of a volume about a plane can be found by extending the two dimensional methods into three dimensions. Work the problem on the next page.

Problem 11-3



Set up the integral for the first moment of a hemisphere about its base. Work it out if you choose.

STOP Return to Frame 11-28

First Moment of Volume about a Line

This topic, while similar to those preceding is more complex and requires the use of multiple integration. An example problem is worked out in some detail in Frames 11-28 through 11-30. Should you wish practice in this area it is suggested that you find Q_y for the same volume.

STOP Return to Frame 11-33